

Computer Graphics

Texture Coordinates

Subodh Kumar, Dept of Computer Sc. & Engg., IIT Delhi

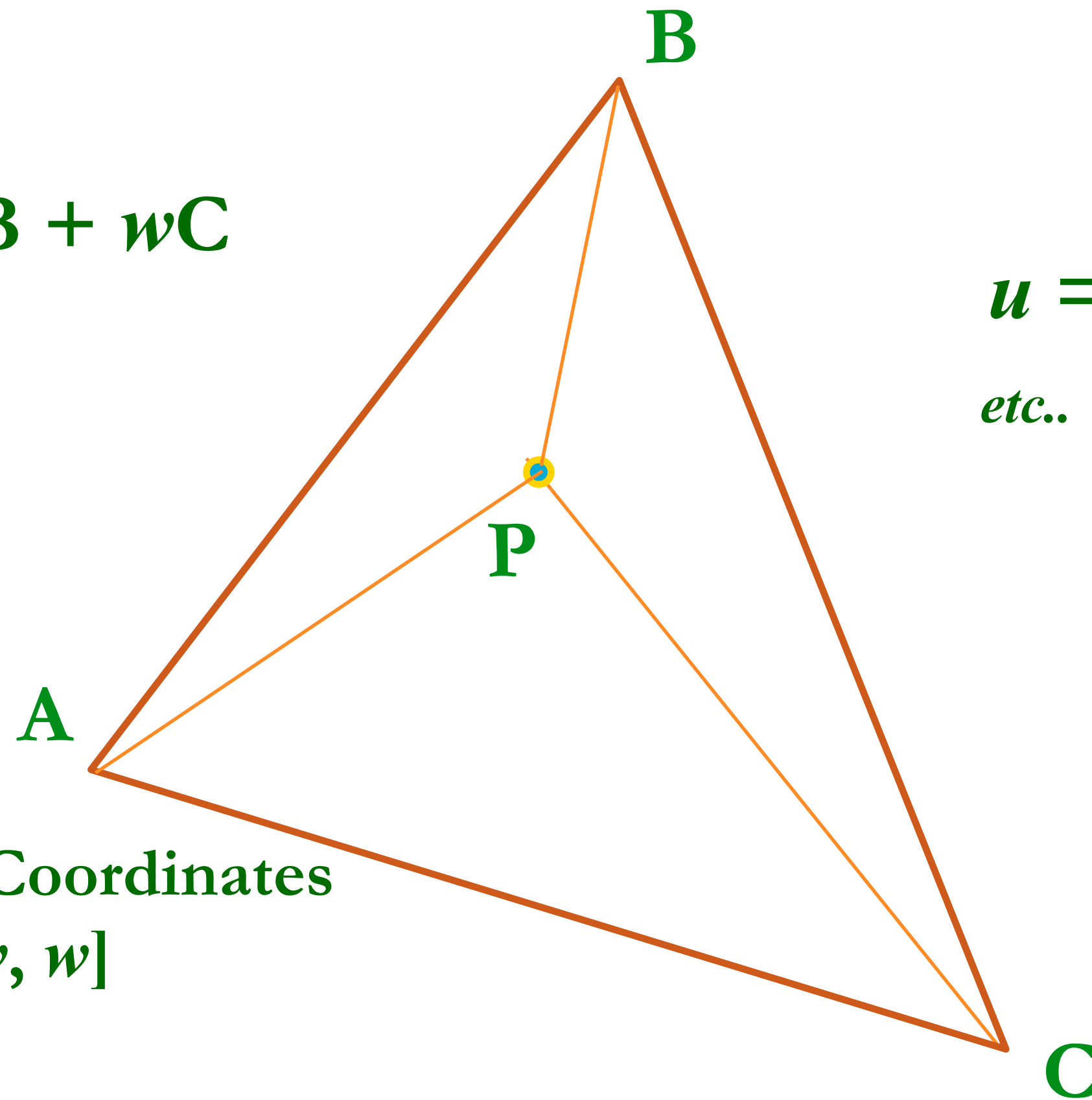


Map from Object Space

$$\mathbf{P} = u\mathbf{A} + v\mathbf{B} + w\mathbf{C}$$

$$u = |\mathbf{PBC}| / |\mathbf{ABC}|$$

etc..



Barycentric Coordinates
of $\mathbf{P} = [u, v, w]$

Should P's texture coordinates be (u, v) ?

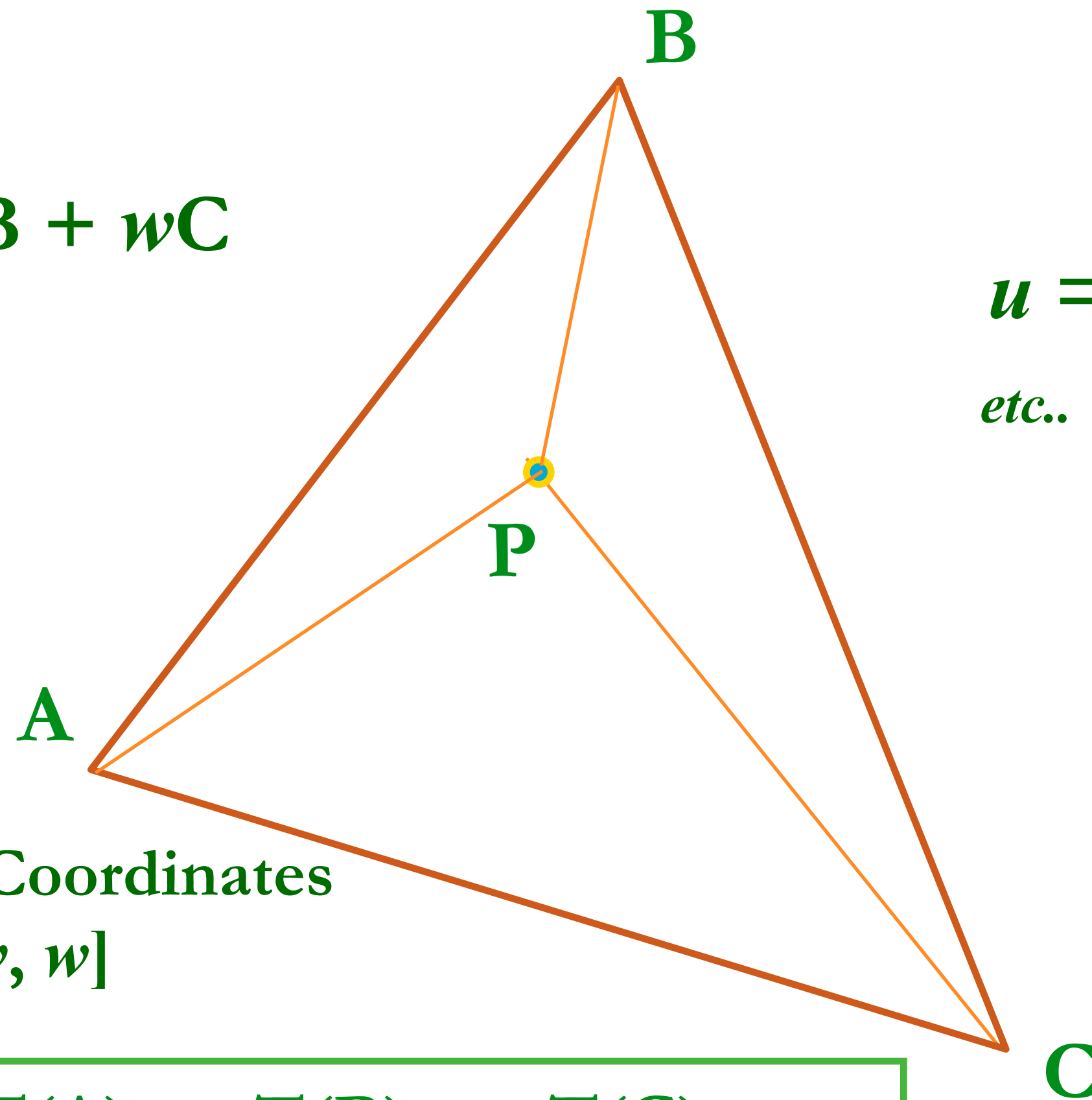


Texture Coordinates

$$\mathbf{P} = u\mathbf{A} + v\mathbf{B} + w\mathbf{C}$$

$$u = |\mathbf{PBC}| / |\mathbf{ABC}|$$

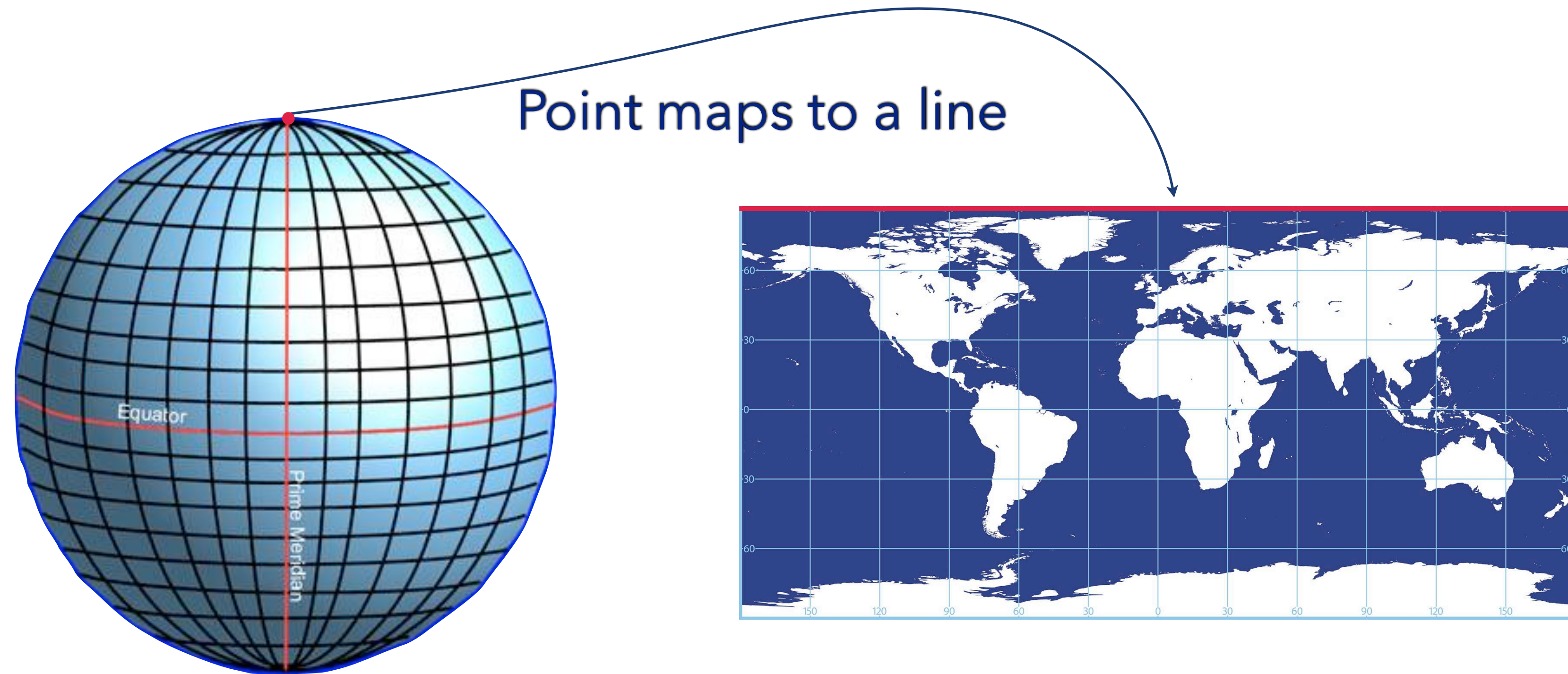
etc..



Barycentric Coordinates
of $\mathbf{P} = [u, v, w]$

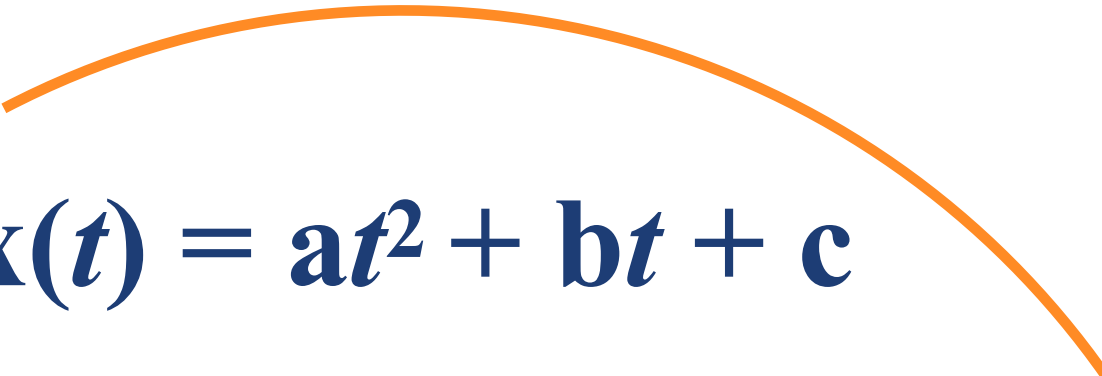
Allow indirection: $\mathbf{T}(\mathbf{P}) = u\mathbf{T}(\mathbf{A}) + v\mathbf{T}(\mathbf{B}) + w\mathbf{T}(\mathbf{C})$,
with some $\mathbf{T}(\mathbf{A})$, $\mathbf{T}(\mathbf{B})$, $\mathbf{T}(\mathbf{C})$

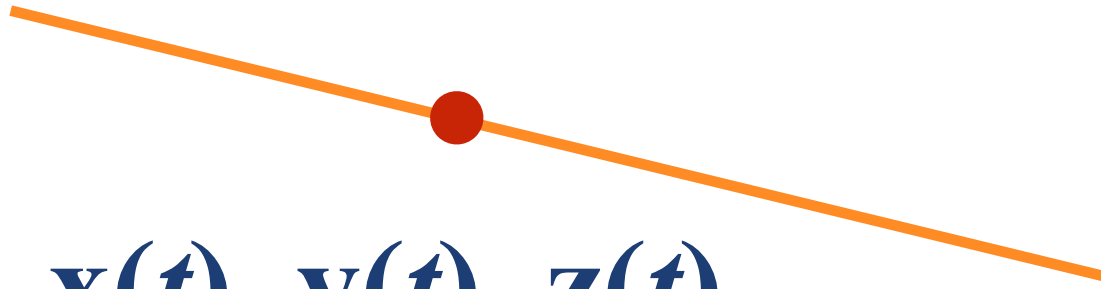
Parameterization

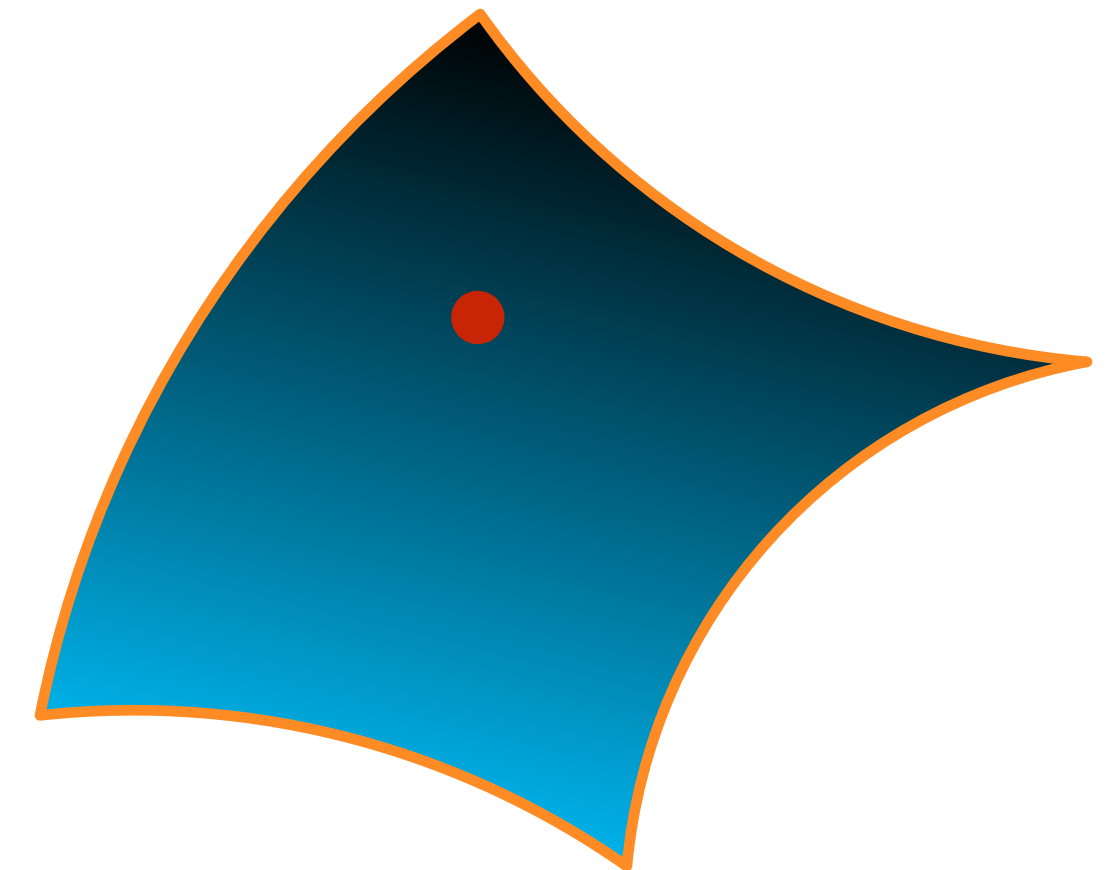


Parametrically defined surface

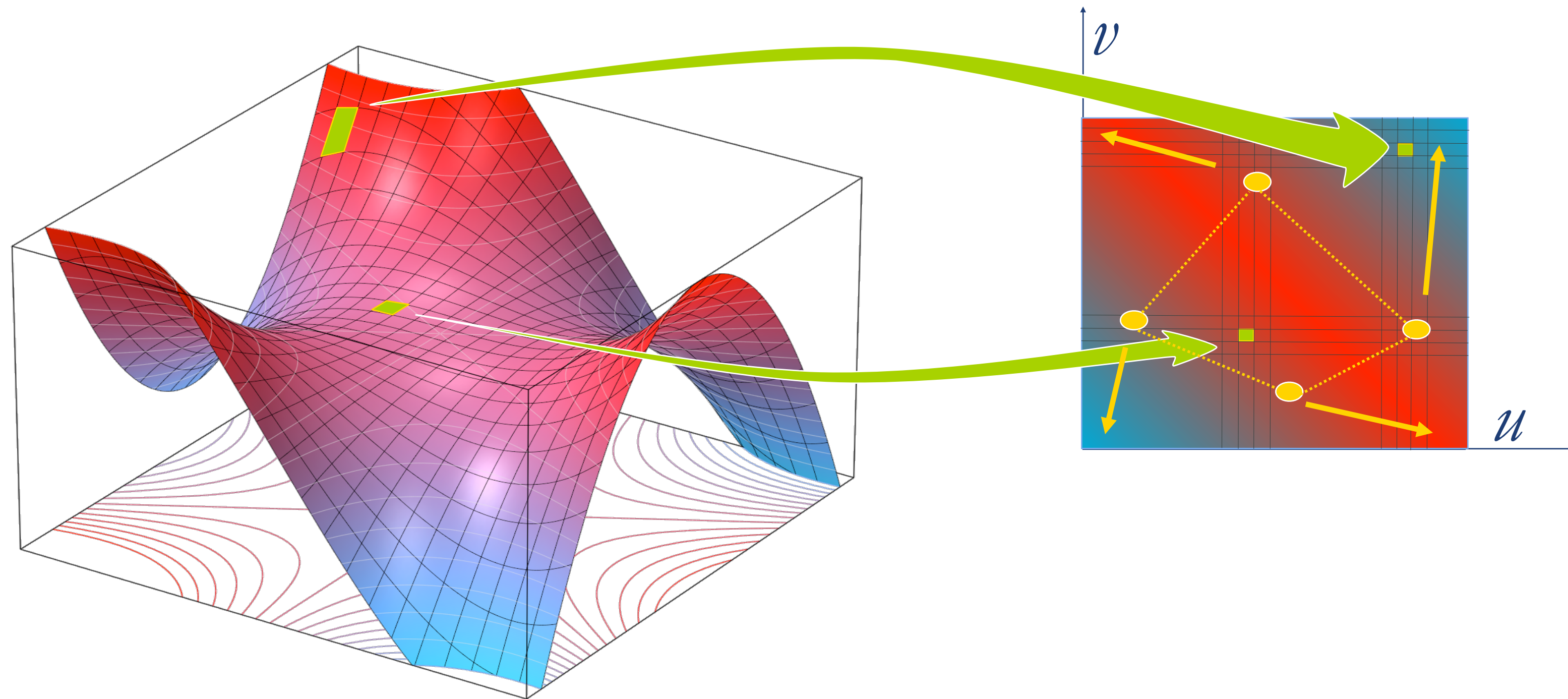



$$\mathbf{x}(t) = at^2 + bt + c$$


$$\mathbf{x}(t), y(t), z(t)$$



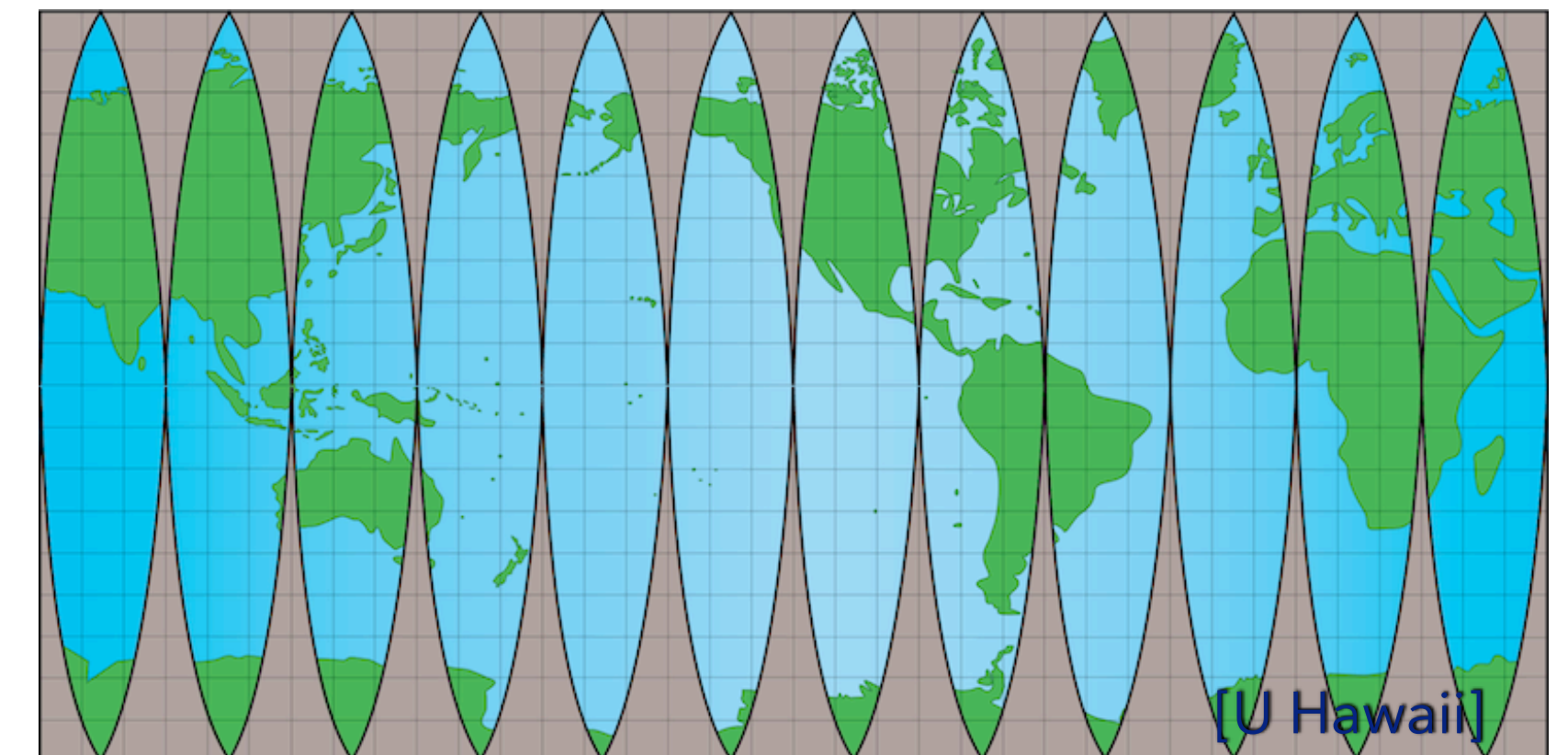
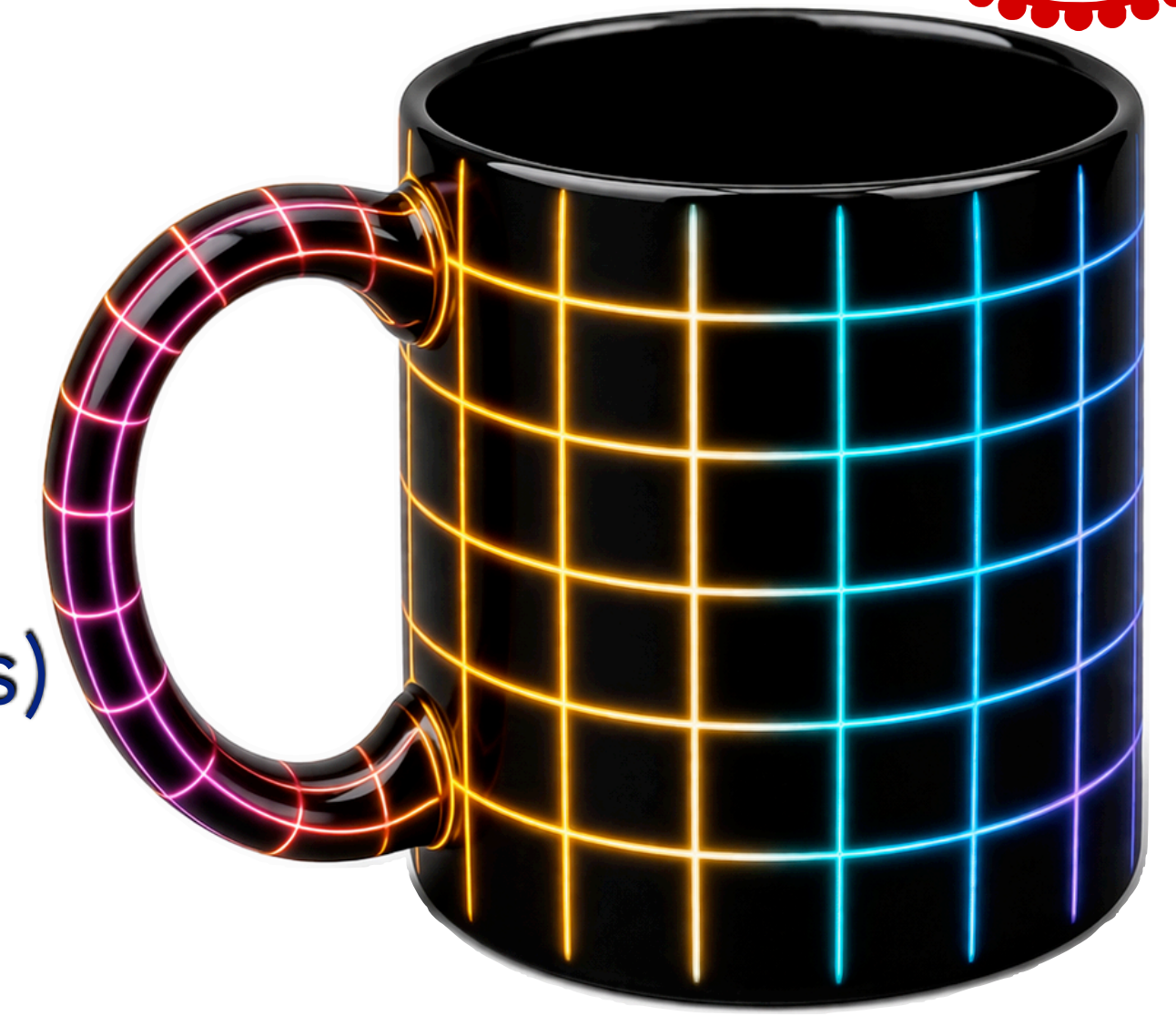
$$\mathbf{x}(u, v), y(u, v), z(u, v)$$



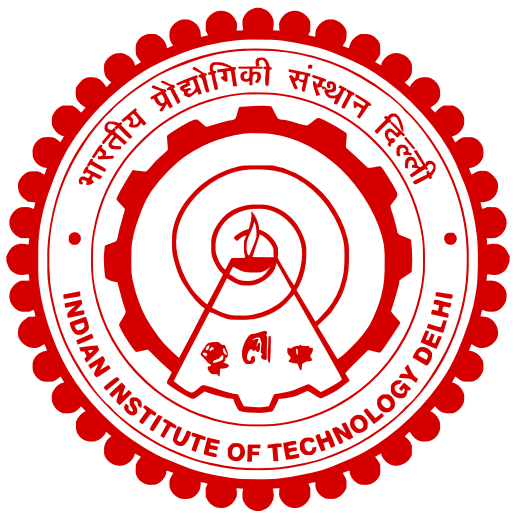
Mapping to Plane



- (Local) Homeomorphism (i.e., bijective, invertible, continuous)
 - ➔ Allows operation in 2D (textures, remeshing, animation, paths)
- Surface should be locally “flat,” (two linearly independent directions)
- 3D to 2D mapping involves distortion
 - ➔ We want to reduce distortion (multiple ways to define)
 - ➔ Posed as local “energy”, leading to a procedure to minimize that energy
- “Flat” surfaces are easier to parameterize
- “Cut” 3D mesh into relatively flat “patches”
 - ➔ Parameterize each patch
 - ➔ Pack 2D shapes on to a plane



[U Hawaii]



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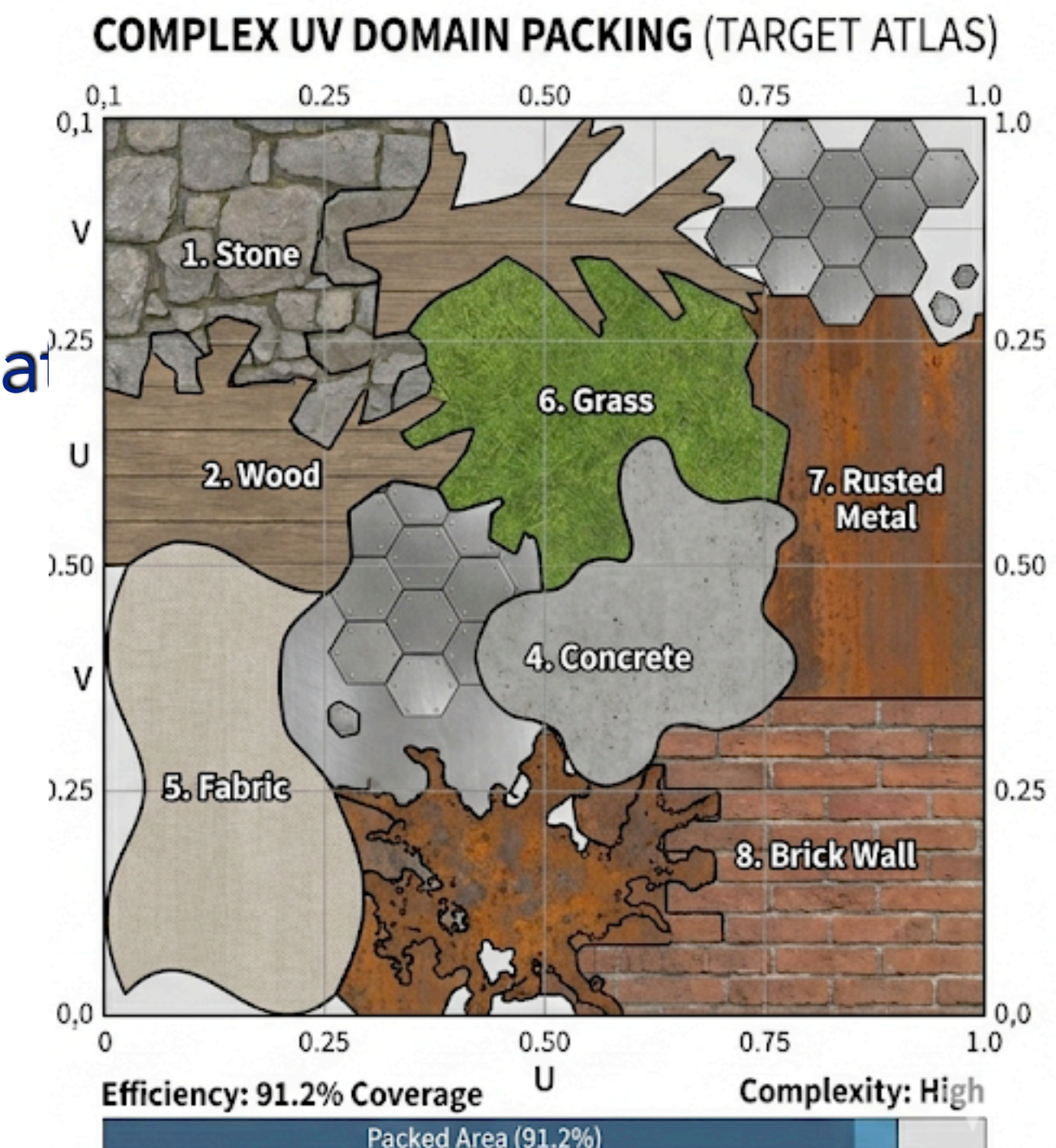
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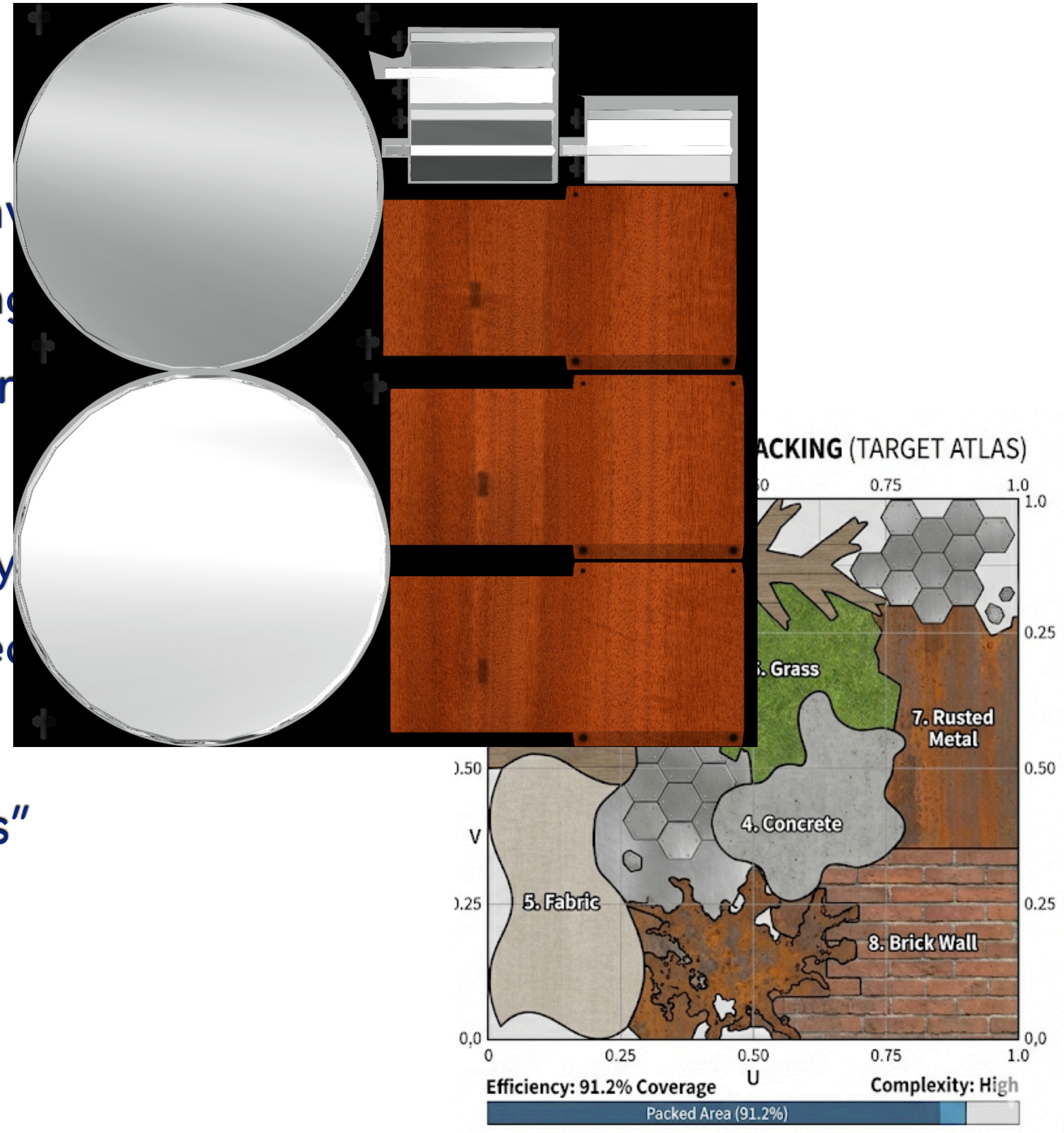
- ➔ Parameterize each patch

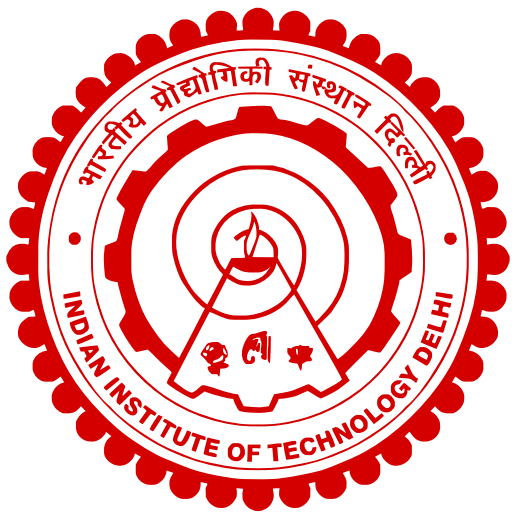
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Mapping to Plane

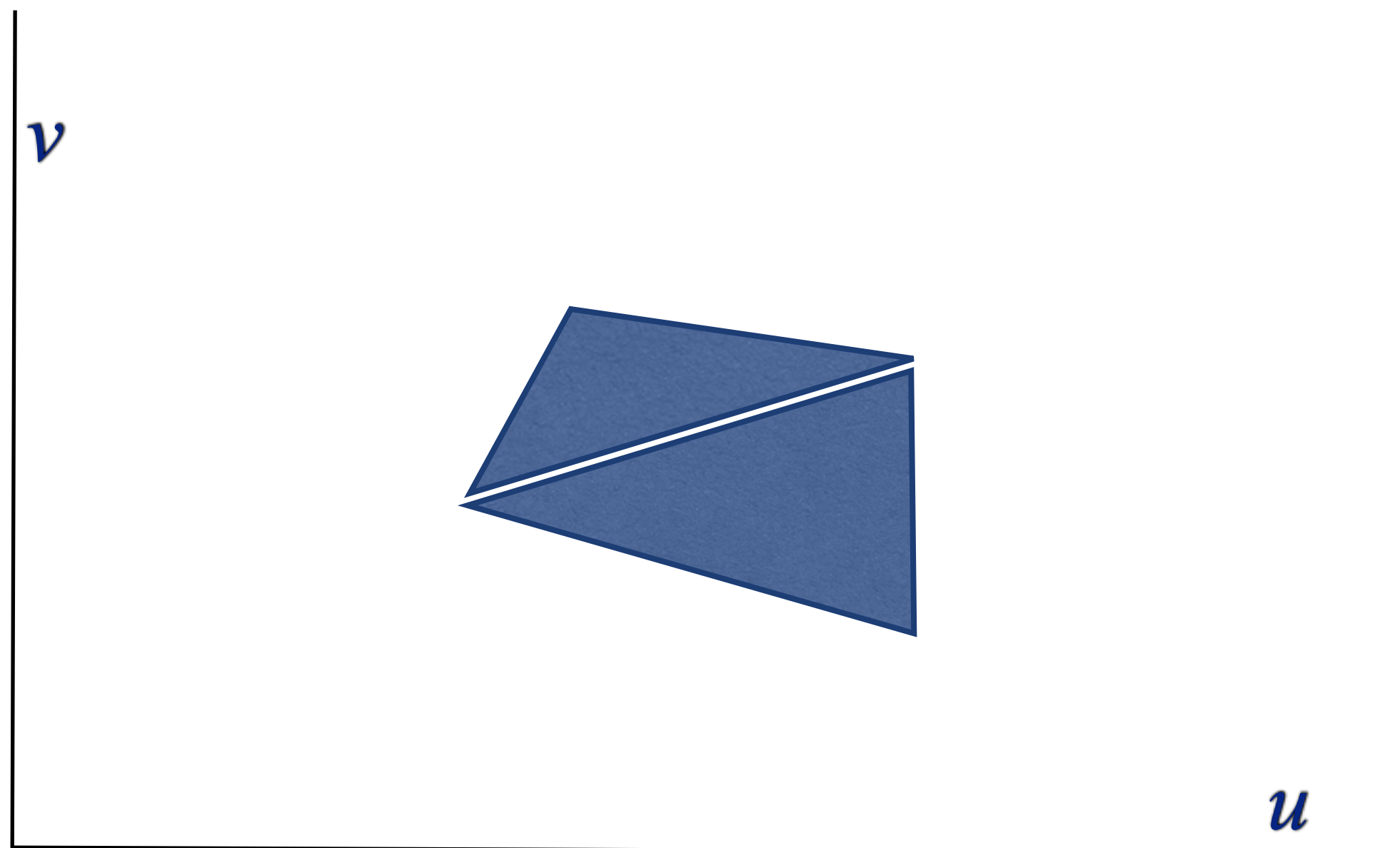
- (Local) Homeomorphism (i.e., bijective, invertible)
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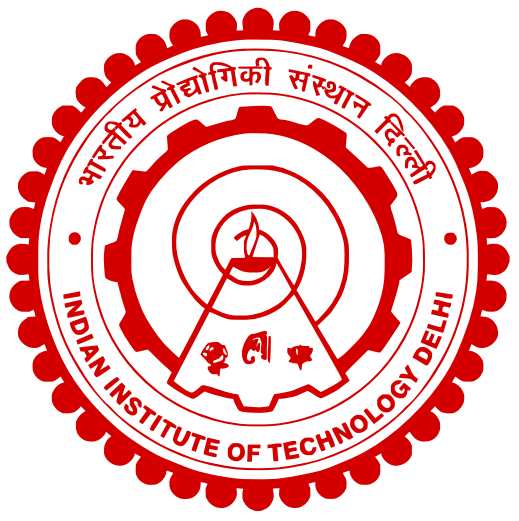




Distortion Types

- Distortion can be defined on length, angle, area, or a combination
- Area distortion
 - ➔ uniform scaling is expected, non-uniform scaling sometimes accepted
- Angle distortion (shear)
 - ➔ Poor quality, avoid



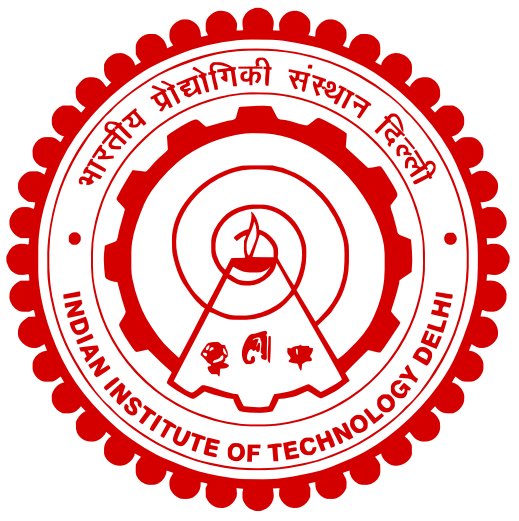


Parameterization Properties

- Isometric mapping
 - ➡ All lengths are preserved
- Conformal mapping
 - ➡ Angles and shapes are locally preserved
 - ➡ No flips
- Harmonic mapping
 - ➡ Minimizes stretching, including low angular distortion
 - ➡ May not be injective
- Reproduction property
 - ➡ For a planar mesh with an affinely mapped boundary, the entire mapping should be affine

Minimizes Dirichlet energy:

$$\int_S |\nabla u|^2 + |\nabla v|^2$$



Mean Value Coordinates

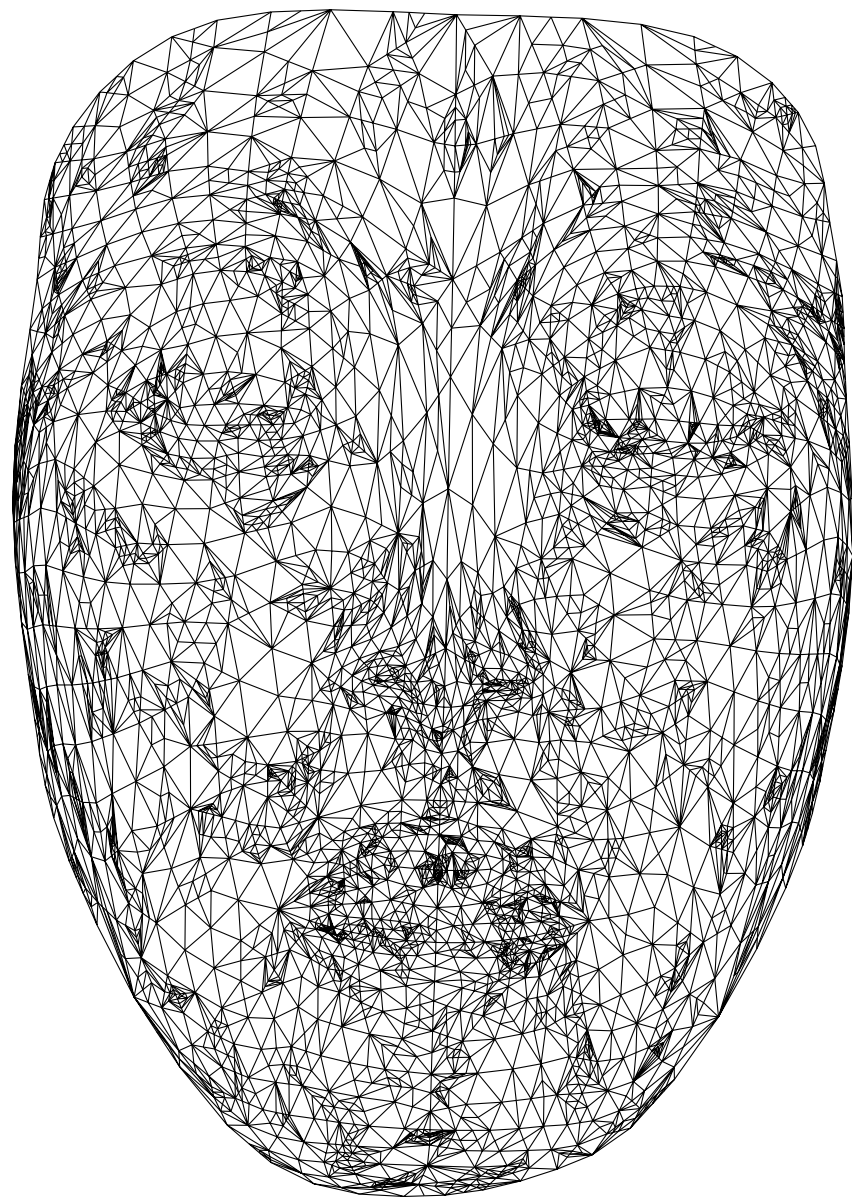
- Harmonic maps satisfy the mean value theorem (MVT):
 - ➡ At a point in the planar domain, the value of a harmonic function is always equal to the average of its values around any circle centered at that point
 - ➡ Find a map satisfying MVT at every interior vertex v , so:
 - ▶ Consider a circle C_i with "small" radius r_i centered at vertex $p_i \Rightarrow T(p_i) = \frac{1}{2\pi r_i} \int_{C_i} T(p) ds$
- Discretization leads to a set of linear equations to solve

$$\sum_{j \in \text{nbr}_i} w_{ij} \left(T(p_j) - T(p_i) \right) = 0 \quad \text{for each edge } p_i p_j$$

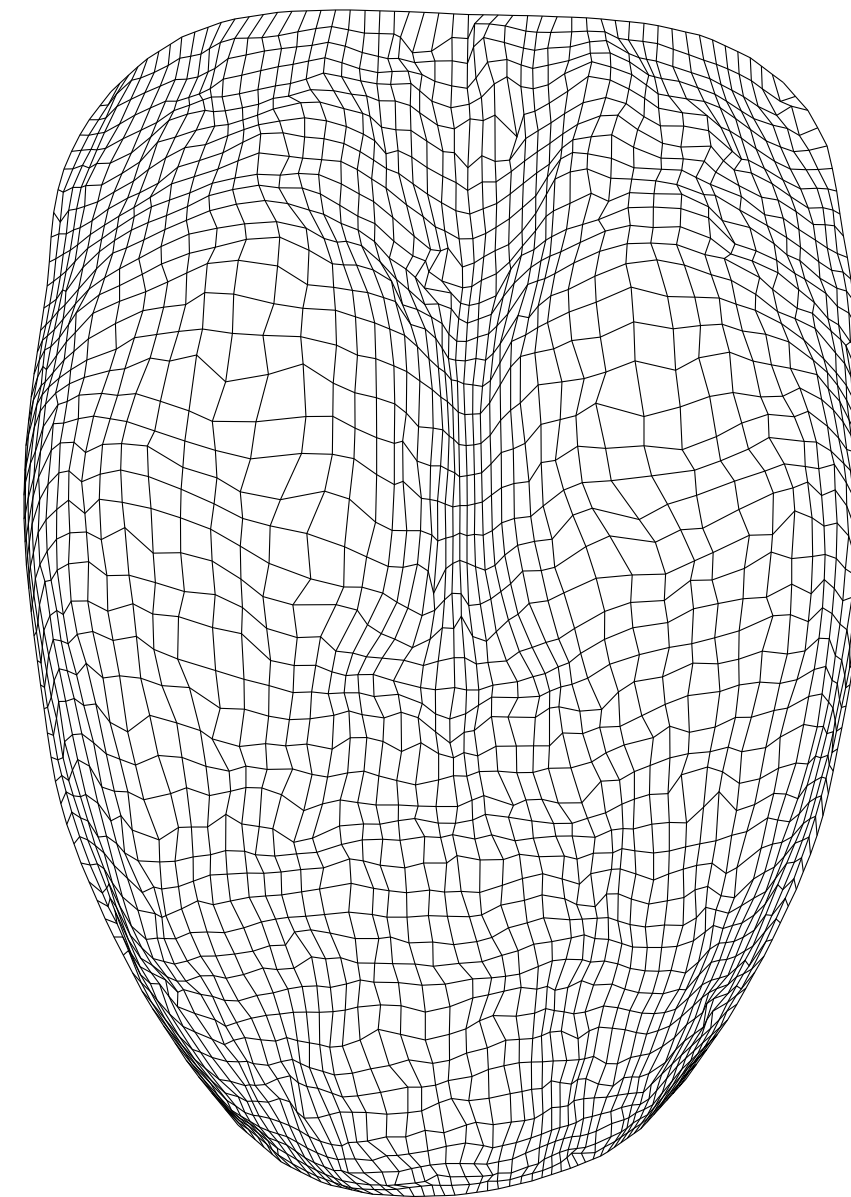
T is texture coordinate (u or v)

Comparison

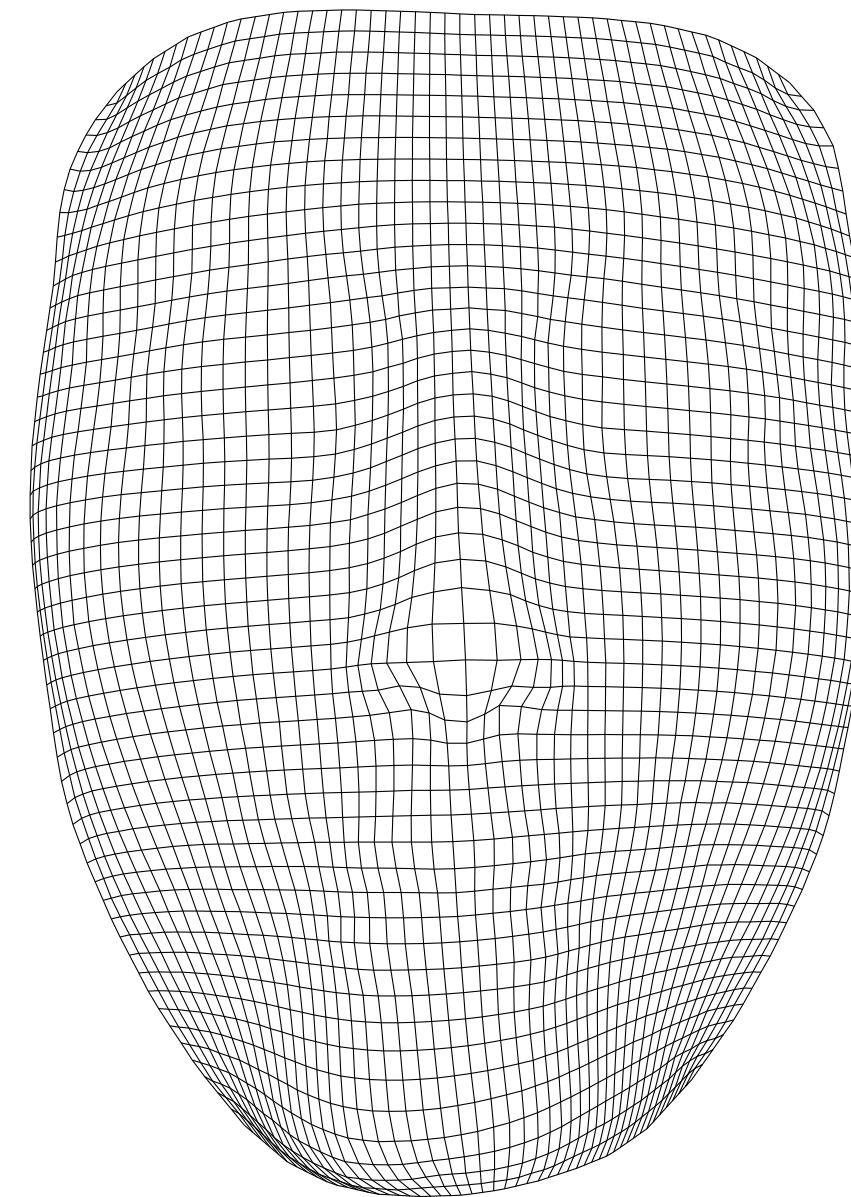
Parameterized, and remeshed by projecting a grid on uv back to 3D



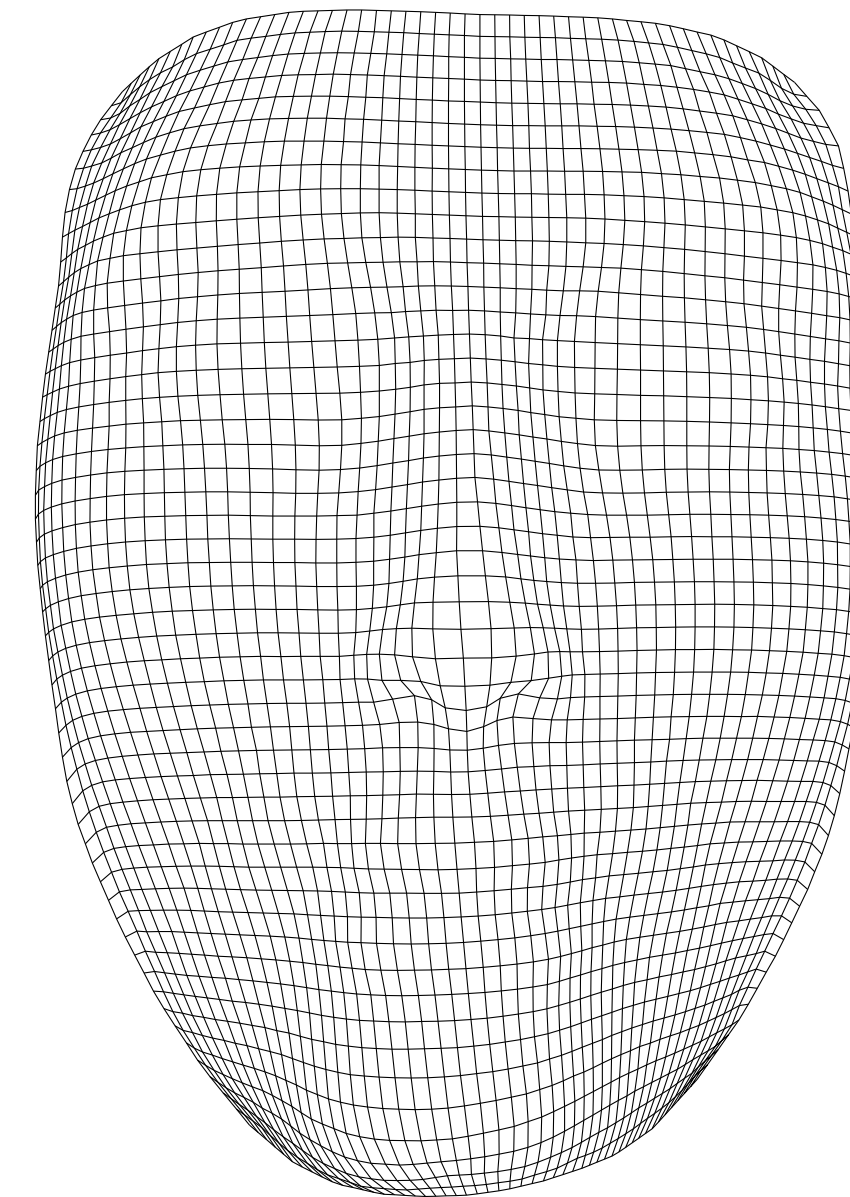
Original
Mesh



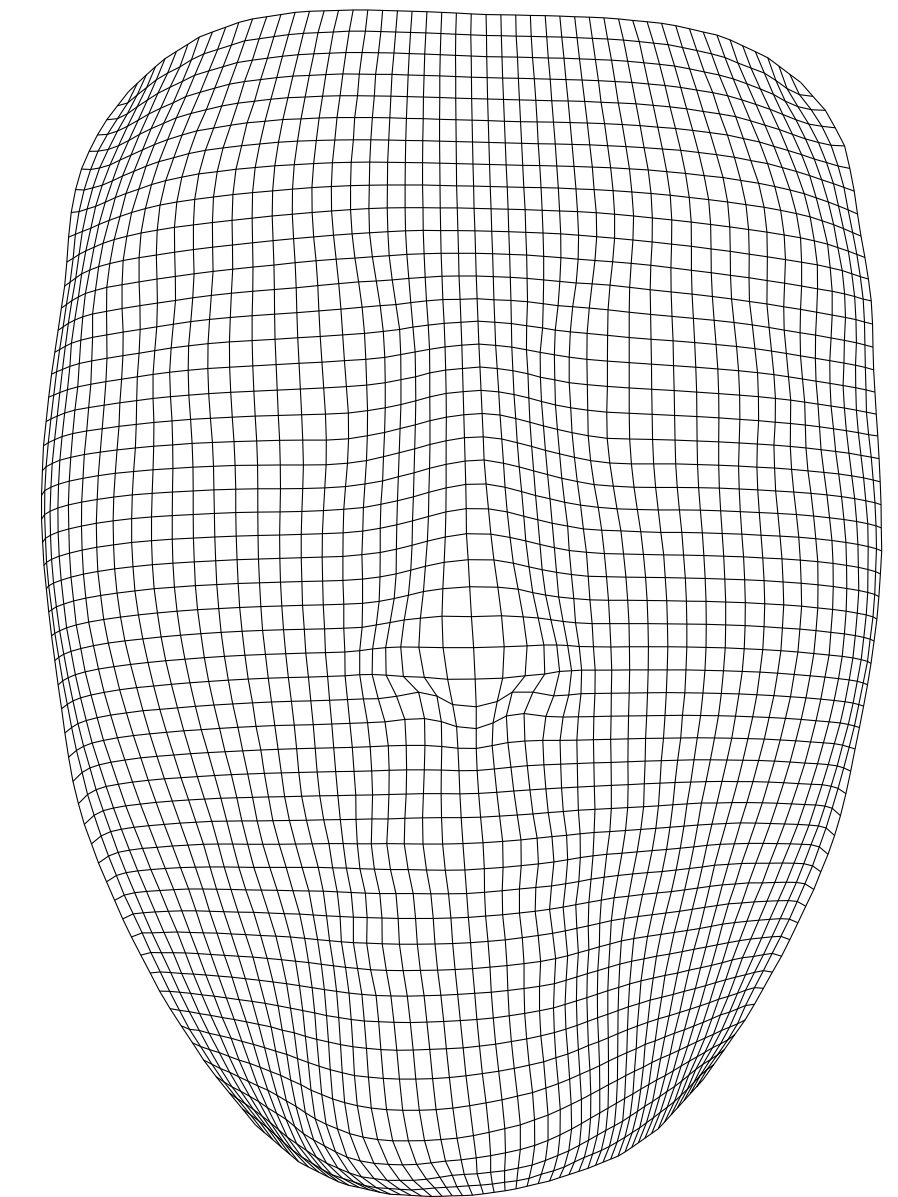
Generalized
Barycentric



Discrete
Harmonic

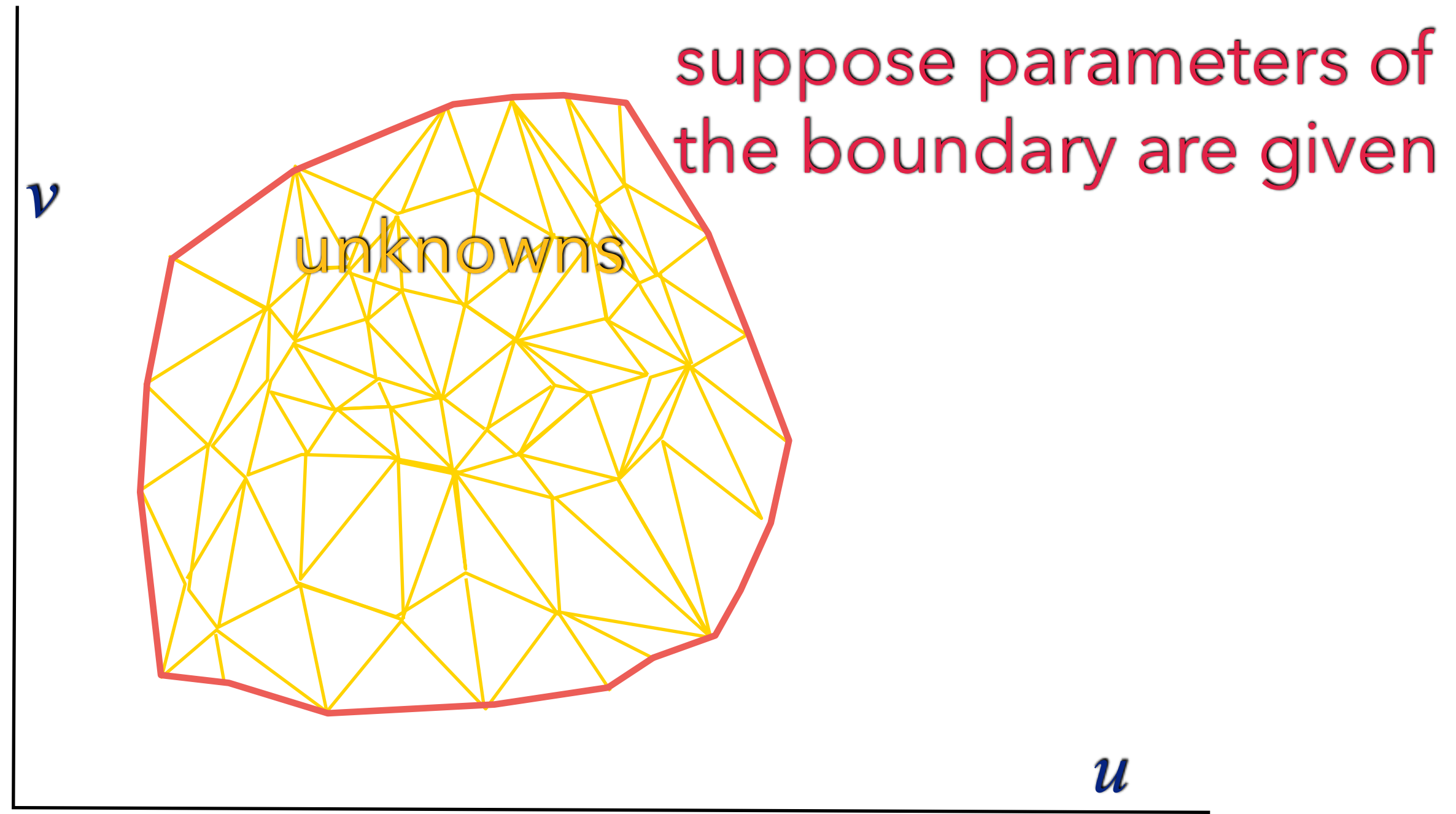
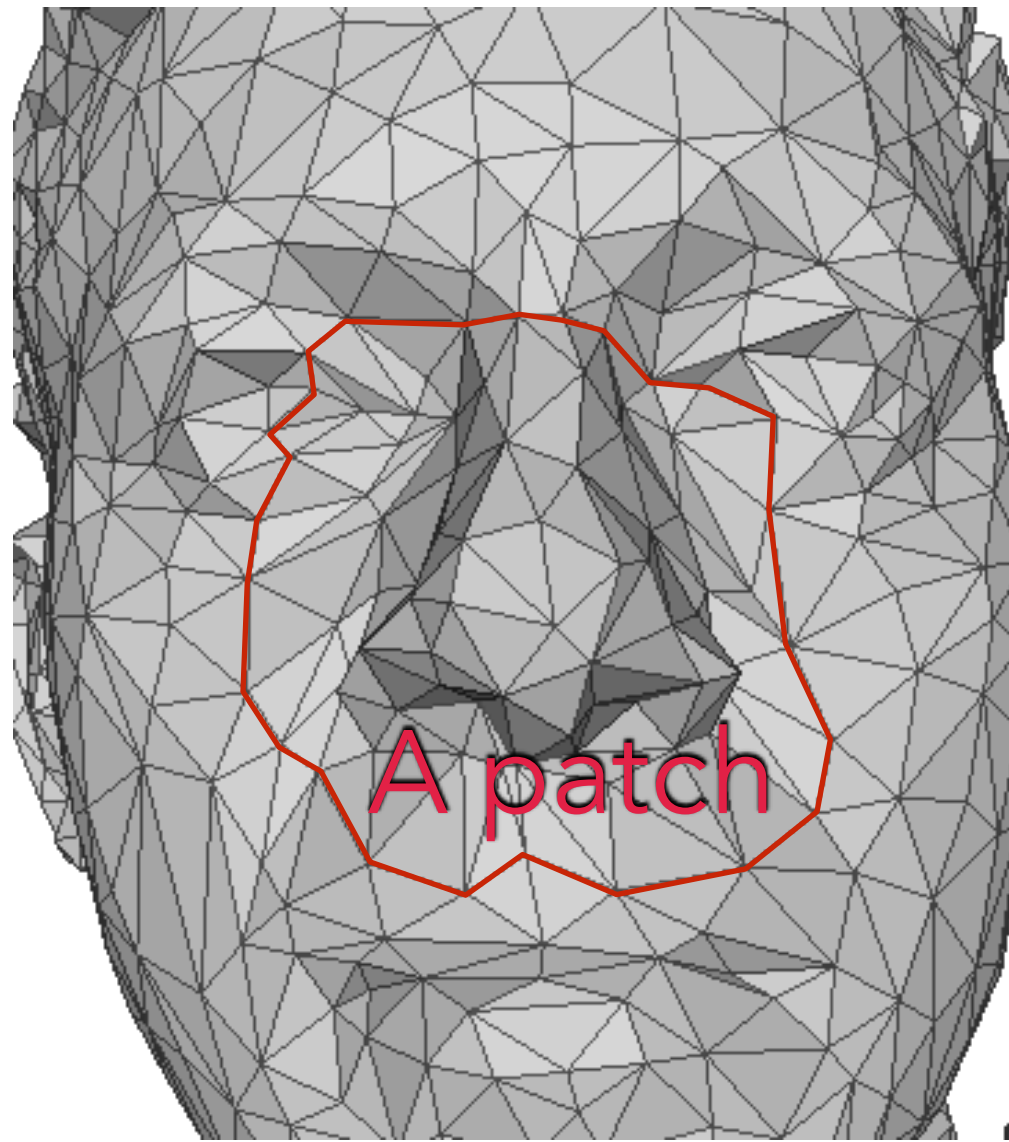


Shape
Preserving



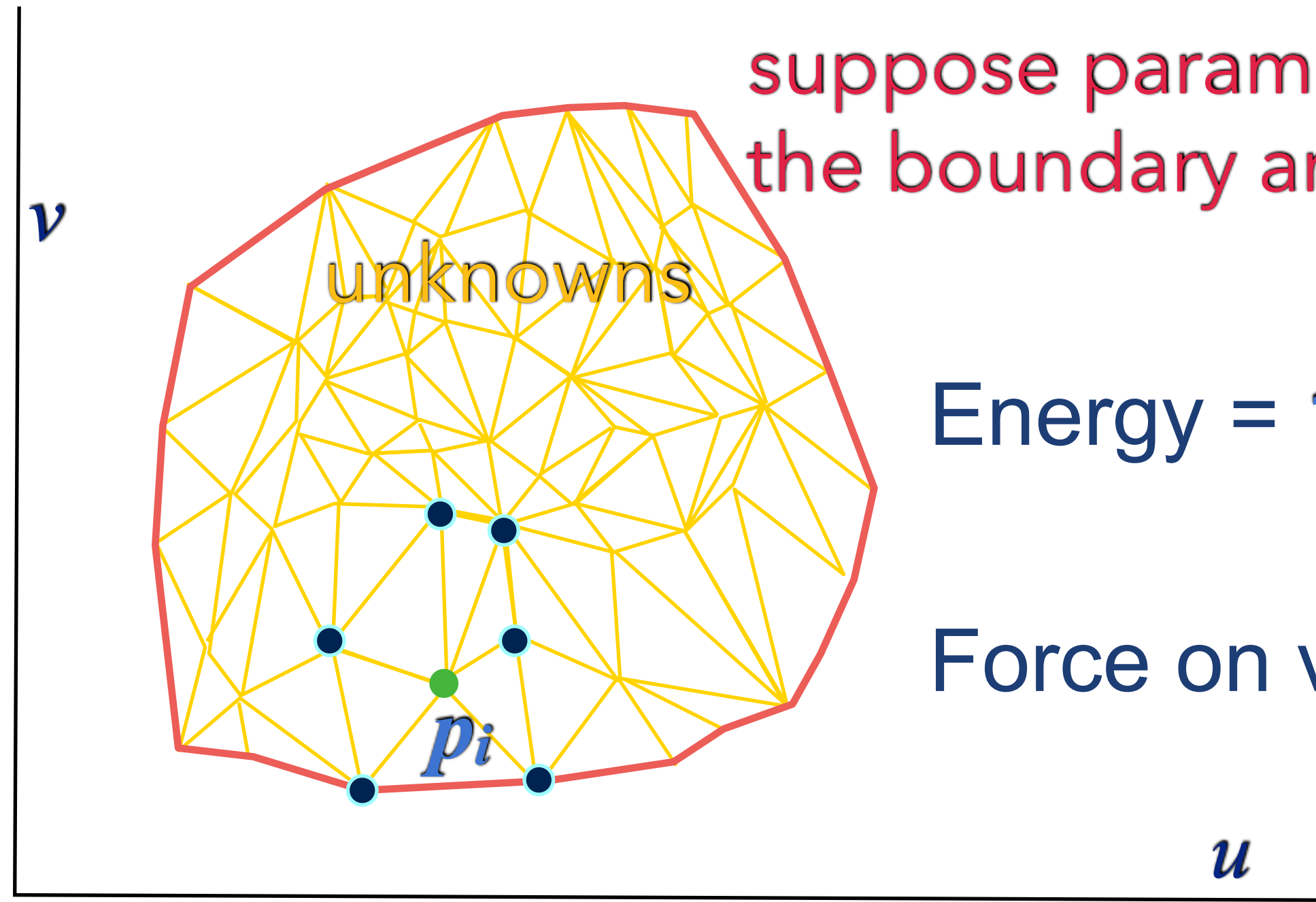
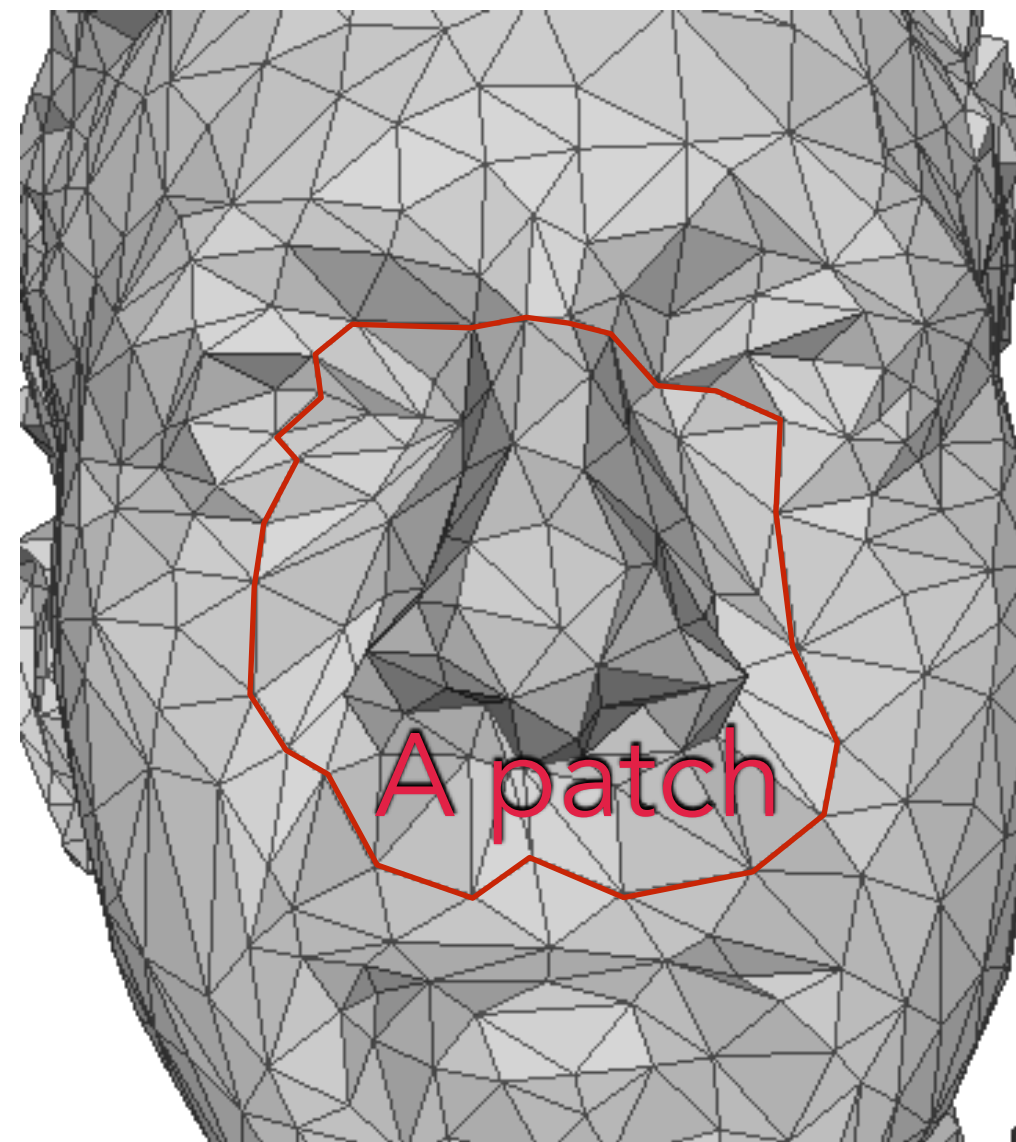
Mean Value

General Parameterization





General Parameterization



suppose parameters of the boundary are given

$$\text{Energy} = \frac{1}{2} \sum_{\text{all edges } ij} k_{ij} l_{ij}^2$$

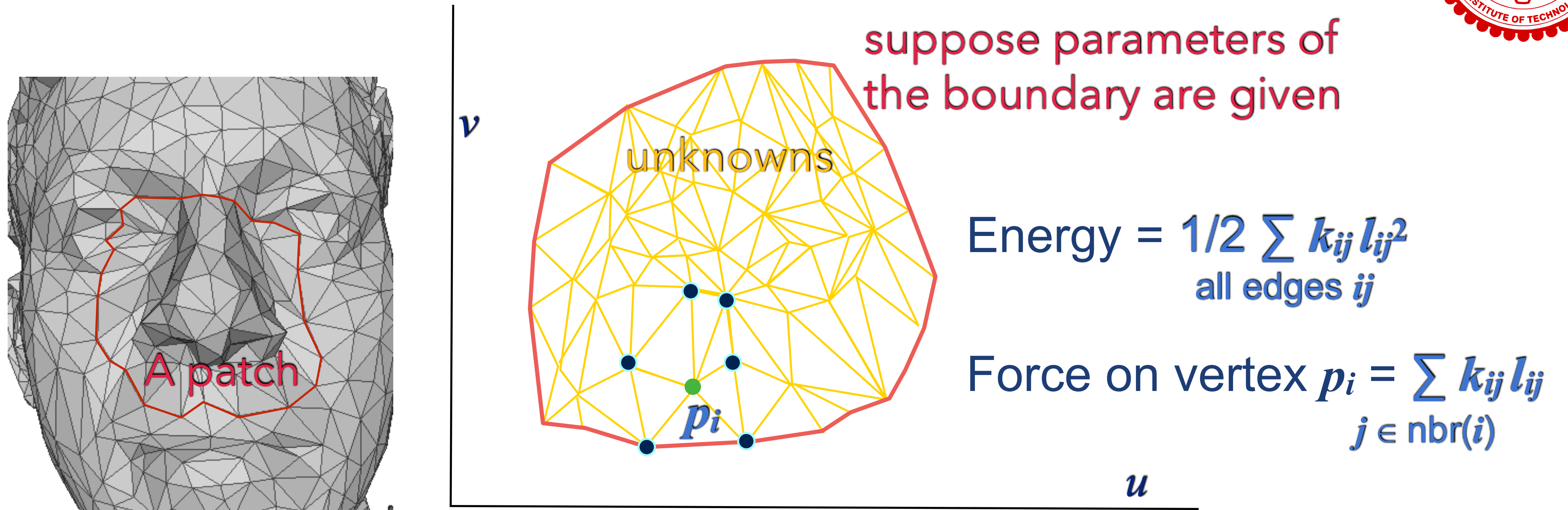
$$\text{Force on vertex } p_i = \sum_{j \in \text{nbr}(i)} k_{ij} l_{ij}$$

$$\text{Net Force on } p_i = \sum_{j \bullet \text{---} i} k_{ij} (u_i - u_j) = 0 \quad \Rightarrow \quad \sum k_{ij} u_i = \sum k_{ij} u_j$$

affine combination $\boxed{u_i = \sum \lambda_{ij} u_j} \quad \lambda_{ij} = k_{ij} / \sum k_{ij}$



General Parameterization



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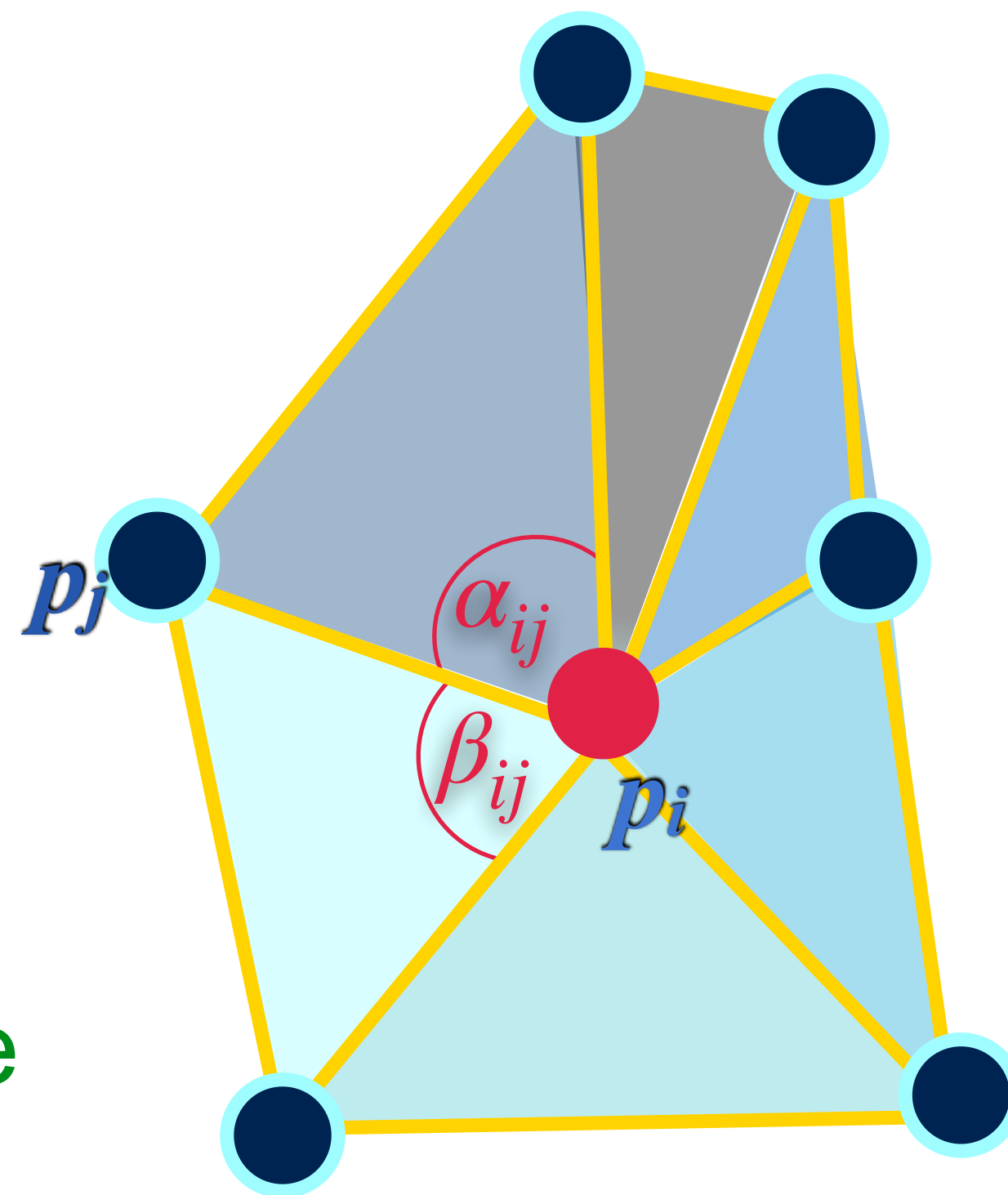
affine combination $u_i = \sum \lambda_{ij} u_j$ $\lambda_{ij} = k_{ij} / \sum k_{ij}$ (Barycentric map: $\lambda_{ij} = 1/\text{deg}_i$)

'Mean' Barycentric Coordinates

Mean value Coordinates

$$k_{ij} = \frac{\tan \frac{\alpha_{ij}}{2} + \tan \frac{\beta_{ij}}{2}}{|p_j - p_i|}$$

On the surface





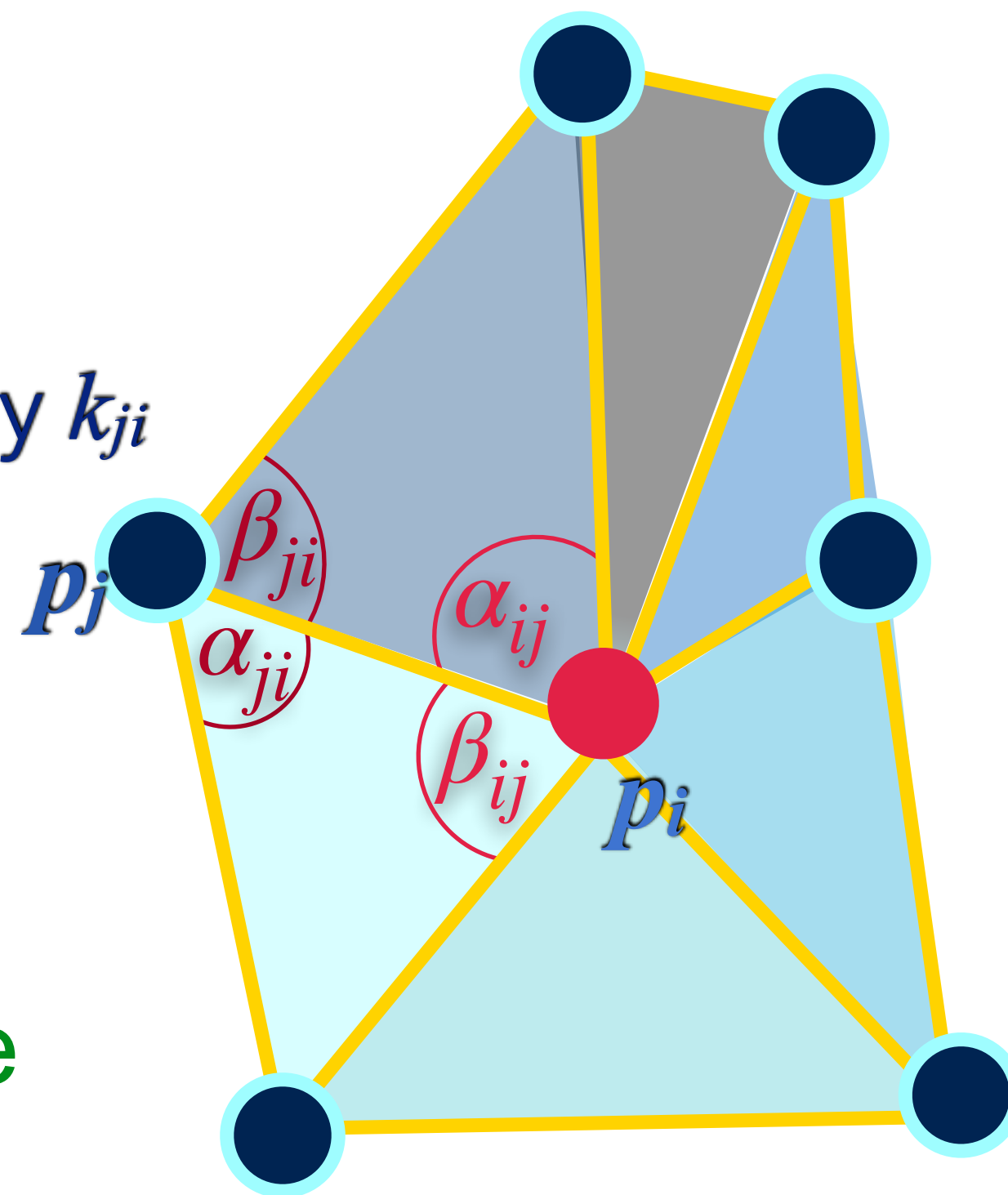
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similarly k_{ji}

On the surface

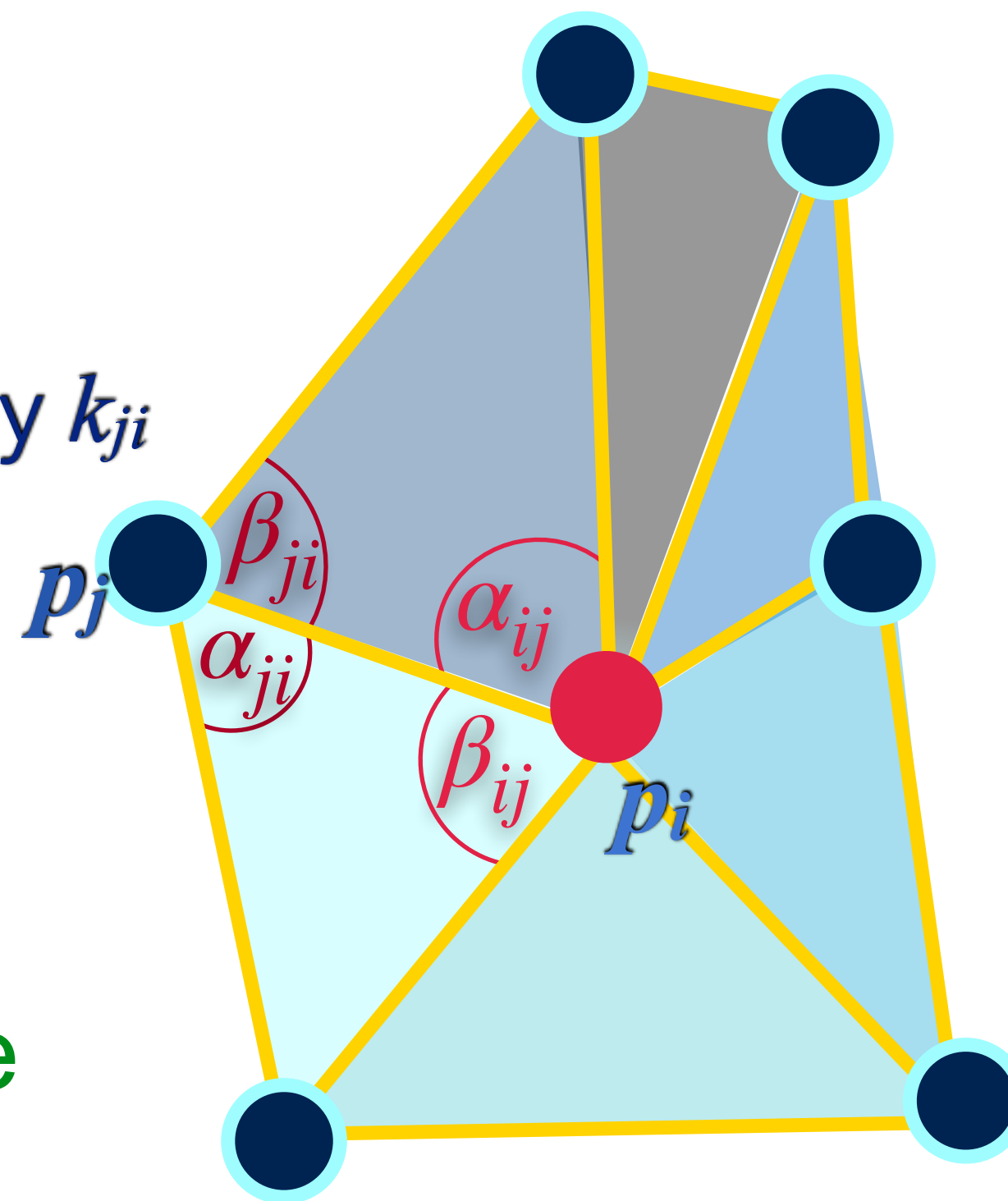


'Mean' Barycentric Coordinates

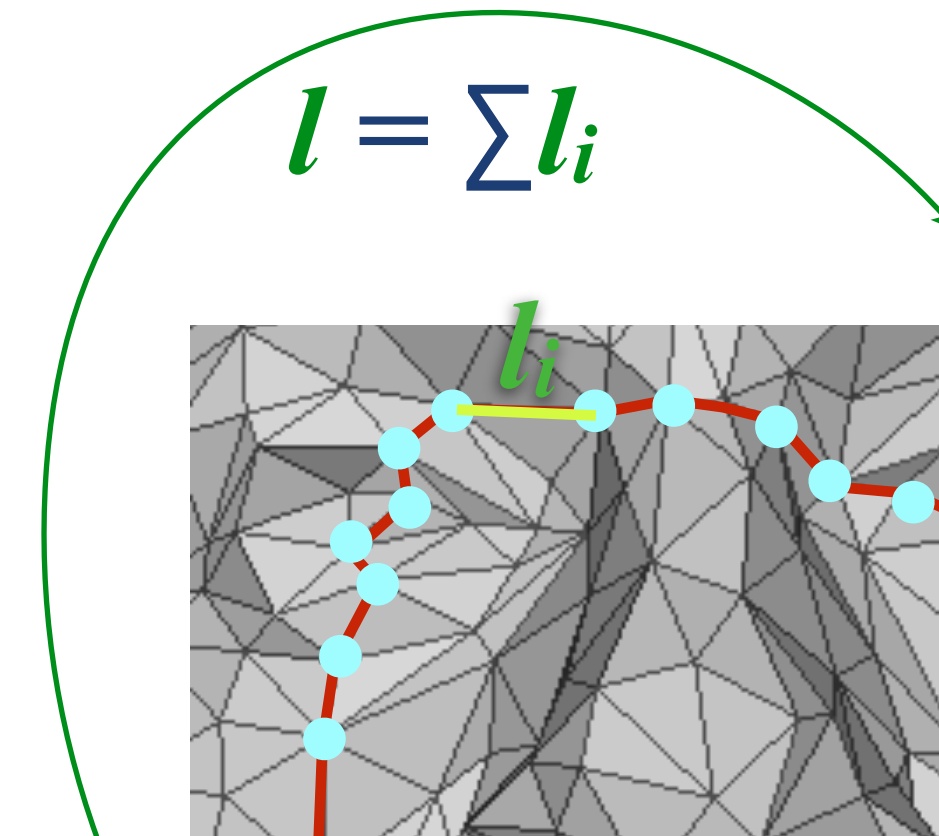
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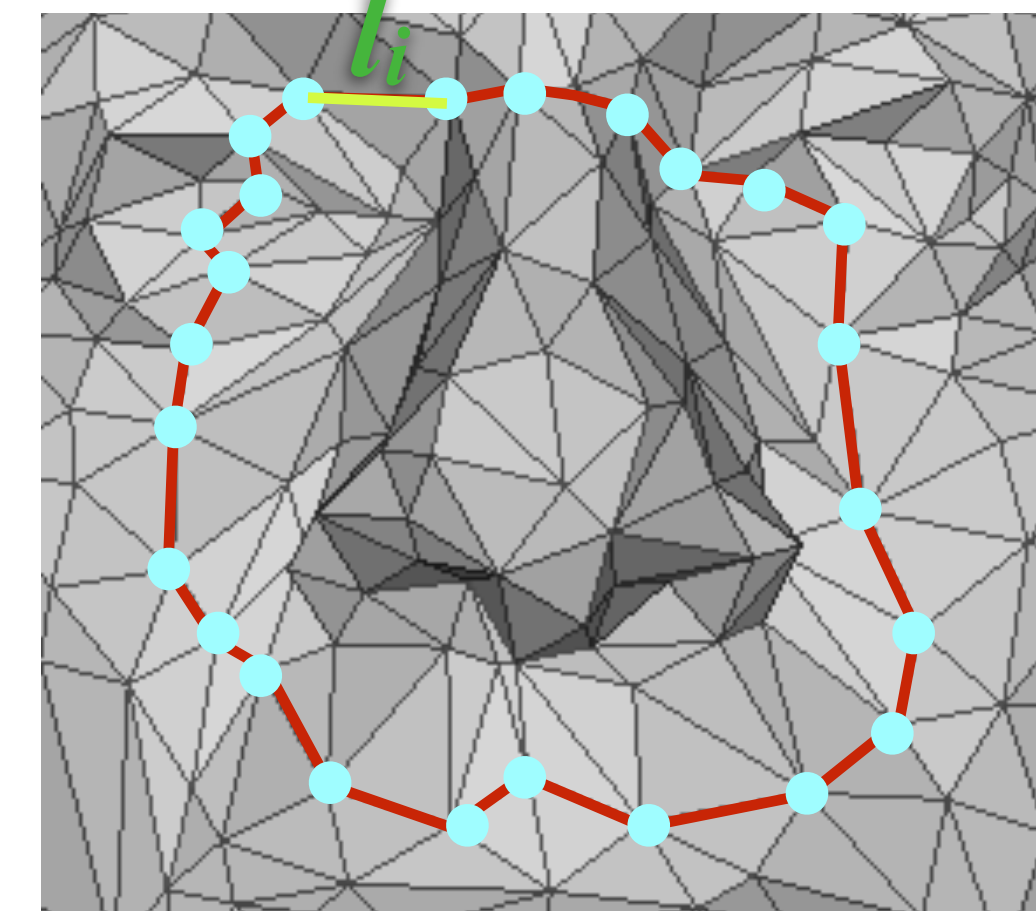


On the surface

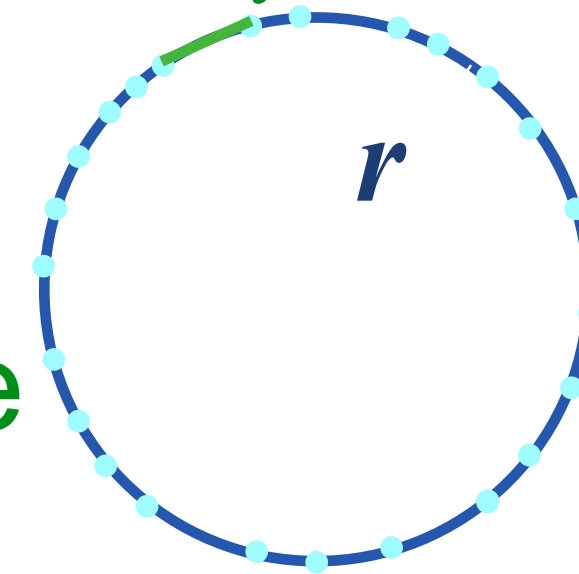


$$l = \sum l_i$$

Parametrize Boundary



$$2\pi r \frac{l_i}{l}$$



Fix shape

Parameterize by chord-length