

Beamforming Optimization via Quantum Algorithms Using VQE and QAOA

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Abstract: This study investigates the application of quantum algorithms, specifically the Variational Quantum Eigensolver (VQE) and the Quantum Approximate Optimization Algorithm (QAOA), to design optimal sensor configurations for beamforming, enhancing signal quality and overall system performance. We propose two distinct optimization formulations: one aimed at maximizing array gain while the other aimed at maximizing Signal-to-Noise-Interference ratio (SINR). Our findings show that the outputs obtained from quantum algorithms are consistent with those derived from classical methods.

1 Introduction

Beamforming is a signal processing technique that directs signals in specific directions to enhance signal quality and reduce interference in sensor arrays [1][2]. Traditional Uniform Linear Arrays (ULA) have been widely used for beamforming design. Delay and sum, MVDR etc. [3][4] [5] have been designed to meet certain performance measure. But recent advancements in sparse arrays have demonstrated improved aperture.

Recently, the literature has explored the concept of sparse arrays, which utilize fewer sensors to reduce hardware requirements. This approach minimizes cost and complexity while still achieving the desired performance in various applications, offering an efficient alternative to traditional sensor arrays. The design of sparse arrays for beamforming can be formulated as a combinatorial optimization problem, necessitating the selection of the most effective configuration from a vast set of potential arrangements. Quantum computers promise to solve these problems by offering solutions that may be exponentially faster than the classical equivalent [6]. Currently, efforts to build a universal quantum computer are limited by the practical difficulties of assembling a large number of qubits and achieving high-fidelity gate operations [7].

To address the daunting resource requirements of many quantum algorithms compared to current availability, the Variational Quantum Eigensolver (VQE) [8] was developed as a quantum-classical hybrid optimization scheme, based on the idea that even minimal quantum resources can be effective when combined with classical routines [6]. It has attracted significant attention as an algorithm suitable for near-term applications, with it being applicable in quantum chemistry and classical combinatorial optimization problems.

In essence, VQE involves preparing a trial quantum state using a parameterized quantum circuit, followed by measuring the expectation value of the Hamiltonian to optimize the parameters through classical algorithms. This iterative process continues until the lowest energy eigenvalue is achieved.

The Quantum Approximate Optimization Algorithm (QAOA) [9] is a specialized variant of the VQE that utilizes a variational Ansatz based on the adiabatic theorem. This approach has the potential to enable practical applications of noisy quantum devices in addressing classical combinatorial optimization problems, including the Max-Cut [9], Traveling Salesman Problem (TSP) [10], Max-Independent Set (MIS) [11] and graph coloring [12].

This work attempts to solve the beamforming problem, a complex combinatorial challenge focused on optimizing sensor arrays to enhance performance. By strategically arranging and adjusting the sensors, we aim to maximize signal quality and improve the effectiveness of the system in real-world applications.

We formulate two optimization problems of beamforming, one

which optimizes Array Gain and the other which optimizes signal-to-interference-plus-noise ratio (SINR), and then transform them into Quadratic Unconstrained Binary Optimization (QUBO) format, ultimately representing them as an Ising Hamiltonian. These formulations are then solved using the VQE and QAOA. We also present simulation results for both algorithms and compare them with classically obtained SINR plots. **We additionally present simulation results by fixing the desired angle and varying the angle of interference in increments of 20 degrees to provide more insight into the impact of interference on system performance.** Equivalent optimum array geometry is obtained using both approaches and the same has maximum SINR.

2 Minimum Variance distortionless Beamformer

2.1 MVDR Beamformer

Consider N omnidirectional antennas arranged in uniform linear array (ULA) with inter-element spacing of $p_n d, p_n \in \mathbb{N}$, with $d = \lambda/2$ and λ being the operating wavelength. Consider this array receiving signals, with a desired signal arriving from direction θ_d and q other interfering signals from various directions $\{\phi_1, \phi_2, \dots, \phi_q\}$. The corresponding steering vectors are given by,

$$\mathbf{u}_a = [e^{jk p_1 d \cos \theta_d} \quad e^{jk p_2 d \cos \theta_d} \quad \dots \quad e^{jk p_N d \cos \theta_d}]^T \quad (1)$$

$$\mathbf{v}_i = [e^{jk p_1 d \cos \phi_i} \quad e^{jk p_2 d \cos \phi_i} \quad \dots \quad e^{jk p_N d \cos \phi_i}]^T \quad (2)$$

where, $i = 1, 2, \dots, q$ and the wavenumber is denoted by $k = 2\pi/\lambda$. The received signal at time instant t is given by,

$$\mathbf{x}(t) = \mathbf{u}_a s(t) + \mathbf{V} \mathbf{v}(t) + \mathbf{n}(t)$$

where $\mathbf{V} = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \dots \quad \mathbf{v}_q]$ is the interference array manifold matrix with a full column rank, $s(t) \in \mathbb{C}$ denotes the desired signal waveform with power σ_s^2 , $\mathbf{v}(t) \in \mathbb{C}^q$ is the vector of q different statistically independent interfering signals with the covariance matrix $\mathbf{R}_v$ and $\mathbf{n}(t) \in \mathbb{C}^N$ is the received noise vector with power σ_n^2 . Noise is assumed to be Gaussian in nature.

The MVDR beamformer minimizes the signal free output power while constraining the array gain in the desired direction to be unity.

$$\underset{\mathbf{w}}{\text{minimize}} \quad \mathbf{w}^H \mathbf{R}_n \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{u}_a = 1$$

where $\mathbf{R}_n = \mathbf{V} \mathbf{R}_v \mathbf{V}^H + \sigma_n^2 \mathbf{I}$ denotes the covariance matrix of interference plus noise. The input SNR is defined as $SNR_i = \frac{\sigma_s^2}{\sigma_n^2}$,

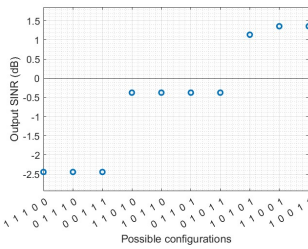
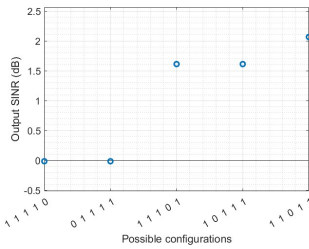
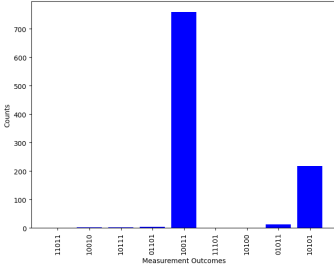
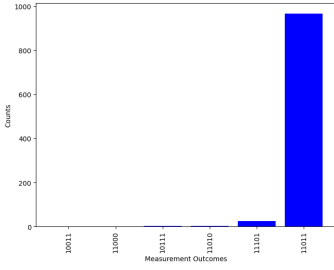
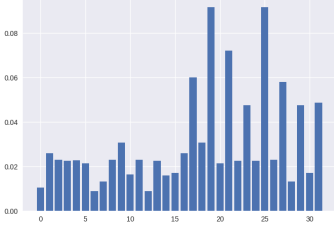
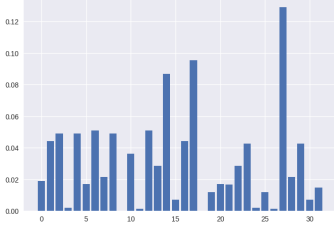
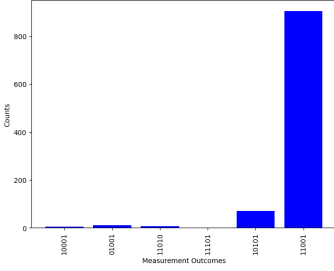
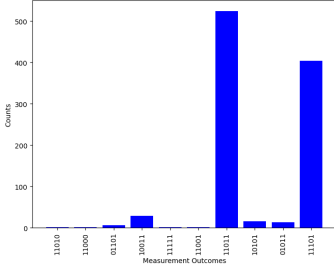
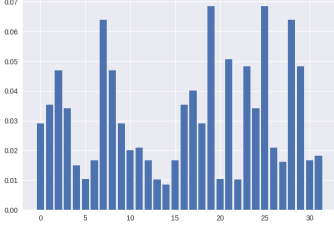
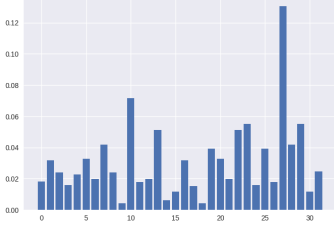
K		3	4
SINR Plots			
Formulation 1	VQE		
	QAOA		
Formulation 2	VQE		
	QAOA		
Optimal Combination		11001 10011	11011

Table 1 Histogram plots for both the VQE and QAOA for each formulation, with θ_d and ϕ_i values of 48° and 37° , respectively, along with the classically computed SINR values which show the optimum array configurations for $N = 5$.

K		3	4
SINR Plots			
Formulation1	VQE		
	QAOA		
Formulation2	VQE		
	QAOA		
Optimal Combination		10101	11011

Table 2 Histogram plots for both the VQE and QAOA for each formulation, with θ_d and ϕ_i values of 40° and 65° , respectively, along with the classically computed SINR values which show the optimum array configurations for $N = 5$.

and output SINR of MVDR beamformer is given as, $SINR_o = \sigma_s^2 \mathbf{u}_a^H \mathbf{R}_n^{-1} \mathbf{u}_a$. Further using block matrix determinant formula [13], correspondingly array gain can be evaluated as,

$$G_{mat} = \frac{SINR_o}{SINR_i} = \frac{\det(\mathbf{V}_a^H \mathbf{V}_a + \mathbf{R})}{\det(\mathbf{V}^H \mathbf{V} + \mathbf{R}_j)} \quad (3)$$

where $\mathbf{V}_a = [\mathbf{V}, \mathbf{u}_a]$ and with $\mathbf{R}_j = \sigma_n^2 \mathbf{R}_v^{-1}$,
 $\mathbf{R} = \begin{bmatrix} \mathbf{R}_j & \mathbf{0}_{q \times 1} \\ \mathbf{0}_{1 \times q} & 0 \end{bmatrix}$.

2.2 Design of Optimum Array

The problem here is to design an array with $K (< N)$ number of sensors which gives the best performance. Designing such an optimum array can be stated as finding the most effective subset of K sensors out of N sensors such that the chosen configuration maximizes the array's performance according to a well-defined performance metric. Define a vector $\mathbf{z} = [z_i, i = 1, \dots, N] \in \{0, 1\}^N$ to represent sensor selection.

2.3 Formulation 1: Optimizing Array Gain

Array gain given in equation (3) is a general expression. For a special scenario when we have one interfering source, equation (3) can be simplified as,

$$G_{mat} = \frac{\det(\mathbf{V}_a^H \mathbf{V}_a + \mathbf{R})}{\det(\mathbf{v}_i^H \mathbf{v}_i + r_j)} \quad (4)$$

where $\mathbf{v}_i$ is the steering vector corresponding to the interfering direction ϕ_i and r_j is the corresponding measure of correlation.

Theorem 1. For a scenario of single interfering source the optimum array of K sensors out of N locations can be obtained by optimizing the array gain as,

$$\max_z \frac{\det(\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a + \mathbf{R})}{\det(K + r_j)} \quad (5)$$

such that $\mathbf{1}^T \mathbf{z} = K$

Proof: By incorporating the sensor selection vector $\mathbf{z}$, array gain given in the equation (4) can be written as,

$$G_{mat} = \frac{\det(\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a + \mathbf{R})}{\det(\mathbf{v}_i^H \text{diag}(\mathbf{z}) \mathbf{v}_i + r_j)} \quad (6)$$

The denominator of the equation (6) is simply $\det(K + r_j)$, which is independent of $\mathbf{z}$. $\square$

Array Gain in Quadratic Form

Theorem 2. The cost function given in (5) can be transformed to the quadratic cost function.

Proof:

The denominator of equation (5) thus is a constant. The numerator of equation (5) can be re-written as:

$$\det(\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a + \mathbf{R}) = \det(\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a) + \det(\mathbf{R}) + \text{tr}(\mathbf{R}^* \mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a) \quad (7)$$

using the identity $\det(\mathbf{A} + \mathbf{B}) = \det(\mathbf{A}) + \det(\mathbf{B}) + \text{tr}(\mathbf{A}^\dagger \mathbf{B})$ for 2×2 matrices, as in [14]. In order to solve equation (7), we write

the first term as:

$$\det(\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a) = \frac{1}{2} \left[\left\{ \text{tr}(\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a) \right\}^2 - \text{tr}^2(\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a) \right] \quad (8)$$

using the identity $\det(\mathbf{A}) = \frac{1}{2} (\text{tr}(\mathbf{A})^2 - \text{tr}(\mathbf{A}^2))$ for 2×2 matrices, as in [15]. Further,

$$\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a = \begin{bmatrix} \sum_{i=1}^N z_i & \sum_{i=1}^N z_i v_{a,i1}^H v_{a,i2} \\ \sum_{i=1}^N z_i v_{a,i2}^H v_{a,i1} & \sum_{i=1}^N z_i \end{bmatrix} \quad (9)$$

Therefore, $\text{tr}(\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a) = 2 \sum_{i=1}^N z_i$ and

$$\text{tr}(\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a)^2 = 4 \sum_{i=1}^N \sum_{j=1}^N z_i z_j \quad (10)$$

Also, from equation (9), computing $(\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a)^2$ and taking its trace, we get $\text{tr}^2(\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a)$

$$\begin{aligned} &= \sum_{i=1}^N \sum_{j=1}^N z_i z_j + \sum_{i=1}^N \sum_{j=1}^N z_i z_j v_{a,i1}^H v_{a,i2} v_{a,j2}^H v_{a,j1} \\ &+ \sum_{i=1}^N \sum_{j=1}^N z_i z_j v_{a,i2}^H v_{a,i1} v_{a,j1}^H v_{a,j2} + \sum_{i=1}^N \sum_{j=1}^N z_i z_j \end{aligned} \quad (11)$$

Equation (10) and (11) are combined together to solve equation (8), which in turn forms the first term of equation (7).

The second term of equation (7), $\det(\mathbf{R})$ is a constant

To solve the third term of equation (7), we use the cyclic property of trace as:

$$\text{tr}(\mathbf{R}^* \mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a) = \text{tr}(\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a \mathbf{R}^*) \quad (12)$$

Further,

$$\mathbf{V}_a \mathbf{R}^* = \begin{bmatrix} v_{a,11} r_j^* & 0 \\ v_{a,21} r_j^* & 0 \\ \vdots & \vdots \\ v_{a,N1} r_j^* & 0 \end{bmatrix} \quad (13)$$

Plugging equation (13) in equation (12), we get,

$$\text{tr}(\mathbf{R}^* \mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a) = r_j^* \sum_{i=1}^N z_i \quad (14)$$

Combining equations (8) - (14), we solve equation (7) as:

$$\begin{aligned} \det(\mathbf{V}_a^H \text{diag}(\mathbf{z}) \mathbf{V}_a + \mathbf{R}) &= \sum_{i=1}^N \sum_{j=1}^N z_i z_j \left(1 + v_{a,i1}^H v_{a,i2} v_{a,j2}^H v_{a,j1} + v_{a,i2}^H v_{a,i1} v_{a,j1}^H v_{a,j2} \right) \\ &+ r_j^* \left(\sum_{i=1}^N z_i \right) \end{aligned} \quad (15)$$

$\square$

Formulation as QUBO :

Since QUBO formulation requires the optimization problem to be unconstrained, we introduce the Lagrange multiplier to incorporate the constraint into the objective function.

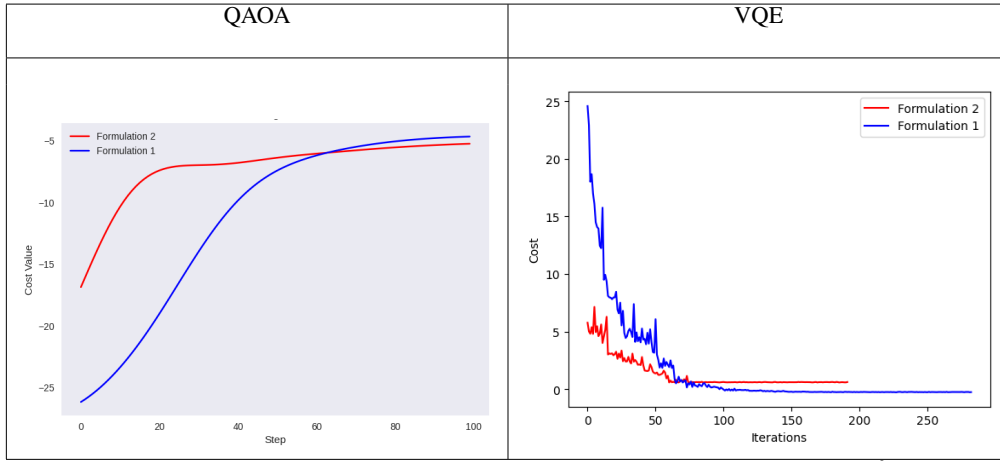


Table 3 Training curves for total number of sensors $N = 5$ and number of active sensors $K = 3$. The desired angle $\theta_d = 48^\circ$ with $SNR = 5dB$ and interference angle $\phi_i = 37^\circ$ with $INR = 0dB$. The curves in 'blue' correspond to Formulation 1, while 'red' are for Formulation 2. The QAOA methodology uses Adam Optimizer with stepsize 0.0005 for Formulation 1, and 0.002 for Formulation 2. VQE uses Coby optimizer for both Formulation 1 and Formulation 2.

For the constraint term of equation (7), we take quadratic penalization factor as:

$$\mu \left(\sum_{i=1}^N z_i - K \right)^2 = \mu \left(\sum_{i=1}^N \sum_{j=1}^N z_i z_j - 2K \sum_{i=1}^N z_i + K^2 \right) \quad (16)$$

where μ is Lagrange multiplier.

Thus, to derive the final Hamiltonian, we combine equation (15) - (16) to get:

$$\sum_{i=1}^N \sum_{j=1}^N z_i z_j \left(\frac{1 + v_{a,i1}^H v_{a,i2} v_{a,j1}^H + v_{a,i2}^H v_{a,i1} v_{a,j1}^H v_{a,j2}}{\det(K + r_j)} + \mu \right) + \sum_{i=1}^N z_i \left(\frac{r_j^*}{\det(K + r_j)} - 2K\mu \right) + constant \quad (17)$$

To derive the Ising Hamiltonian, we substitute $z_p = \frac{1 - Z_p}{2}$ where Z_p is the Pauli-Z operator acting on qubit p , which is then used in the QAOA and VQE algorithm. We can ignore the constant term in the above Hamiltonian since it does not have any effect on the relative energy differences among quantum states.

2.4 Formulation 2: Optimizing SINR

An alternative formulation is also possible, in which we utilize the signal-to-interference-plus-noise ratio (SINR) to optimize the sensor array configuration. In the context of optimizing sensor array configurations, maximizing SINR allows for improved detection and processing capabilities.

Theorem 3. To optimize sensor configuration for maximizing the signal-to-interference-plus-noise ratio (SINR), the optimal configuration of K sensors selected from N uniformly spaced locations can be determined for a scenario involving a single interfering source by formulating the optimization problem as:

$$\begin{aligned} \max \quad & SINR \\ \text{such that} \quad & \mathbf{1}^T \mathbf{z} = K \end{aligned} \quad (18)$$

Proof:

As stated in 2.1, the SINR of MVDR beamformer is given by:

$$SINR = \sigma_s^2 \mathbf{u}_a^H \mathbf{R}_n^{-1} \mathbf{u}_a \quad (19)$$

where $\mathbf{R}_n = \mathbf{V} \mathbf{R}_V \mathbf{V}^H + \sigma_n^2 \mathbf{I}$.
The constraint being $\mathbf{1}^T \mathbf{z} = K$.

Substituting $\mathbf{R}_n$ in $SINR$,

$$\begin{aligned} SINR &= \sigma_s^2 \mathbf{u}_a^H [\mathbf{V} \mathbf{R}_V \mathbf{V}^H + \sigma_n^2 \mathbf{I}]^{-1} \mathbf{u}_a \\ &= \frac{\sigma_s^2}{\sigma_n^2} \mathbf{u}_a^H \left[\mathbf{I} + \frac{\mathbf{V} \mathbf{R}_V \mathbf{V}^H}{\sigma_n^2} \right]^{-1} \mathbf{u}_a \end{aligned} \quad (20)$$

We restrict ourselves to one interfering signal.

For $q = 1$, $\mathbf{R}_V = \sigma_V^2$, i.e. variance of the signal $\mathbf{v}(t)$. Thus, Equation (20) becomes,

$$SINR = \frac{\sigma_s^2}{\sigma_n^2} \mathbf{u}_a^H \left[\mathbf{I} + \frac{\sigma_V^2}{\sigma_n^2} \mathbf{V} \mathbf{V}^H \right]^{-1} \mathbf{u}_a \quad (21)$$

We utilize Woodbury Matrix Identity [16] expressed as $(\mathbf{I} + \mathbf{U} \mathbf{V})^{-1} = \mathbf{I} - \mathbf{U} (\mathbf{I} + \mathbf{V} \mathbf{U})^{-1} \mathbf{V}$ to expand equation (21) as :

$$SINR = \frac{\sigma_s^2}{\sigma_n^2} \mathbf{u}_a^H \left[\mathbf{I} - \frac{\sigma_V^2}{\sigma_n^2} \mathbf{V} \left(\mathbf{I} + \frac{\mathbf{V}^H \sigma_V^2 \mathbf{V}}{\sigma_n^2} \right)^{-1} \mathbf{V}^H \right] \mathbf{u}_a \quad (22)$$

Since we assumed $q = 1$, so the term $\mathbf{V}^H \mathbf{V}$ is consequently of dimension 1×1 .

Thus,

$$SINR = \frac{\sigma_s^2}{\sigma_n^2} \left[\mathbf{u}_a^H \mathbf{u}_a - \frac{\sigma_V^2}{\sigma_n^2} \frac{\mathbf{u}_a^H \mathbf{V} \mathbf{V}^H \mathbf{u}_a}{1 + \frac{\sigma_V^2}{\sigma_n^2} \mathbf{V}^H \mathbf{V}} \right] \quad (23)$$

Thus, maximizing $SINR$ would be equivalent to:

$$\max \frac{\sigma_s^2}{\sigma_n^2} \left[\mathbf{u}_a^H \text{diag}(\mathbf{z}) \mathbf{u}_a - \frac{\sigma_V^2}{\sigma_n^2} \frac{\mathbf{u}_a^H \text{diag}(\mathbf{z}) \mathbf{V} \mathbf{V}^H \text{diag}(\mathbf{z}) \mathbf{u}_a}{1 + \frac{\sigma_V^2}{\sigma_n^2} \mathbf{V}^H \text{diag}(\mathbf{z}) \mathbf{V}} \right] \quad (24)$$

We can substitute $\mathbf{u}_a^H \text{diag}(\mathbf{z}) \mathbf{u}_a = K$. Thus the optimization problem becomes:

$$\max \frac{\sigma_s^2}{\sigma_n^2} \left[K - \frac{\sigma_V^2}{\sigma_n^2} \frac{\mathbf{u}_a^H \text{diag}(\mathbf{z}) \mathbf{V} \mathbf{V}^H \text{diag}(\mathbf{z}) \mathbf{u}_a}{1 + \frac{\sigma_V^2}{\sigma_n^2} K} \right] \quad (25)$$

Thus, $\max SINR$ becomes equivalent to :

$$\min \mathbf{u}_a^H \text{diag}(\mathbf{z}) \mathbf{V} \mathbf{V}^H \text{diag}(\mathbf{z}) \mathbf{u}_a \quad (26)$$

□

θ_i	SINR	VQE	Optimal Configuration
60°			11011
80°			11011
100°			11110 01111
120°			11110 01111 11011
140°			11110 01111 11011

Table 4 Plots showing the histograms obtained from VQE algorithm using Formulation 2 at certain initial parameters for $N = 5$, $K = 4$. The desired angle θ_d is fixed at 65° , while the initial angle θ_i is varied from 60° to 140° in increments of 20° to observe changes in the plots as the angle of interference changes. Corresponding SINR plots are also included.

Formulation as QUBO : To derive the QUBO formulation, we rewrite equation (26) as:

$$\sum_{i=1}^N \sum_{j=1}^N z_i z_j (u_{a_i}^* v_i v_j^* u_{a_j}) \quad (27)$$

We use the Lagrange multiplier to make it unconstrained problem. Incorporating the constraint as quadratic penalization factor as shown in equation (16) in equation (27), we get the optimization formulation as:

$$\sum_{i=1}^N \sum_{j=1}^N z_i z_j (u_{a_i}^* v_i v_j^* u_{a_j} + \mu) - 2K\mu \sum_{i=1}^N z_i + \text{constant} \quad (28)$$

To derive Ising Hamiltonian from Equation (28), we substitute $z_p = \frac{1-Z_p}{2}$ where Z_p is the Pauli-Z operator acting on qubit p , which is then used in the QAOA and VQE algorithm. We can ignore the constant term in the above Hamiltonian since it does not have any effect on the relative energy differences among quantum states.

3 Results and Simulations

In this work, we formulate two optimization problems of beamforming - first, by maximizing the array gain, and second, by enhancing the signal-to-interference-plus-noise ratio (SINR), aiming to find an optimized subset of sensors that can achieve performance similar to a setup using all available sensors, assuming we have prior knowledge of the source direction of arrival (DOAs).

The formulation obtained in Equation (17) and (28) are used as the Cost hamiltonians for VQE and QAOA for the results shown in Table 1,2. These Tables illustrate the performance of both algorithms QAOA and VQE using specific parameter values. The classically obtained SINR values are also plotted for ease of comparison.

For QAOA, we choose a simple Mixer Hamiltonian defined as $\sum_{i=1}^N X_i$ where X_i denotes the Pauli-X operator acting on the i -th qubit. We have used Adam Optimizer as the classical optimizer in QAOA for tuning the circuit parameters. We choose depth of the circuit p to be 2, i.e, the quantum circuit consists of two alternating layers: one layer for the mixed Hamiltonian followed by one layer for the cost Hamiltonian, and this sequence is repeated twice.

For VQE, we choose Cobyla Optimizer as the classical optimizer. The EfficientSU(2) ansatz[17] was employed for the optimization process with *reps* set to 1, indicating that the structure of a rotation layer followed by an entanglement layer is repeated only once.

The columns of Table 1 show the cases for the number of active sensors K to be 3 and 4, while keeping the desired signal angle θ_d fixed at 48° with SNR of 0dB, the angle of interfering signal ϕ_i at 37° with INR of 5dB and the total number uniformly spaced sensors N to be 5. Table 2 is also generated in a similar way, with the desired signal angle θ_d being 40° with SNR of 0dB and interfering signal angle ϕ_i being 65° with INR of 5dB. The parameters of both algorithms for all cases were randomly initialized. We used Qiskit [18] for the simulations of both algorithms. To execute the quantum circuit, QAOA leveraged the *default.qubit* backend [19], while VQE utilized the *AerSimulator*[20].

Table 3 shows the training curves of both Formulation 1 and Formulation 2 for a particular case of $N = 5, K = 3, \theta_d = 48^\circ$ with $SNR = 5dB, \phi_i = 37^\circ$ with $INR = 0dB$. The training curves indicate that the models have reached a point of optimal performance with minimal fluctuations in loss.

Table 4 shows the histogram plots of the VQE algorithm using Formulation 2 with θ_d being fixed at 65° while θ_i varies from 60° to 140° in increments of 20° , showing changes in the plots as the angle of interference changes, along with the corresponding SINR plots for the ease of comparison. This analysis highlights how varying θ_i while keeping θ_d constant affects the performance and the resulting SINR. This comparison of quantum results with SINR plots serve as the state-of-the-art benchmark for performance evaluation.

We see, for QAOA, for specific initial parameters, the optimal states

obtained from histogram matches precisely with the configuration of highest SINR. For VQE, although the highest probability state matches with the highest SINR, it is unable to distinguish between degenerate states (or quasi-degenerate)[21]. However, the highest probability state for both algorithms suggest one of the optimum configurations for all cases.

4 Conclusion

In this work, we formulated beamforming using quantum optimization algorithms to achieve optimal sparse array configurations. We discussed two formulations of optimization and compared them with classical outputs. Notably, the highest probability state consistently identified one of the optimal configurations. The results demonstrate the potential of quantum algorithms to tackle complex optimization challenges in beamforming, highlighting their relevance in advancing modern communication systems.

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