

A Generalized Transform Methodology using Quantum Computation and its Application for ECG classification

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Abstract:

Transforms play a pivotal role in the study of signals and images. With the advent of quantum systems, analyzing signals in the quantum domain is of particular interest. In this work, we aim to build a generalized transform circuit in the quantum domain. We have shown the working of this circuit for Wavelet transform, Fourier transform, and Discrete Cosine Transform(DCT) on 1D vector, 2D matrix, and an image respectively. We also take inverse transforms in each of the cases to match with the given initial input. We use simulators for this work, and the results obtained are favorable. We further use this circuit for classification of heart beats as it is an essential task in detection of cardiac diseases. We utilise both classical and quantum computation to classify beats of ECG signals into normal and not-normal beats. This novel architecture to carry out transform is quite general and can be used for any arbitrary transform.

1 Introduction

Signals play an important role in our everyday life. Examples include the sound waves we hear when listening to music or a conversation, the air temperature, the electrical activity of the heart recorded by placing sensors on the chest, the price history of a company's stock, and the sequences of images making up a movie. In this work, we analyze ECG signals, which are an important tool for monitoring the functioning of the heart[1].

According to WHO, cardiovascular diseases(CVDs) are the leading cause of death globally, taking an estimated 17.9 million lives each year[2]. ECG or Electrocardiogram is one of the most common and painless test to monitor the heart. A group of cells in the right atrium of the heart known as Sinoatrial node (SA node) send out regular electrical signals to control heartbeat. ECG is measure of the changes in these electrical signals at different places of skin. The rhythm, the shape and size of these ECG waves helps in the detection of many heart ailments like myocardial infection or heart attack, narrowing of coronary arteries etc.

ECG signal comprises of five main waves i.e. P,Q,R,S and T wave. The P wave occurs due to atrial depolarization when both left and right atria are full of blood. The three waves of the Q, R, S generally known as QRS complex represent depolarization of both the lower chambers of the heart i.e. left and right ventricles. The T wave represents the re-polarization of the ventricles.

With the development of quantum computers, a new technology has evolved which provides much more computational power than the traditional classical computers[3]. Even in its nascent stage, quantum systems have shown the potential to solve problems which are computationally hard for classical systems to solve. The exponential speed-up achieved in a quantum system allows faster simulations and analysis on larger dataset[4].

Analyzing signals in quantum domain is of interest due to the fact that this exploration might bring new possibilities to the field. Transformation of a signal allows us to view a signal in a domain different from its own natural domain. This allows many of its properties to become noticeable which otherwise would not have been prominent, thus allowing it to be processed and analyzed more efficiently. For example, MP3 format uses Fourier Transform for audio compression.

Quantum Fourier Transform(QFT)[5], the backbone of several

important quantum algorithms, is exponentially faster than the classical FFT(Fast Fourier Transform). The basic principle is QFT acts on quantum states to map them to other states based on the Fourier transform formula using quantum computations.

But it is very difficult to derive a generalized transform circuit from this conventional QFT implementation. This work aims at proposing a circuit which can work for any transform, given the transformation matrix. In the further sections, we first present the theoretical framework of the proposed circuit and then discuss its implementation for Wavelet transform and Fourier Transform. We then use this generalized transform circuit to analyze ECG signals. We use Qiskit Statevector Simulator [6] and Unitary Simulator [7] for the simulations.

Researchers have suggested many ways/ algorithms to study or detect abnormalities in heart using ECG signal. Literature is full of many state of the art contributions[8]. Madan et al.[9] proposed a hybrid deep learning-based approach to automate the detection and classification process based on 2D scalogram images. They combined 2D convolutional neural network (CNN) and the Long Short Term Memory (LSTM) network to achieve 98.7%, 99%, and 99% accuracy for Cardiac Arrhythmias (ARR), Congestive Heart Failure (CHF), and Normal Sinus Rhythm (NSR), respectively.

Li et al. [10] implemented a parallel general regression neural network (GRNN) to classify the heartbeat, and achieved accuracy of 88% compared to the specific ECG data of patients in an online learning program.

In pace-makers and also for people recovering ICU etc, these ECG signals are collected 24 hours. To process them real time and understand the dynamics we require huge compute power. Sridevi et al.[11] proposed the use of Quantum Convolutional Neural Network to classify the scalogram images of ECG signals to recognize Arrhythmia. To utilize the enhanced computational power of quantum computation, in this paper we propose a hybrid quantum classical model comprising of quantum convolutional neural networks. The pre-processing of the data as well as the wavelet transform is being implemented in quantum domain.

Here, heart beats are classified into normal and not-normal beats (all other beats which are not normal including abnormal beats and non beats). We use the MIT-BIH arrhythmia dataset consisting of 48 ECG recordings annotated by cardiologists.

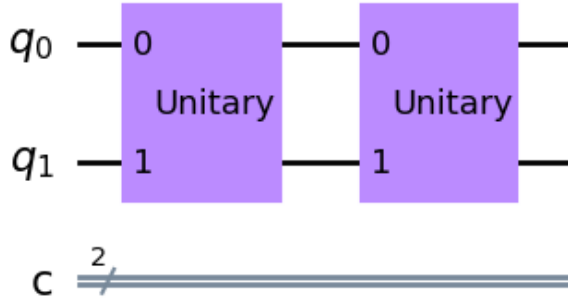


Fig. 1: Circuit diagram of the proposed generalized transform method. The qubits are initialized to zero. The unitary transformation block on the left and right corresponds to A and $B^\dagger$ respectively (from Eq. 7).

The simulation results of the designed quantum wavelet transformation show that it matches exactly with the exact transform. The proposed technique for ECG signal classification has resulted in the high precision of 99.13% for the MIT-BIH arrhythmia database.

2 Problem Formulation

Classifying heart beats correctly is essential to early diagnosis of several underlying diseases. ECG signal represents electrical activity measured in terms of electrical potential due to cardiac muscle contraction. Five deflection points P, Q, R, S and T in cardiac cycle have become useful screening tool for detection and diagnosis of various cardiac abnormalities. ECG signal gives us large amount of information about the structure and functioning of the heart. In this work aim is to classify R points as normal and abnormal, using Classical quantum hybrid architecture. To classify the R points, we first need to enhance ECG signal by removing the noise. As ECG signal has special structure and can more better represented in time frequency domain, we use wavelet based denoising techniques for signal enhancement. Here the wavelet transforms is computed using a quantum circuit. This newly designed quantum circuit is capable of computing most of the signal transform. To the best of authors knowledge this is first generalized quantum circuit to compute any arbitrary transform in quantum domain. Denoised ECG signal is further process using quantum machine learning model for the classification. After classification measurement is performed for diagnosis.

3 Generalized Transform Circuit Design

3.1 Framework for a Generalized Transformation

In this section, we extend the basic quantum state and amplitude encoding idea of Markov et al. [12], towards designing a generalized transform. The starting point is to express a generalized transformation as the matrix multiplication of a matrix with the original signal.

The philosophy of the designed transformer circuit is based on the fact that the transformer can be expressed as matrix multiplication of signal with the transformation matrix. Further this matrix multiplication can be seen as inner products of row/column of the corresponding matrix with input signal arranged in a vector form. Let $x[N]$ be $1 \times N$ 1D signal vector defined as, $x[N] = [x_1, x_2, \dots, x_N]$. Now the N -point transform of $x[N]$ can be evaluated by taking the inner product of it with the corresponding columns/row of the transform matrix. This procedure helps us in computing any arbitrary transform provided the transformation matrix is known.

To understand the inner product in quantum domain define two arbitrary vectors for which inner product needs to be evaluated as,

$$|\psi_A\rangle = A|0\rangle_n \quad (1)$$

$$|\psi_B\rangle = B|0\rangle_n \quad (2)$$

From equation 2, taking conjugate transpose and rewriting it as,

$$\langle\psi_B| = \langle 0|_n B^\dagger \quad (3)$$

Then, we can evaluate inner product as,

$$\langle\psi_B|\psi_A\rangle = \langle\psi_B|BB^\dagger|\psi_A\rangle \quad (4)$$

where $B^\dagger$ represents the conjugate transpose of B . As B is a unitary matrix, so $BB^\dagger = I_n$, thus LHS and RHS of equation 4 are the same.

Substituting equation (3) in equation (4), we get:

$$\langle\psi_B|\psi_A\rangle = \langle 0|B^\dagger BB^\dagger|\psi_A\rangle \quad (5)$$

Substituting equation (1) to the above, we get:

$$\langle\psi_B|\psi_A\rangle = \langle 0|B^\dagger BB^\dagger A|0\rangle = \langle 0|B^\dagger A|0\rangle \quad (6)$$

Further, we can expand $B^\dagger A|0\rangle$ by defining its projection on various basis functions of quantum domain i.e. $|i\rangle$'s as:

$$B^\dagger A|0\rangle = \gamma_0|0\rangle + \sum_{i=1}^{N-1} \gamma_i|i\rangle \quad (7)$$

where, the first term on the RHS corresponds to $|0\rangle$ projection, while the second term corresponds to the rest of the elements. (Fig. 1 shows a generalized circuit diagram for the proposed quantum generalized transform, showing the two blocks for A and $B^\dagger$, respectively.) On replacing this in equation (6), the second term reduces to zero. Thus using linearity we can write,

$$\langle\psi_B|\psi_A\rangle = \langle 0|\gamma_0|0\rangle + \langle 0|\sum_{i=1}^{N-1} \gamma_i|i\rangle = \gamma_0 \quad (8)$$

as $\langle 0|X|i\rangle = 0$ for $i \neq 0$,
 $= 1$ for $i = 0$

This is true for any X . This concept helpful in computing any inner product. As linear transformation can be decomposed into a series of inner products, therefore we can use above equation to evaluate inner product and hence transformation.

Let $\vec{x}[n] \in \mathbb{C}^n$ be an N dimensional input vector and matrix T is the matrix corresponding to the transformation.

$$\vec{x}[n] = [x[n]x[n-1]\dots x[n-N+1]]^T \quad (9)$$

and

$$T = [\mathbf{t}_1^T \mathbf{t}_2^T \dots \mathbf{t}_N^T]^T \quad (10)$$

$\mathbf{t}_i$ is the i th row of transformer matrix T . To evaluate the signal transformed output $y[n]$ can be evaluated as

$$y[n] = T\vec{x}[n] \quad (11)$$

Here transformed output $y[n] = [y[n]y[n-1]\dots y[n-N+1]]^T$. Further the above equation can be written as,

$$y[n-i] = \langle \mathbf{t}_{i-1}, \vec{x}[n] \rangle \quad (12)$$

The signal transformation is seen as the inner product of the input vector $\vec{x}[n]$ and the corresponding column of the transformation

expected	8+0j	-1.5-1.5j	-1+0j	-1.5+1.5j
quantum outcome	8. +0.0000000e+00j	-1.5-1.5000000e+00j	-1.0 -5.5109106e-16j	-1.5+1.5000000e+00j

Table 1 Table showing the expected outcome and output of our quantum circuit for Fourier transform on the data [2, 3, 5, 6]

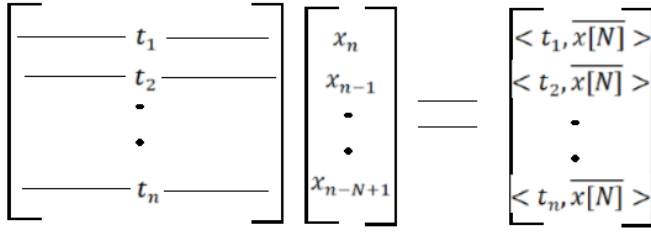


Fig. 2: Matrix transformation of a vector can be decomposed into a series of inner products between the given vector and the constituent vectors of the matrix.

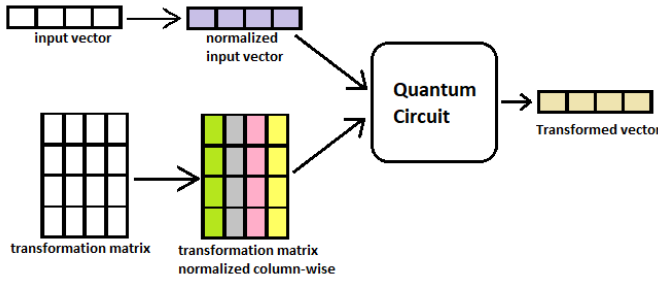


Fig. 3: Flowchart showing the method to compute the transform

matrix T . To evaluate the N -point transformation N inner products are evaluated corresponding to each column of the transformation matrix with input signal vector $x[n]$.

It should be noted that this methodology can be used in obtaining any signal transformation. Therefore this formulation can be used in evaluating the signal transformation in quantum domain. We use this concept and develop the generalized signal transformation in quantum domain. A complete circuit is rigged to evaluate this transformation which is independent of any transformation. Figure 2 shows the working diagrammatically. Figure 3 shows the general overview of the method through a flowchart.

3.2 Designing the circuit

We first design a quantum circuit to obtain the inner product according to equation (8). The circuit is shown in figure 4.

For computing the transform in quantum domain first the quantum representation of the signal needs to be calculated. Here we are using amplitude encoding to represent the signal in qubits, storing the information in the relative amplitudes of the probability of qubit states.

Inner product can be evaluated by using the methodology of Markov et. al [12] by applying unitary transformation corresponding to amplitude encoded first vector A followed by $B^\dagger$ amplitude encoded second vector on a set of qubits initialized to zeros. The amplitude of the resulting state of $|0\rangle_n$ gives the inner product of two quantum states $|\psi_A\rangle$ and $|\psi_B\rangle$.

This concept of inner product of input vector and the corresponding row/column of the transformation matrix will be used in evaluating the transform. The Unitary matrices corresponding to signal (A) and transformation vectors (B) are used to evaluate corresponding quantum states i.e. $|\psi_A\rangle$ and $|\psi_B\rangle$, as seen in figure 4-a,b. The final circuit to evaluate the inner product is shown in figure 4-c. The state of the $|0\rangle$ of the output gives the required inner product.

To extend this inner product circuit to the transform circuit, we repeat this circuit for all the vectors/columns of the transformation matrix, i.e. several combinations of the given vector and columns/rows of the transformation matrix.

The present formulation gives a generic representation for the transform, the circuit can be used to evaluate any transform. The only constraint in this circuit construction is that both the input vector as well as the vectors of the transformation matrix must be normalized. The amplitudes encoding requires that the sum of the square of amplitudes of all qubit states should be equal to one i.e. all operations we perform are unitary operations. Owing to the length-preserving nature of unitary operations, the resultant states are also guaranteed to have a sum of squares of amplitudes equal to one. This guarantees the sum of probabilities of all the resultant states is equal to one making it a viable output. Thus, for arbitrary input the normalization is done before computing the transform, and again we scale the outputs with corresponding normalized factors to get the final results.

4 Implementing the circuit for different transforms

To test the algorithm we take a length 4 array and a 4×4 image matrix and compute different transforms. On the transformed matrix the inverse transform is computed and we compare it with the given input image to understand the efficacy of the proposed system.

4.1 Fourier Transform

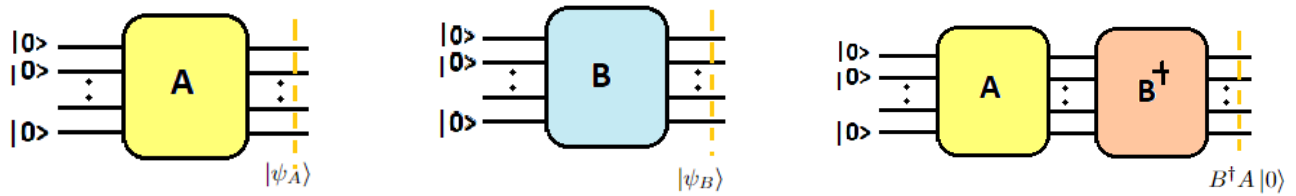
We check the same algorithm for Discrete Fourier Transform. We choose an arbitrary input array: [2, 3, 5, 6]. The Fourier transform matrix can be found at [13].

Accordingly, we calculate the Fourier transform classically using $F_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} f_k e^{2\pi j n k / N}$, where f and F are the input and output array of length N respectively. We also compute it using our proposed method. Figure 5 shows the expected and computed transformed values on input [2, 3, 5, 6].

$$\begin{aligned}
 F_0 &= \frac{1}{\sqrt{4}}(2e^0 + 3e^0 + 5e^0 + 6e^0) = 8 \\
 F_1 &= \frac{1}{\sqrt{4}}(2e^{-2\pi j \cdot 1 \cdot 0/4} + 3e^{-2\pi j \cdot 1 \cdot 1/4} + 5e^{-2\pi j \cdot 1 \cdot 2/4} \\
 &\quad + 6e^{-2\pi j \cdot 1 \cdot 3/4}) = 1.5 + 1.5j \\
 F_2 &= \frac{1}{\sqrt{4}}(2e^{-2\pi j \cdot 2 \cdot 0/4} + 3e^{-2\pi j \cdot 2 \cdot 1/4} + 5e^{-2\pi j \cdot 2 \cdot 2/4} \\
 &\quad + 6e^{-2\pi j \cdot 2 \cdot 3/4}) = -1 \\
 F_3 &= \frac{1}{\sqrt{4}}(2e^{-2\pi j \cdot 3 \cdot 0/4} + 3e^{-2\pi j \cdot 3 \cdot 1/4} + 5e^{-2\pi j \cdot 3 \cdot 2/4} \\
 &\quad + 6e^{-2\pi j \cdot 3 \cdot 3/4}) = -1.5 + 1.5j
 \end{aligned}$$

Table 1 shows the expected values of the Fourier transform coefficients, and the quantum outcome of the same, with our representative experiments with the Qiskit Statevector Simulator [6]. The results on the Unitary Simulator [7] are quite similar.

Figure 5 shows the transform computed on the array. We also take an inverse Fourier transform on $[F_0, F_1, F_2, F_3]$ to compare with the input. Both the transformed coefficients and inverse is exactly same as obtained with classical Fourier transform. The expected values are the same as the quantum outcomes.



(a) According to equation 1,
A maps from $|0\rangle$ to $|\psi_A\rangle$

(b) According to equation 2,
B maps from $|0\rangle$ to $|\psi_B\rangle$

(c) According to equation 7,
designing the circuit

Fig. 4: Transforms for generalized transformation circuit corresponding to equations 1-7.

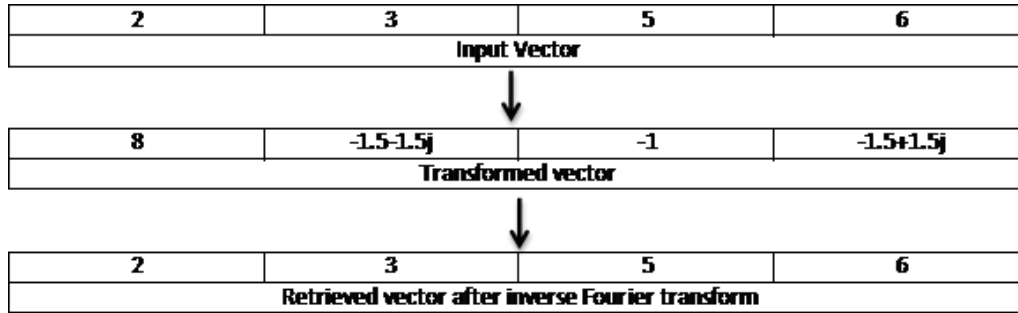


Fig. 5: We use our proposed mechanism to calculate Fourier Transform of the array [2, 3, 5, 6]. On the original signal, we take the fourier transform to get the transformed coefficients. On the transformed signal, we compute the inverse fourier transform following our methodology. The output obtained is very close to the input signal. Both the transforms, the fourier transform as well as its inverse, have been computed totally in the quantum domain following our methodology.

4.2 Wavelet Transform

We use Haar Wavelet Transform[14] here.

Haar wavelets can be defined using Haar functions[15] which are given by:

$$\begin{cases} 1 & 0 \leq x < 1/2 \\ -1 & 1/2 \leq x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

The scaling function of the wavelet is:

$$\begin{cases} 1 & 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

The results are shown in Figure 6.

The fourth-order Haar function wavelet matrix is given by

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 \end{bmatrix} \quad \text{When the input array of Figure 6a is acted}$$

upon by this transform matrix, we get the transformed output as shown in Figure 6b. We also compute the inverse transform to verify

with the provided input. It is seen that the resultant transform from the suggested method is exactly the same as classical method.

4.3 DCT on an image

We take an input image of size 8×8 and compute DCT on the first 4×4 pixels of the image. We again compute inverse DCT and compare it with the input given. The results are shown in figure 7 for an arbitrary image.

For this work, we take a 4×4 sub-image to compute the DCT due to the hardware restrictions of quantum computers. Using the same methodology, we can compute transforms for larger images provided we have sufficient hardware for the required number of qubits.

5 Methodology for ECG signal classification

The main aim of this paper to design a classifier for ECG signal. Heart beats reveal the possibility of several cardiovascular issues and early diagnosis increases the chances of recovery in the patients. Thus it is important to classify heart beats to extract information about the health of a person.

5	10	5	6
1	23	-15	32
5	-1	0	14
23	-10	0	5

(a) Original matrix

26	4	-5	-1
41	7	-22	-47
18	-10	6	-14
18	8	33	-5

(b) Transformed matrix

5	10	5	6
1	23	-15	32
5	-1	0	14
23	-10	0	5

(c) After taking inverse

Fig. 6: Computing Haar wavelet on an arbitrary 4×4 matrix

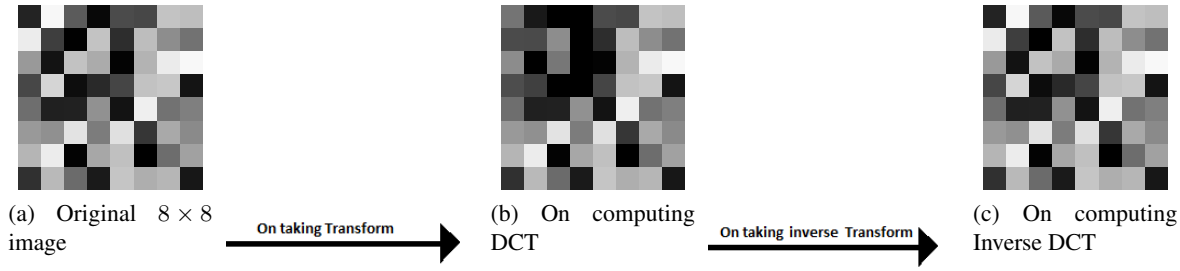


Fig. 7: On the 8×8 input image, we compute DCT on the top left 4×4 area of the image. We again compute inverse DCT to compare with the input.

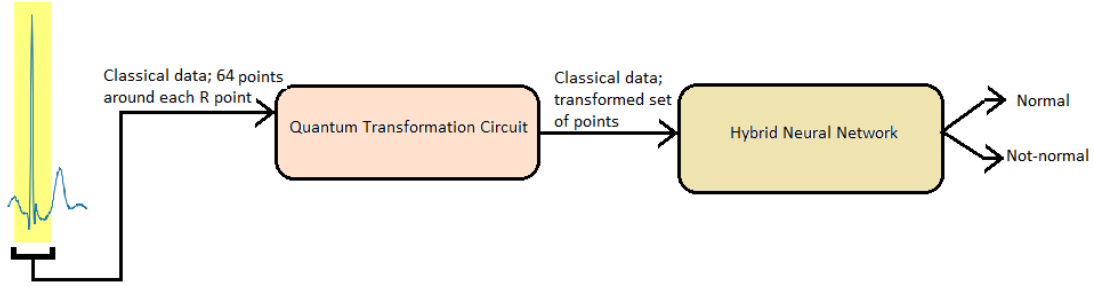


Fig. 8: Block Presentation of the complete algorithm

ECG signals are comprised of P, Q, R, S, and T waves. A hybrid quantum-classical model employing quantum convolutional neural networks for enhanced computational power is designed to classify R points into normal and not-normal beats. Preprocessing and wavelet transform are executed in the quantum domain, to get real quantum advantage.

5.1 Block presentation of complete algorithm

Figure 8 shows the overall block diagram for the algorithm. Around each R point, we choose 64 points at a sampling rate of 360Hz i.e. signal of 178ms. In adults, the QRS complex normally lasts 80 to 100 ms; in children it may be shorter [16].

This classical data is mapped to quantum domain. On the quantum representation, we apply wavelet transform using the transformation circuit discussed earlier. The output from the quantum circuit is processed using hybrid (classical and quantum) neural network to classify the R point as normal or abnormal.

5.2 Designing Model

The transformed values are fed into a machine learning model which has been designed using the following few ideas:

5.2.1 Classical-to-Quantum Transfer Learning: Transfer Learning refers to reusing a trained network for related but different tasks. This “transfers” the knowledge gained from one domain to another. Here, we use the Classical-to-Quantum Transfer Learning approach to study the performance of the hybrid architecture for our classification task. The Hybrid Neural Network of Figure 8 employs this architecture. It has a classical layer which takes in input from the classical domain, produces some intermediate classical output, which is converted to the quantum domain. The quantum circuit then processes this output, which is further converted to classical measurement for human interpretation.

5.2.2 Leave-one-out-cross-validation (LOOCV): This is a type of k -fold cross-validation where the data is split into n blocks and we train the model on $n - 1$ blocks while validate using the other block. We do this iteratively, selecting the training and validation blocks in a round-robin fashion. This approach is good when the data is limited and using this we can design a good system.

5.3 Wavelet Transform Used

For this work, we restrict ourselves to Gabor Transform[17]. Mathematically, this can be represented as: [18] Let $\hat{f}(\omega)$ denote the Fourier transform

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} f(t) dt,$$

and let $g_{\alpha}(t)$ denote the Gaussian function

$$g_{\alpha}(t) = \frac{1}{2\sqrt{\pi\alpha}} e^{-t^2/(4\alpha)}, \alpha > 0.$$

Then the Gabor transform of $f \in L^2(\mathbf{R})$ is defined by

$$(G_b^{\alpha} f)(\omega) = \int_{-\infty}^{\infty} [e^{-i\omega t} f(t)] g_{\alpha}(t - b) dt,$$

where the real parameter b is used to translate the “window” $g_{\alpha}(t)$

5.4 Dataset and Model

The MIT-BIH Arrhythmia database is available at [19]. It has ECG recordings of 48 patients, Figure 9 shows one such ECG signal for patient 103 of the database. Here we use 735 heart beats for this work, discarding samples at very beginning and end which did not have enough R points around them.

These samples are then fed to the Quantum Transformation Circuit corresponding to the wavelet transform. The transformed vectors are further processed using a hybrid neural network.

We train classical-to-quantum transfer learning model. The transfer learning models designed have a classical network comprising of Convolution, ReLU, Dropout, Flatten layers sequentially, and a quantum network as shown in Figure 10 on 4 qubits. The Hadamard layer creates a state of equal superposition initially. The data encoding layer uses amplitude encoding to convert the classical input data to the quantum domain to further process it. The variational layer comprises of circuit parameters which we update as the model is trained. Fundamentally, the variational layer consists of layers of CNOTs and Rotation gates such that the rotation angles are the circuit’s parameters. The different layers are shown in Figure 11. To

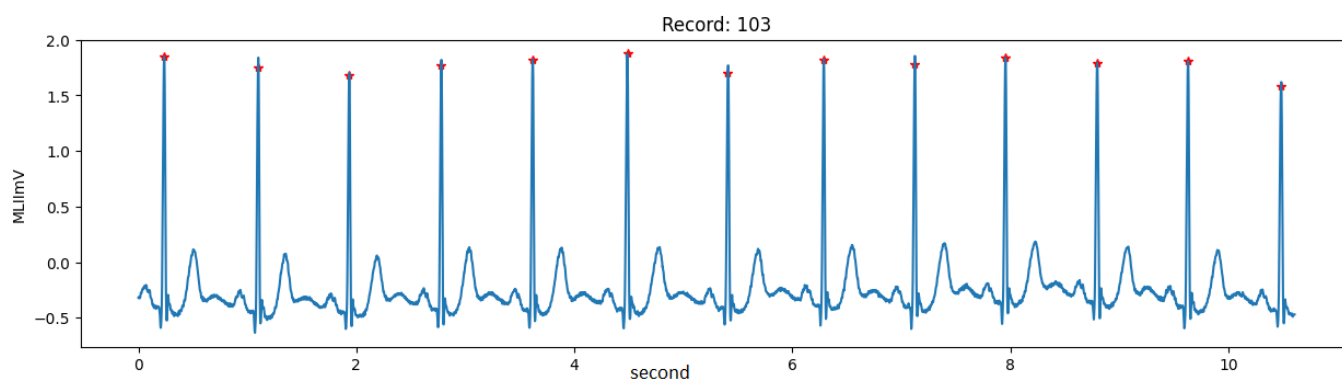


Fig. 9: ECG signal of patient number 103 from the MIT-BIH Arrhythmia database, with the heart beats marked in red asterisk

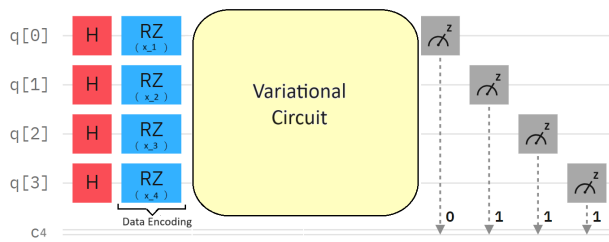


Fig. 10

obtain the output from the model, the last layer of the model is a linear layer which maps from $4 \rightarrow 2$, as there are 4 qubits and 2 output classes. As we train the model, both the classical weights as well as the quantum circuit parameters are updated.

We follow two strategies for our models:

5.4.1 Hold out: We split the dataset into a 'train' and 'test' set. We train the model using 'train' set, while test the model using 'test' set.

Dataset: Out of the 735 beats, we preserve 150 beats for testing our model, while the rest 585 beats are used to train the model. But this training set was not balanced. In order to balance the dataset, we perform oversampling, i.e. we randomly select samples from the not-normal class and duplicate that in the training set. After balancing the training set, the total size becomes 750 samples, containing roughly same number of normal and not-normal samples.

Models used: We train different quantum architectures following this strategy:

- **Model I:** The quantum layer has a structure as shown in Figure 10, where the variational circuit is comprised of 3 layers of the block shown in Figure 12a.
- **Model II:** The quantum layer has a structure as shown in Figure 10, where the variational circuit is comprised of 3 layers of the block shown in Figure 12c.
- **Model III:** The quantum layer has a structure as shown in Figure 10, where the variational circuit is comprised of 3 layers of the block shown in Figure 12e.

5.4.2 Leave-one-out-cross-validation(LOOCV): We train the model k times where k is dataset size, every time leaving one block for validation while the rest $k - 1$ are used to train our model.

Dataset: To have a balanced dataset, we take 200 normal and 200 not-normal samples from the original dataset.

Model used: For a batch size b , we have $400/b$ folds. In this work, we took batch size to be 40, 10 and 5, thus the number of folds were 10, 40, and 40 respectively. While training on $k - 1$ blocks, we used 150 epochs to train our model. The model has a quantum layer as shown in Figure 10, where the variational circuit is comprised of 3 layers of the block shown in Figure 12a.

6 Results Obtained for the complete model

The results obtained corresponding to various models and different hyperparameters are shown in Table 2 and the training curves are shown in Figure 12.

7 Conclusion

In this work, we propose a methodology for computing any arbitrary transform entirely in the quantum domain. Most of the earlier work focused on building circuits dedicated to particular transforms, not to a generalized transform.

We portrayed the working of our methodology for Wavelet transform, Fourier transform, and DCT. We also used 1D vector, 2D matrix as well as an image to show our results. We further use

this circuit to process ECG signals in quantum domain, and further classify R points into normal and not-normal categories using Classical-Quantum Transfer Learning approach.

From the results obtained, the accuracies were decent. The results obtained are very promising since quantum is still in the nascent stage, and as the system develops, we expect better results.

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- 18 Encyclopedia of Mathematics. :http://encyclopediaofmath.org/index.php?title=Gabor_transform&oldid=50367
- 19 <https://physionet.org/content/mitdb/1.0.0/>

	Parameters	Confusion matrix				Accuracy	Precision
Hold Out	Model:I, epochs:150, b'size:5	Actual	not-normal	Predicted		95.33%	96.63%
			normal	not-normal	normal		
				28	4		
				3	115		
Hold Out	Model:I, epochs:250, b'size:2	Actual	not-normal	Predicted		98%	99.14%
			normal	not-normal	normal		
				31	1		
				2	116		
Hold Out	Model:II, epochs:150, b'size:5	Actual	not-normal	Predicted		92.67%	98.19%
			normal	not-normal	normal		
				30	2		
				9	109		
Hold-Out	Model:III, epochs:170, b'size:5	Actual	not-normal	Predicted		96.67%	98.29%
			normal	not-normal	normal		
				30	2		
				3	115		
Hold Out	Model:III, epochs:350, b'size=2	Actual	not-normal	Predicted		97.33%	99.13%
			normal	not-normal	normal		
				31	1		
				3	115		
LOOCV	epochs:150, b'size:10, folds:40	Actual	not-normal	Predicted		92.25%	86.87%
			normal	not-normal	normal		
				170	30		
				1	199		
LOOCV	epochs:150, b'size:10, folds:40	Actual	not-normal	Predicted		94.5%	90.09%
			normal	not-normal	normal		
				178	22		
				0	200		
LOOCV	epochs:150, b'size:5, folds:80	Actual	not-normal	Predicted		96.25%	93.43%
			normal	not-normal	normal		
				186	14		
				1	199		

Table 2 Table showing results obtained for both Hold Out and LOOCV strategies.

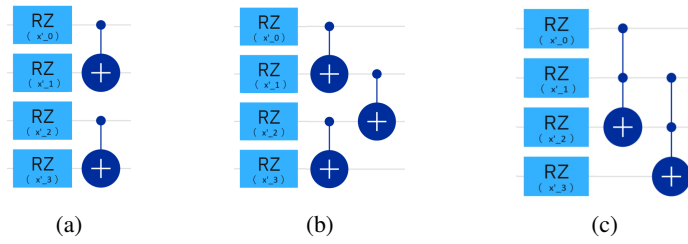


Fig. 11: Layers used as a part of the variational circuit of Figure 10. The layers are repeated 3 times for this work

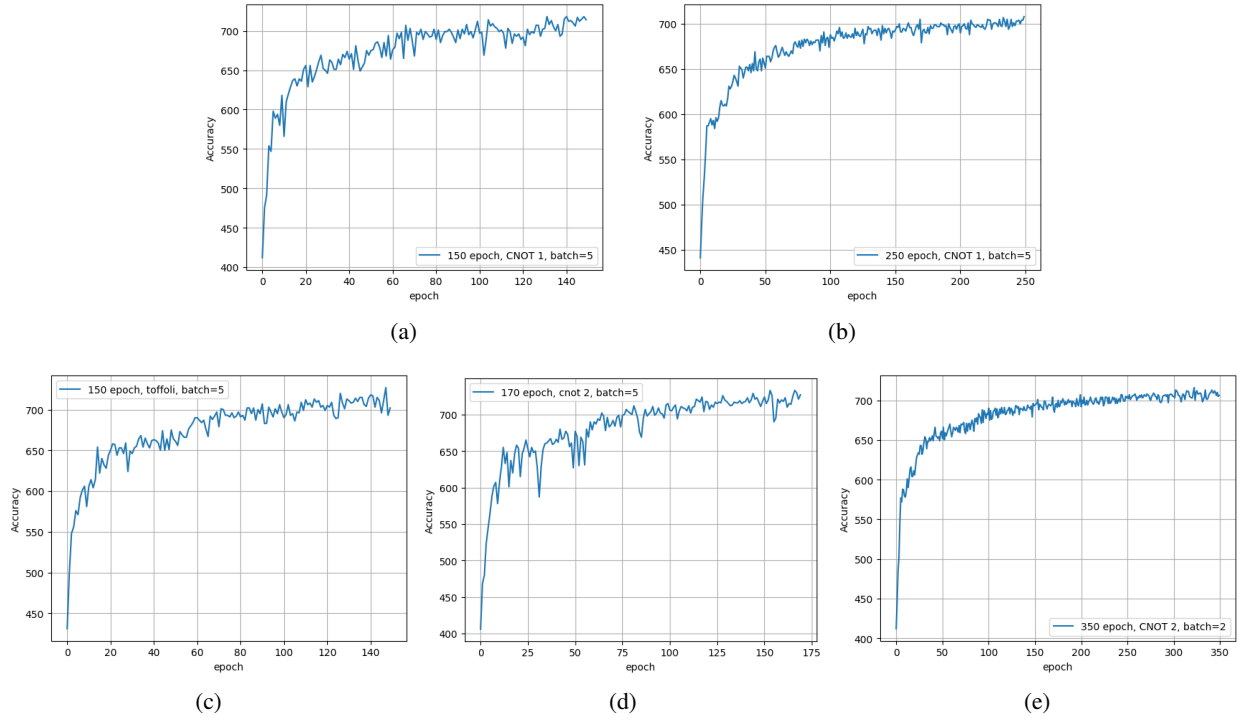


Fig. 12: Training curves