

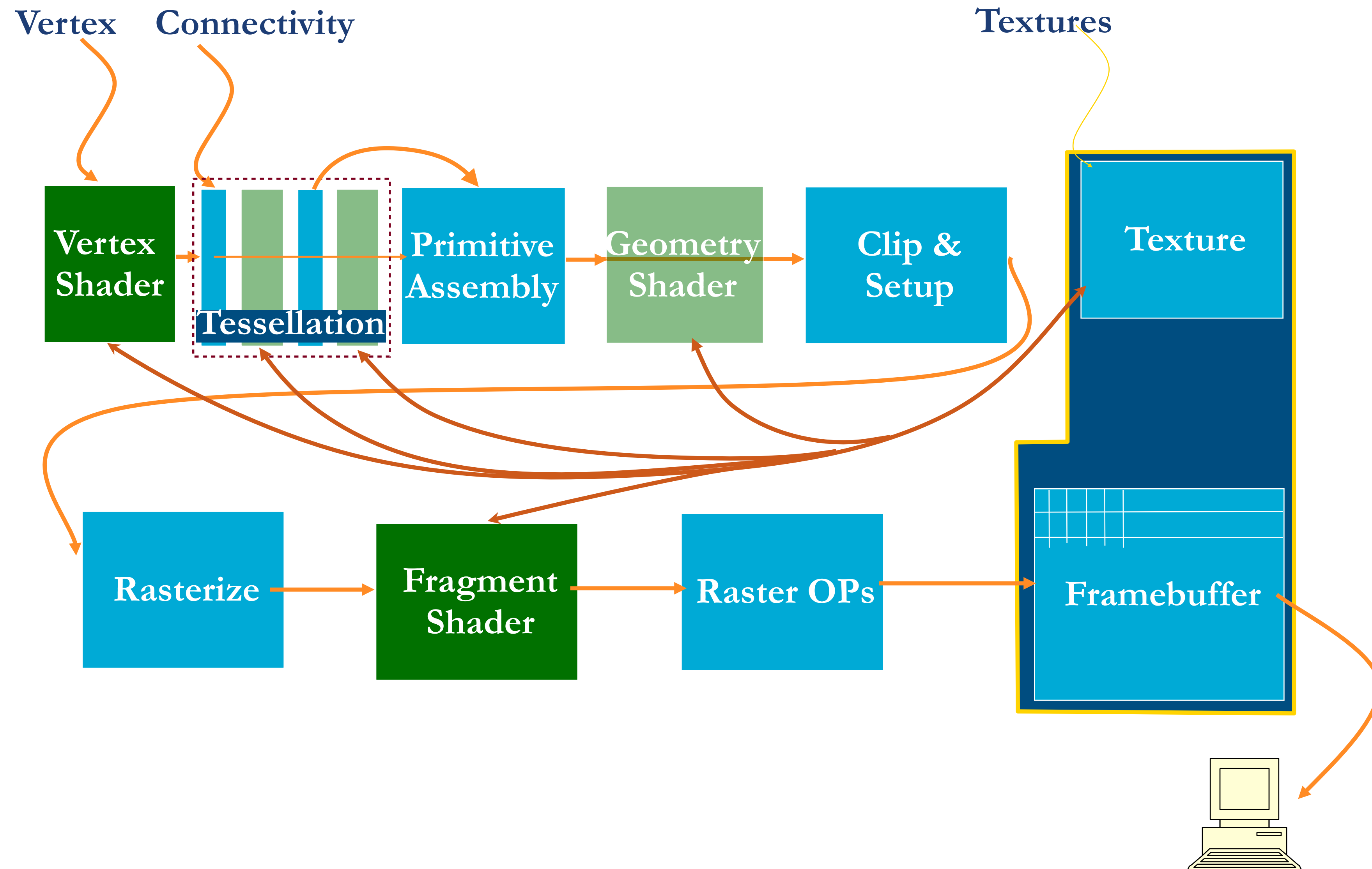
Computer Graphics

Attribute Interpolation

Subodh Kumar, Dept of Computer Sc. & Engg., IIT Delhi



Graphics Pipeline





Core Clipping Tasks

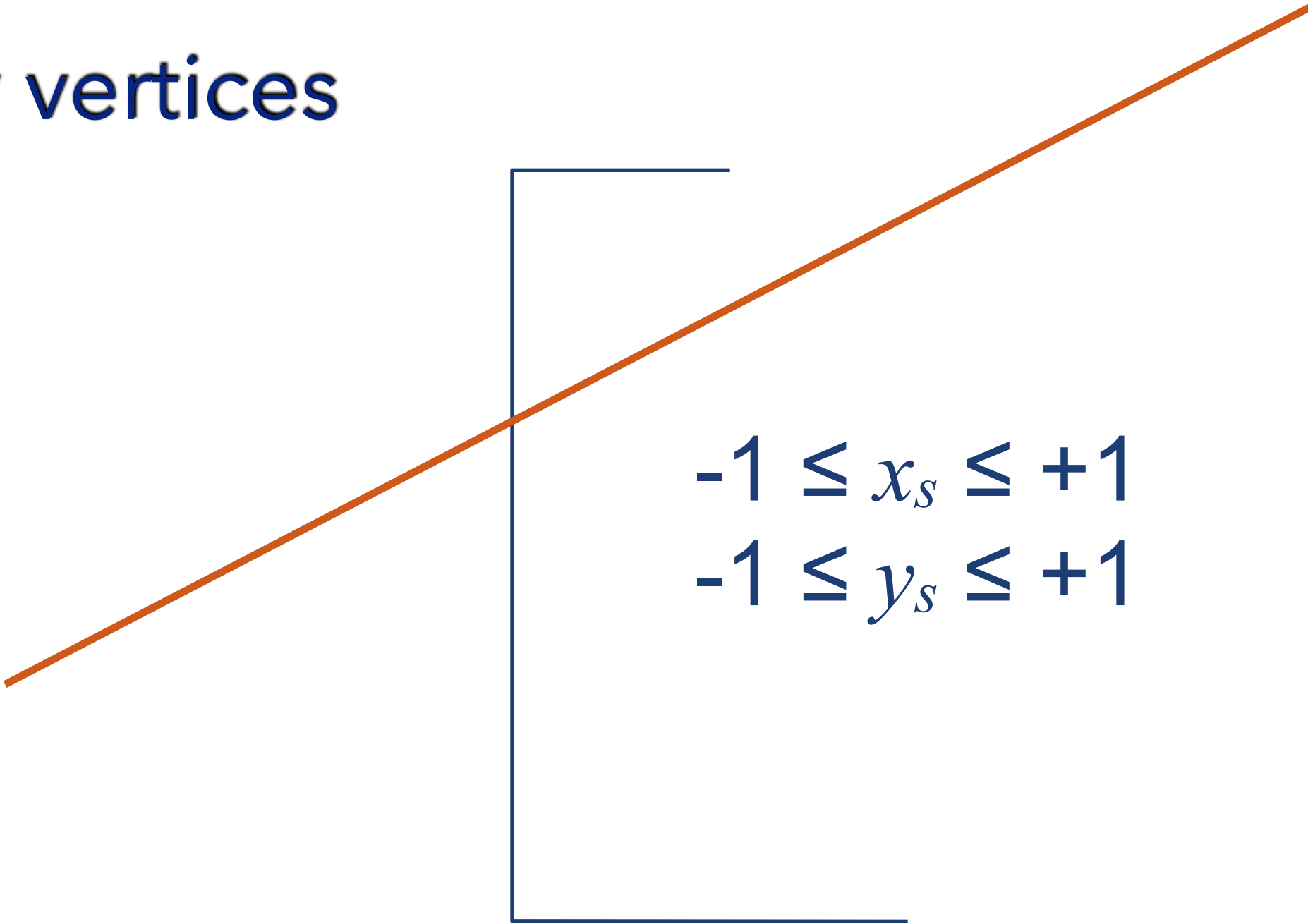
- Find vertices outside the window
- Replace outsiders with boundary vertices

➡ x, y

➡ z , other attributes

$$\begin{bmatrix} v' \end{bmatrix} = \begin{bmatrix} PMV \end{bmatrix} \begin{bmatrix} v \end{bmatrix}$$

$$\begin{aligned} x_s &= v_x' / v_w' \\ y_s &= v_y' / v_w' \\ z_s &= v_z' / v_w' \end{aligned}$$



The diagram shows a blue rectangular clipping window. An orange line passes through the window from the bottom-left to the top-right. To the right of the window, the normalized coordinates of the line are given as $-1 \leq x_s \leq +1$ and $-1 \leq y_s \leq +1$.

$$\begin{aligned} -1 &\leq x_s \leq +1 \\ -1 &\leq y_s \leq +1 \end{aligned}$$



Core Clipping Tasks

- Find vertices outside the window
- Replace outsiders with boundary vertices

➡ x, y

➡ z , other attributes

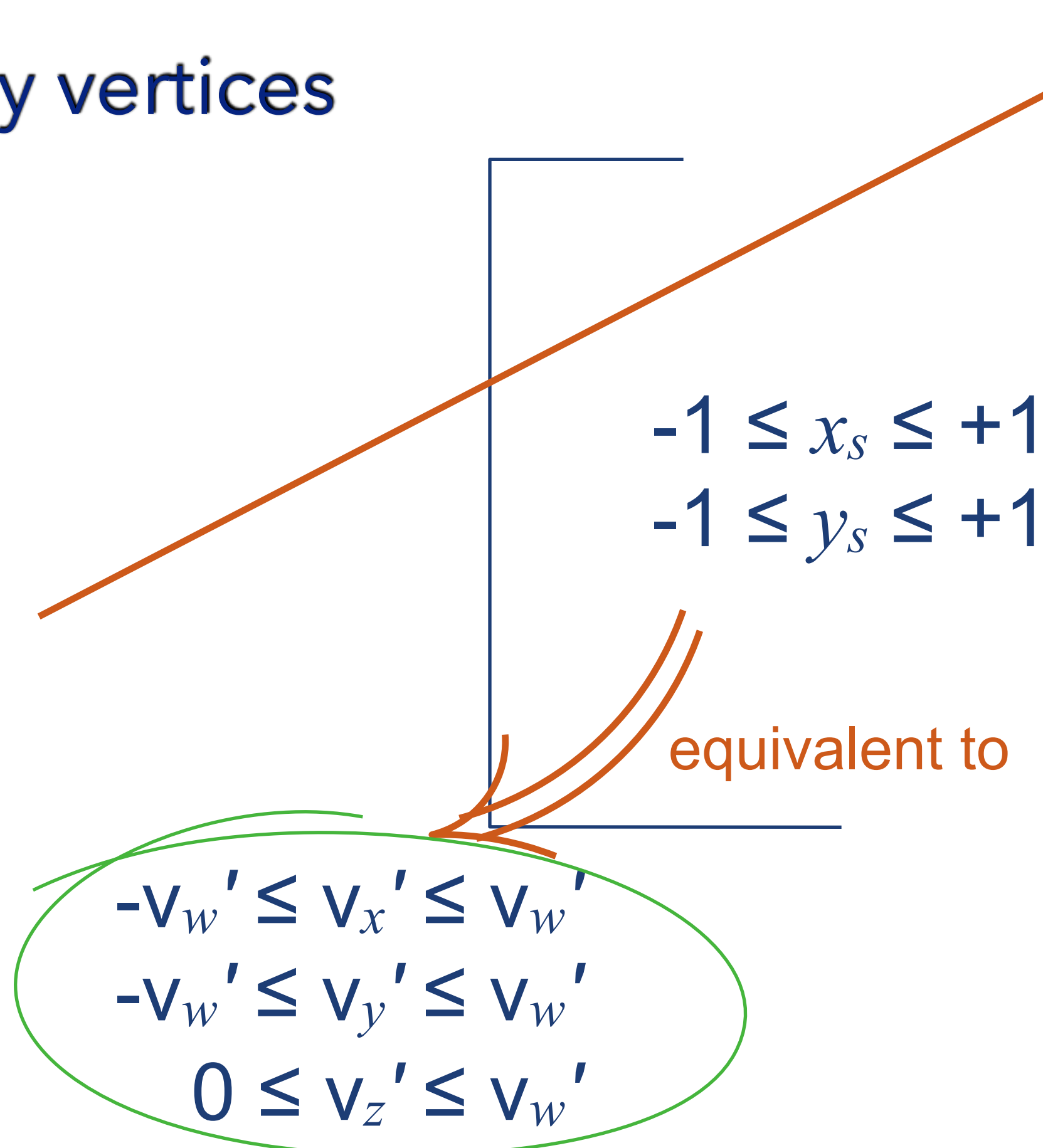
Clip Coordinates

$$\begin{bmatrix} v' \end{bmatrix} = \begin{bmatrix} PMV \end{bmatrix} \begin{bmatrix} v \end{bmatrix}$$

$$x_s = v_x' / v_w'$$

$$y_s = v_y' / v_w'$$

$$z_s = v_z' / v_w'$$





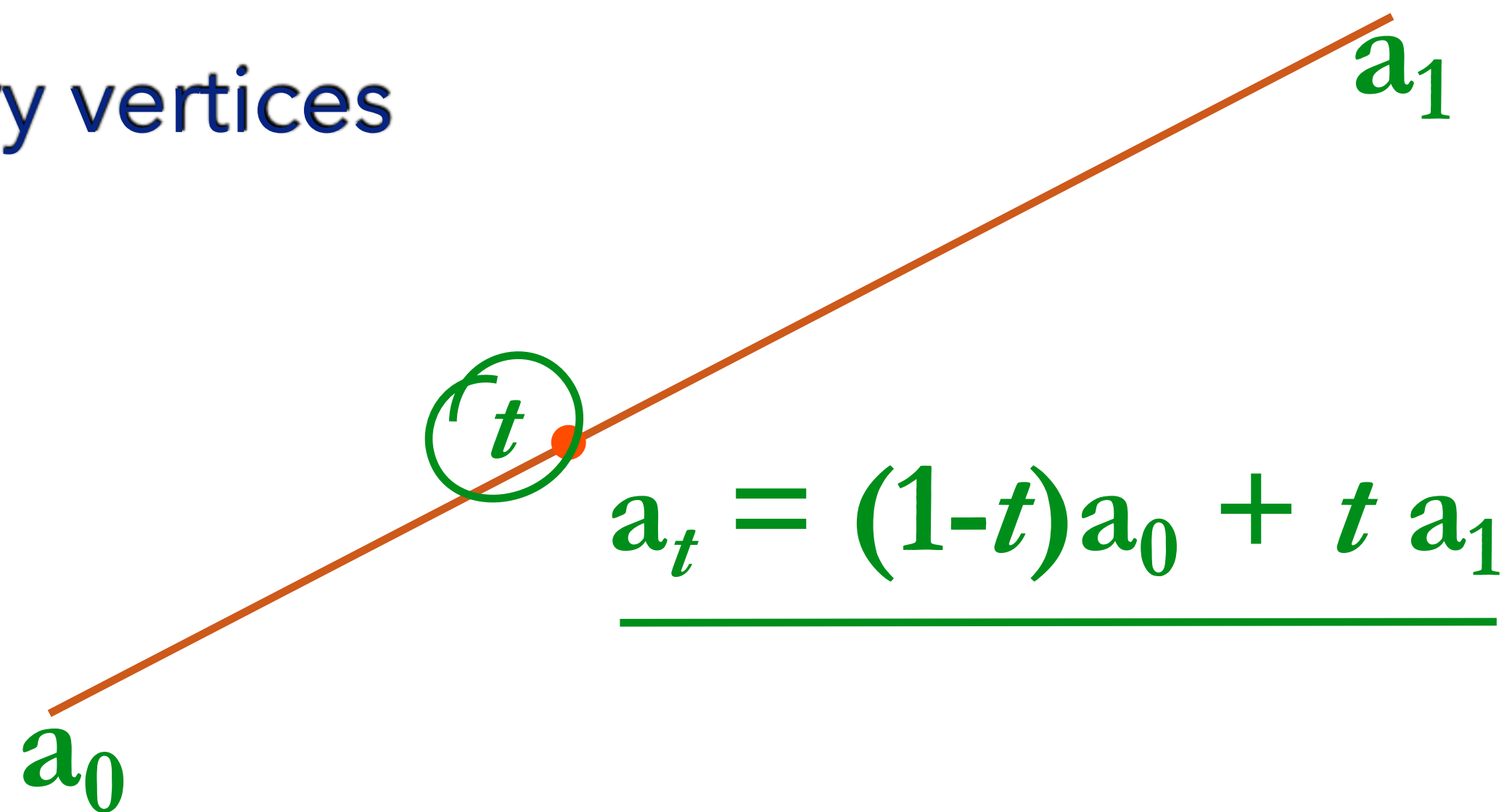
Core Clipping Tasks

- Find vertices outside the window
- Replace outsiders with boundary vertices

➡ x,y

➡ z, other attributes

$$\begin{bmatrix} v' \end{bmatrix} = \begin{bmatrix} PMV \end{bmatrix} \begin{bmatrix} v \end{bmatrix}$$



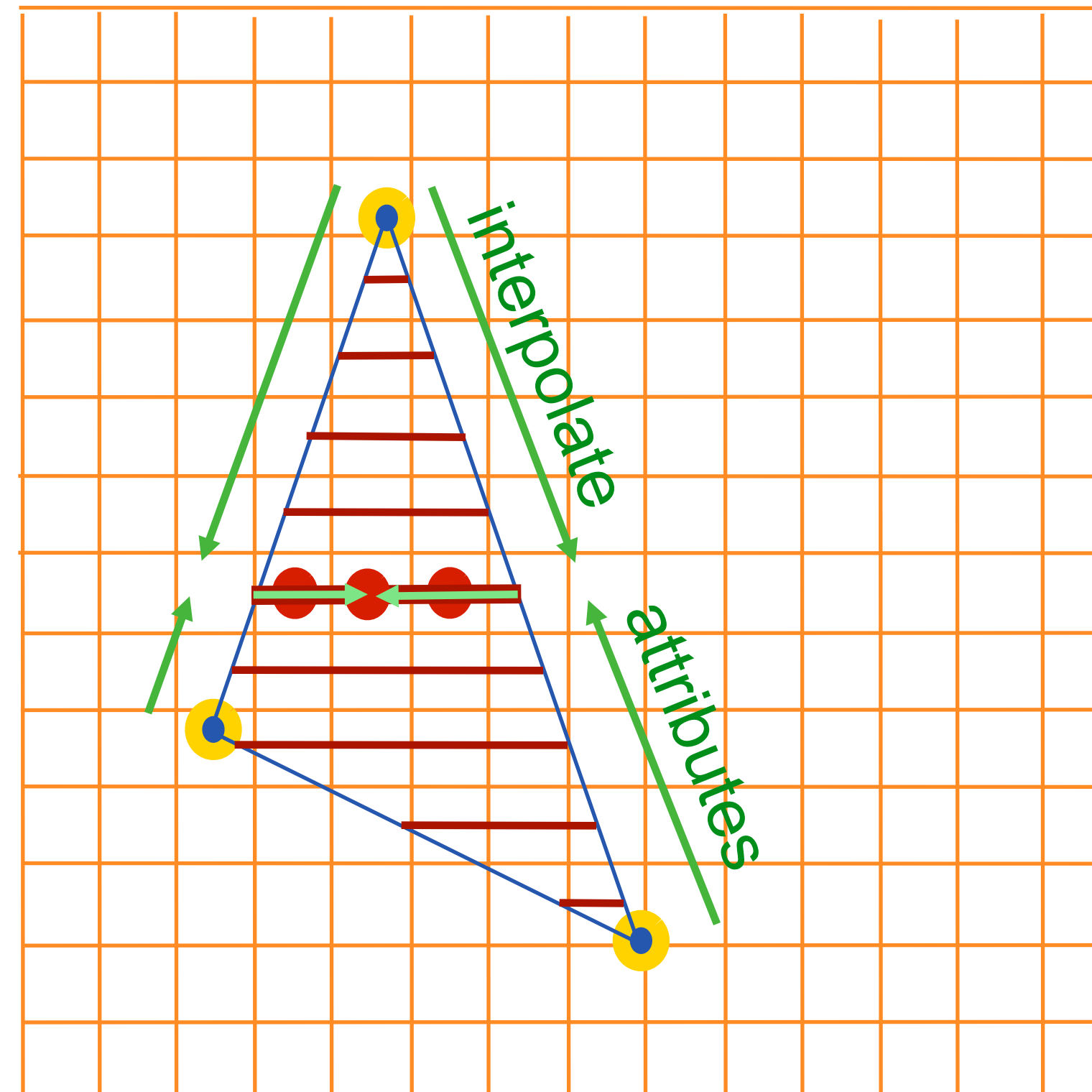
$$-v_w' \leq v_y' \leq v_w'$$

$$-v_w' \leq v_y' \leq v_w'$$



Basic Rasterization

- For scanline = lo to hi
 - ➡ Find Left End
 - ➡ Find Right End
 - ➡ Fill **span**
 - ▶ Interpolate **Attributes**
 - ▶ $a_1t + a_0(1-t)$
 - ▶ Call fragment shader

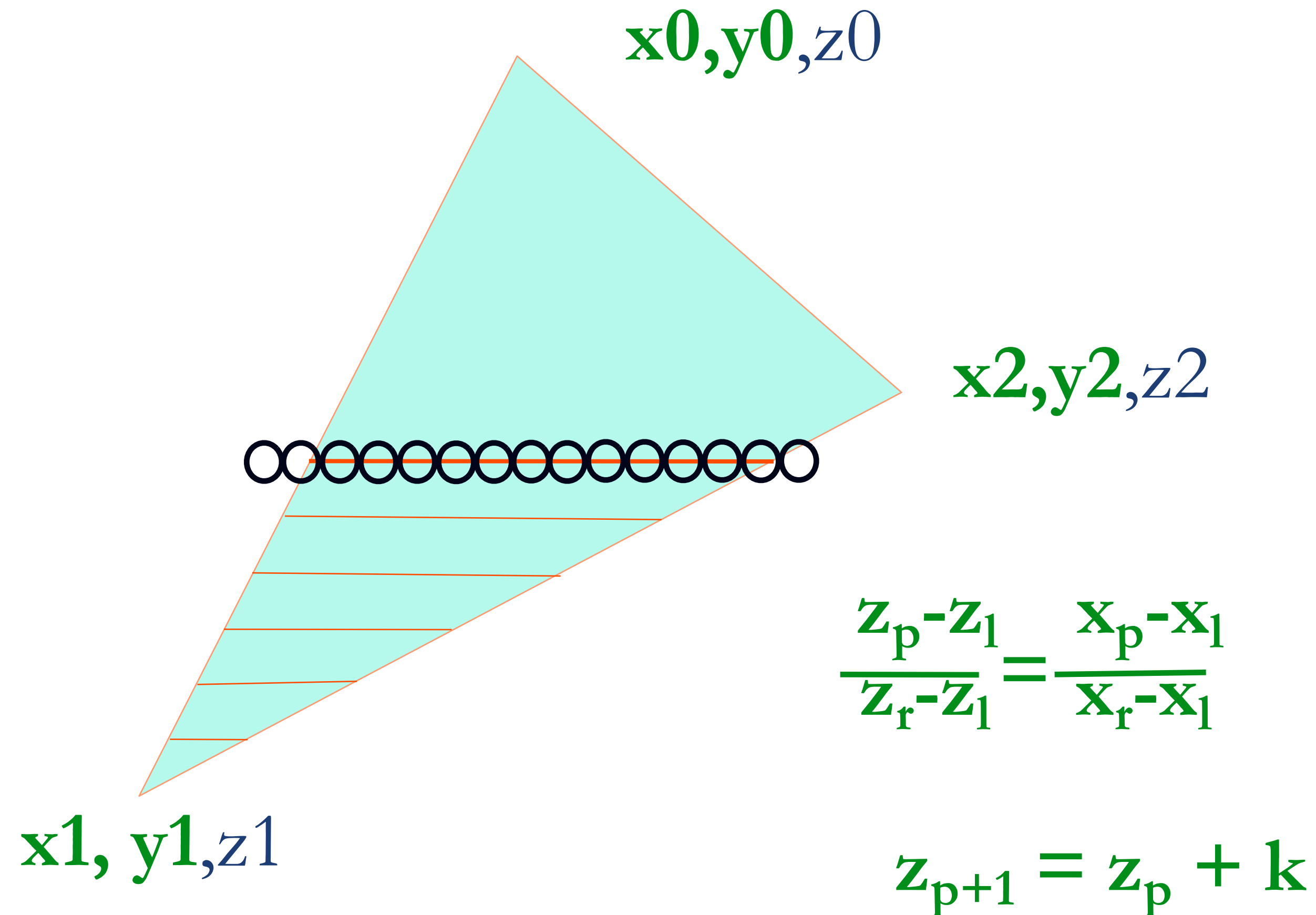




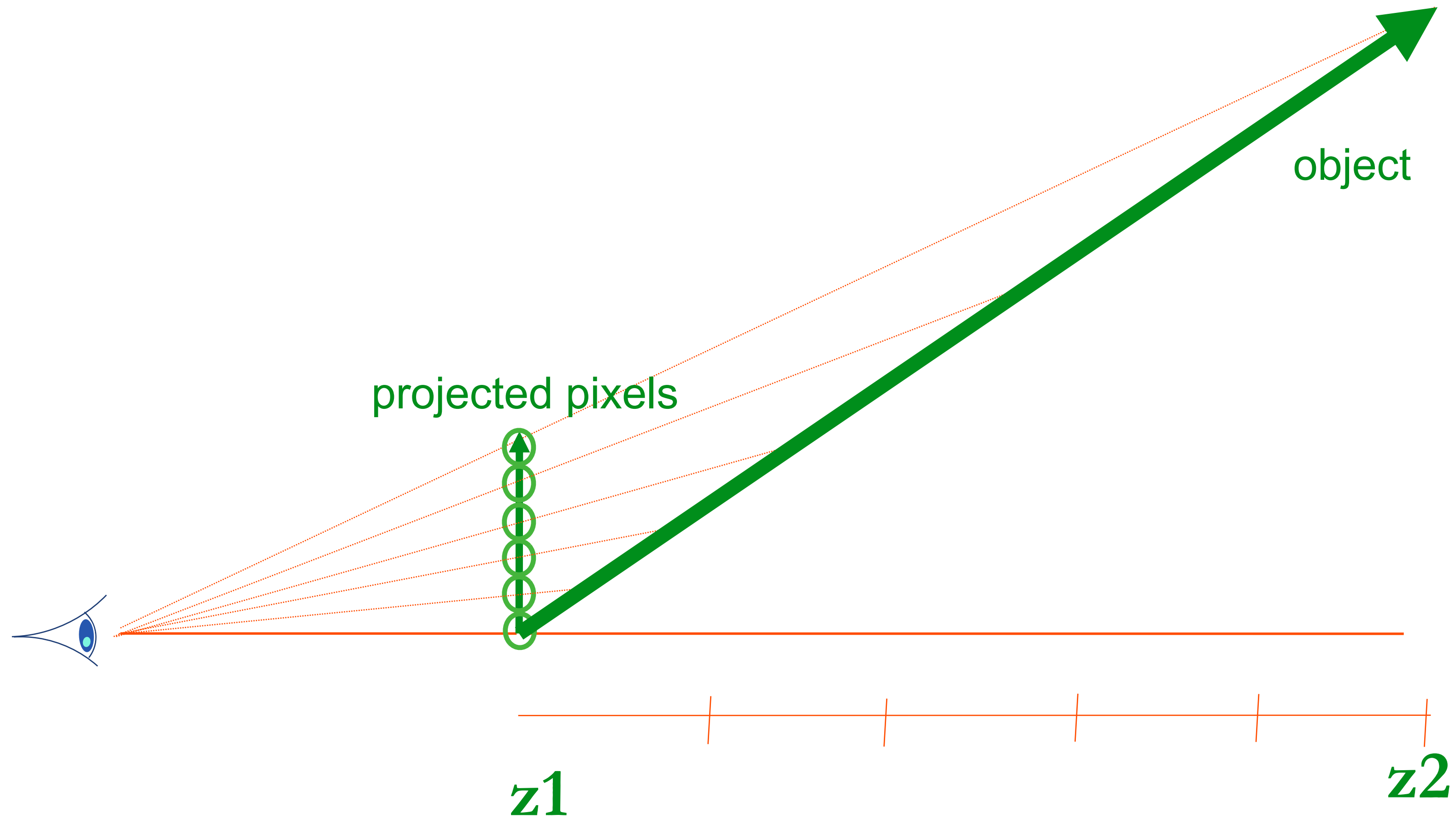
Attribute Interpolation

$$x = ay + b$$

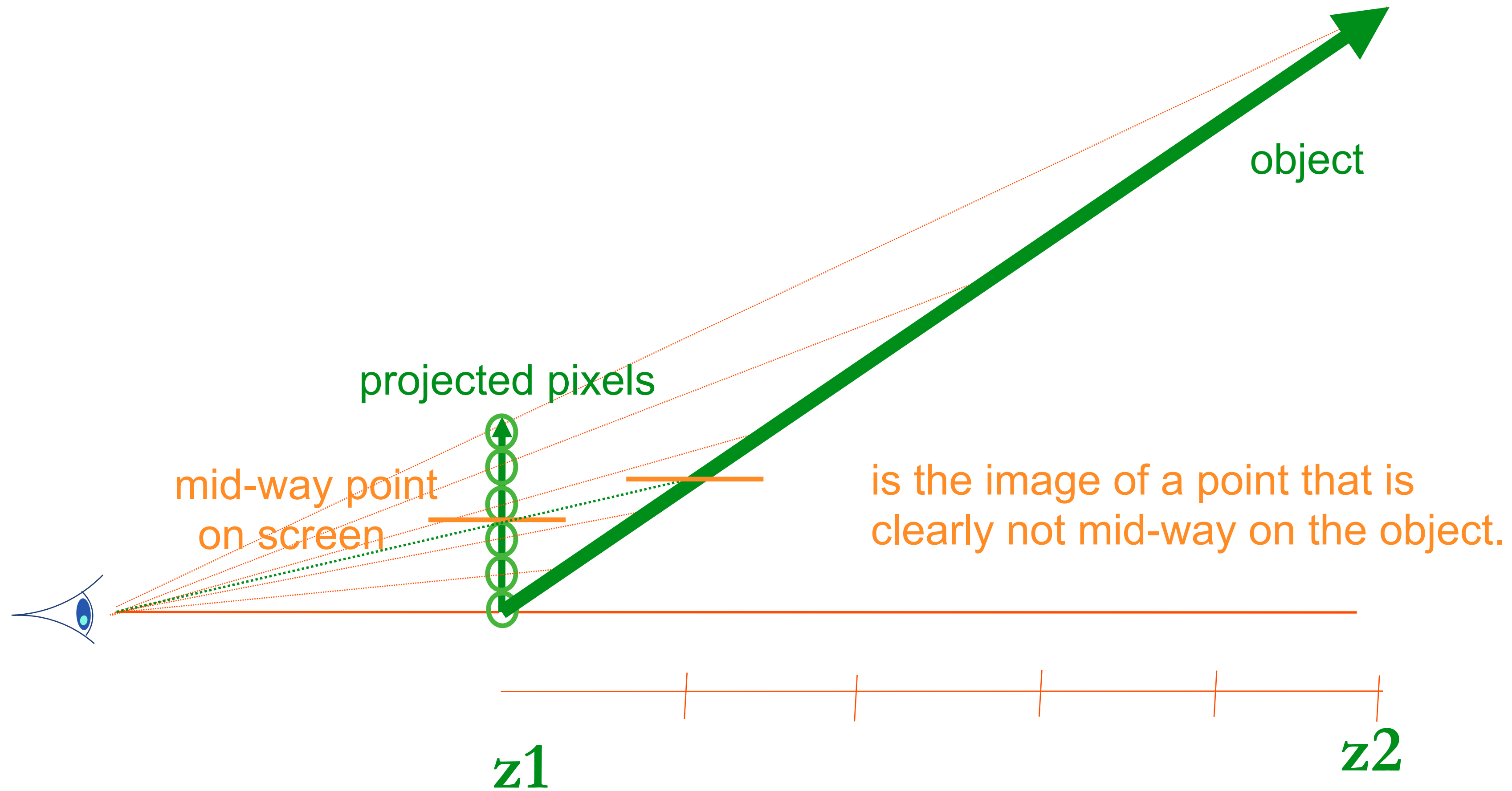
$$\begin{aligned}x_{+1} &= a(y+1) + b \\ &= x + a\end{aligned}$$



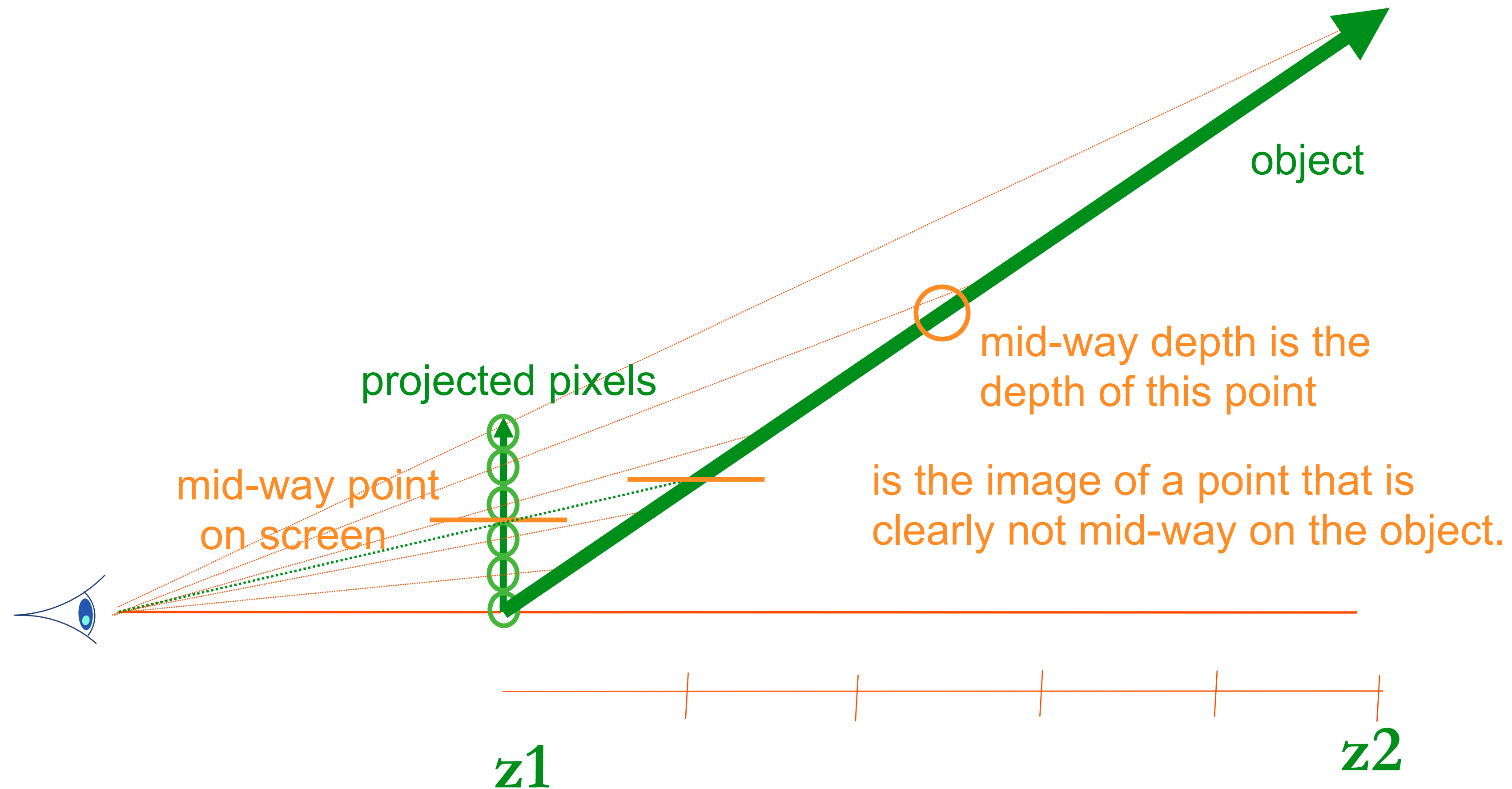
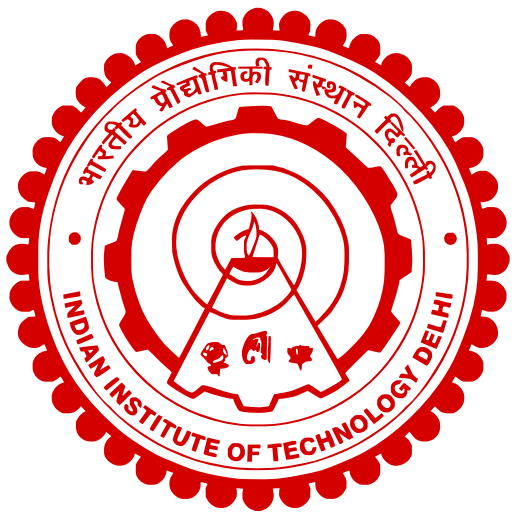
Perspective Distortion



Perspective Distortion

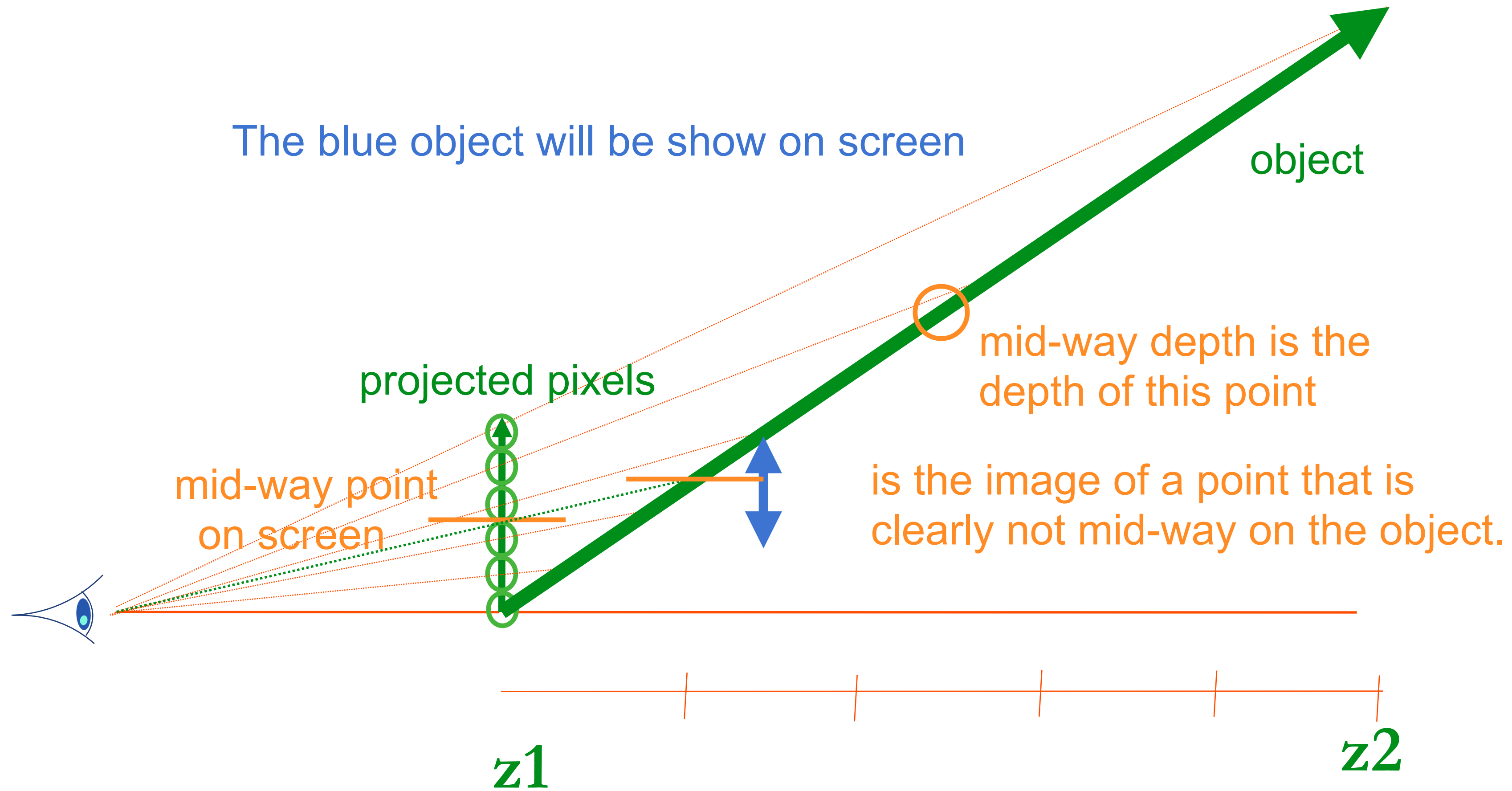


Perspective Distortion

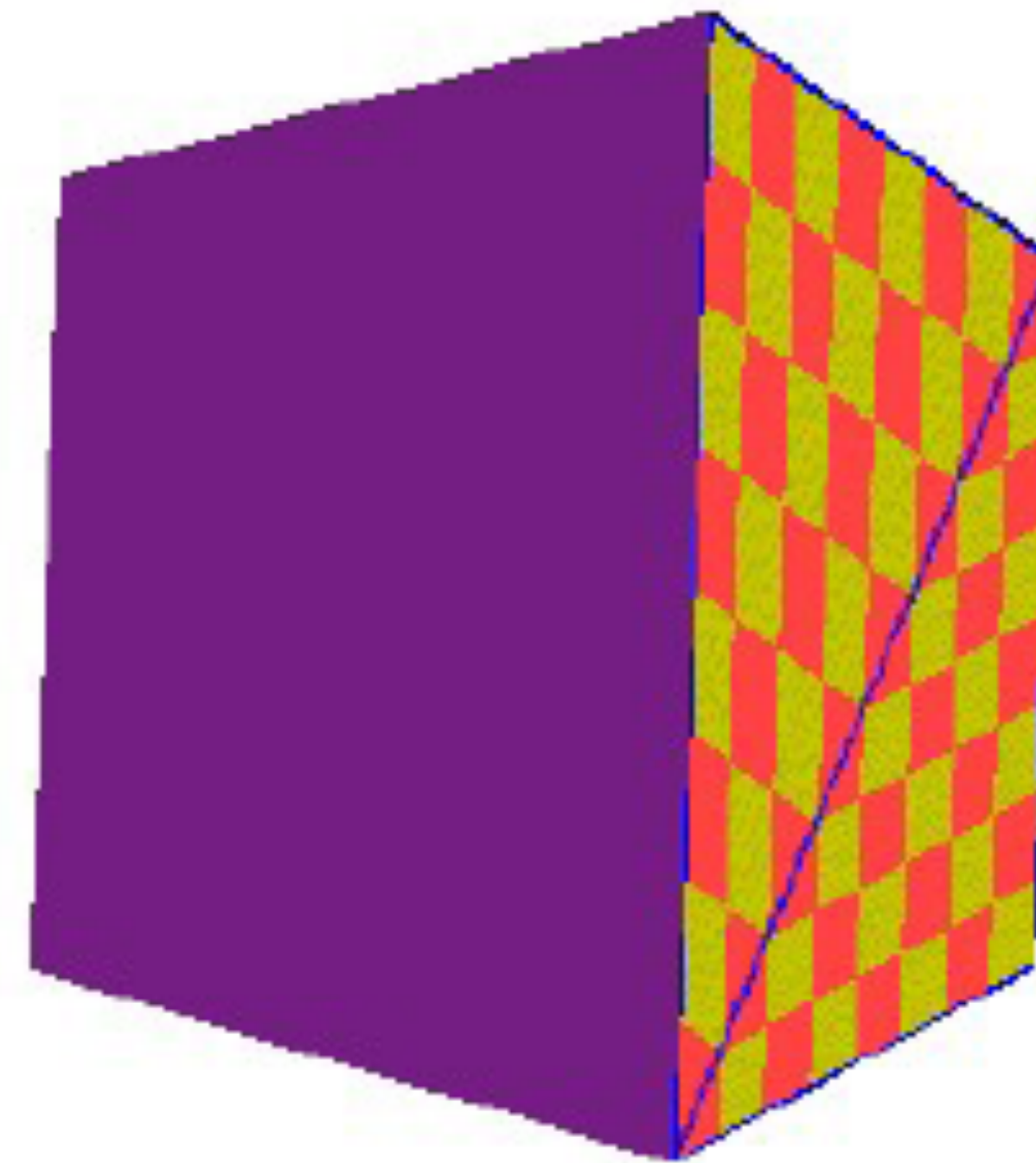
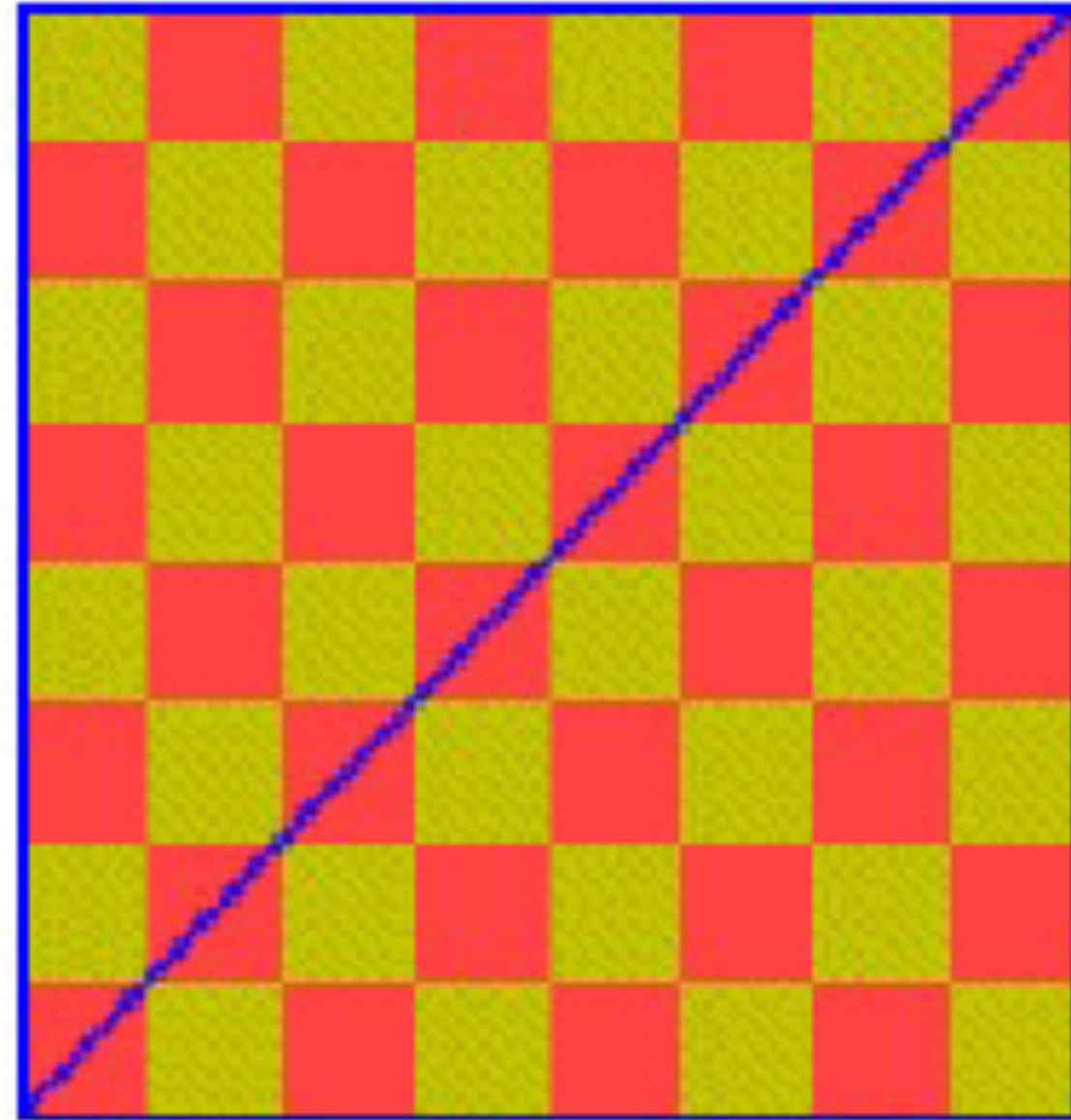




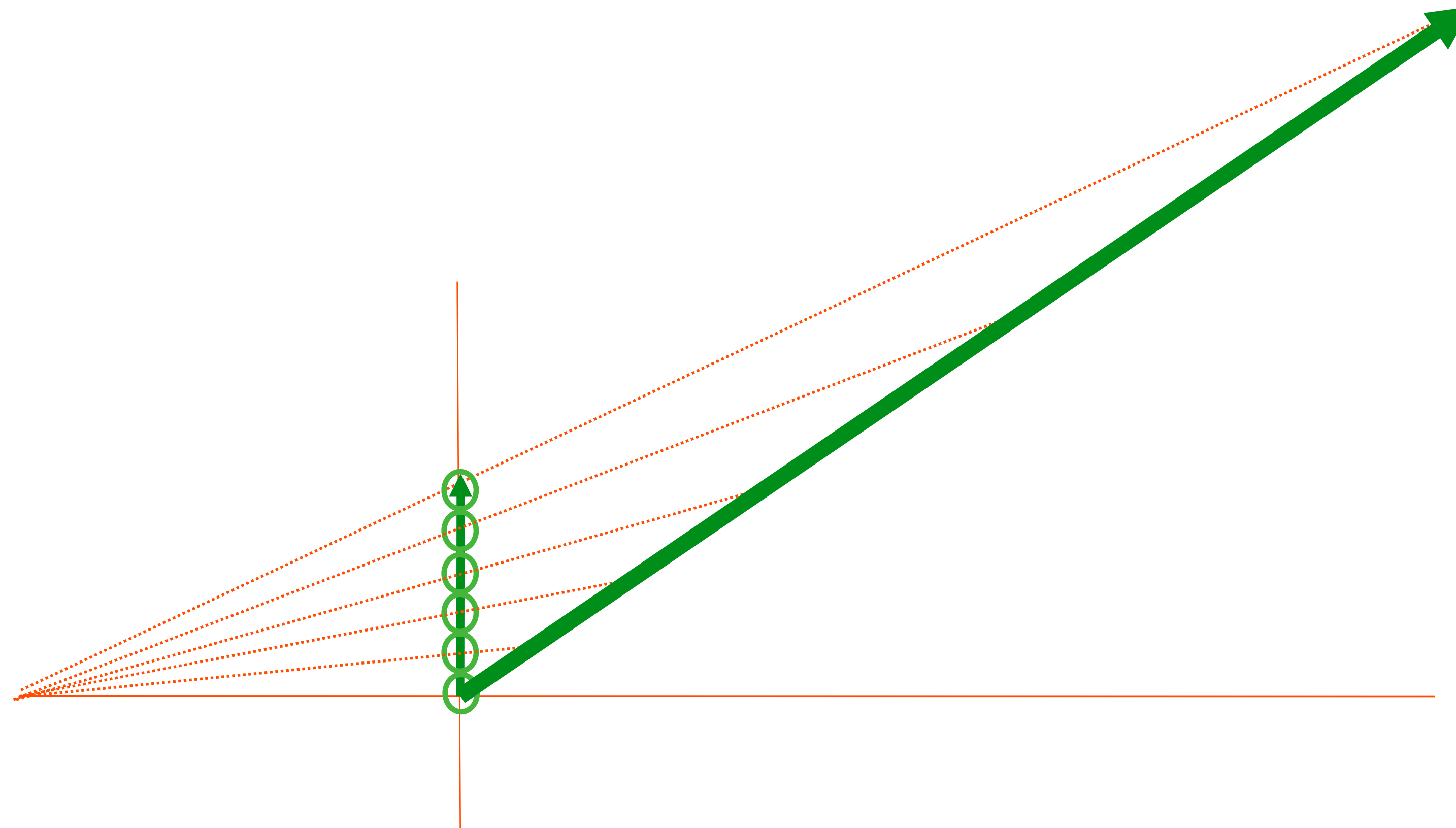
Perspective Distortion



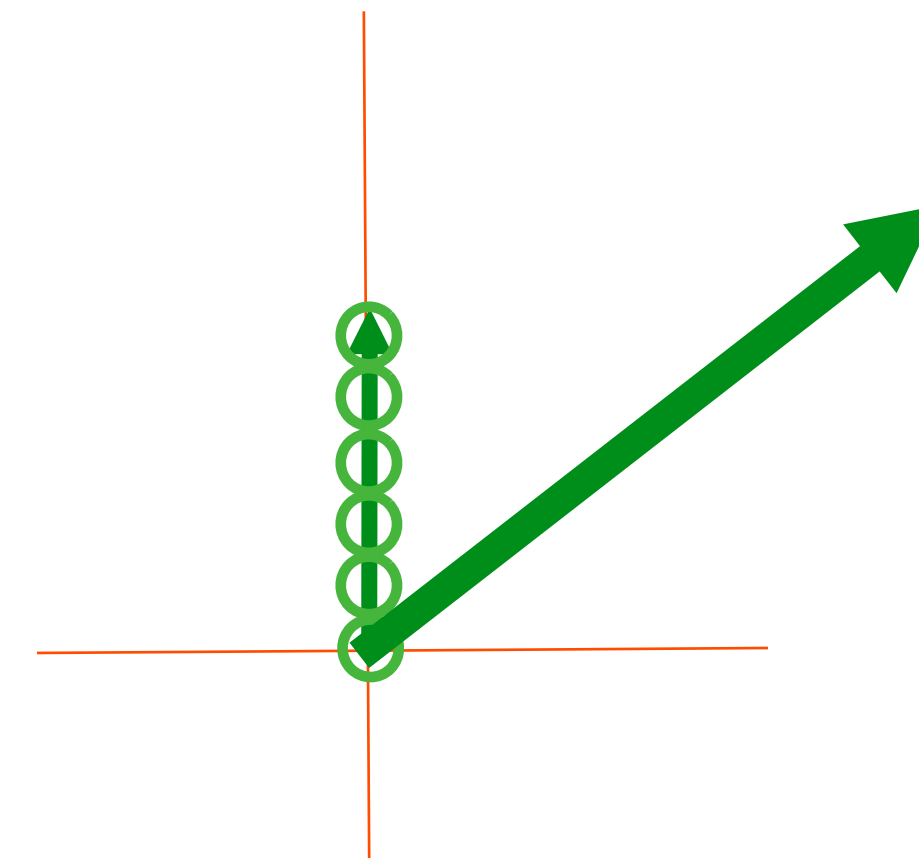
Perspective Distortion



Perspective Distortion



Perspective Correction





Perspective Correction

- Linear interpolation:

$$\Rightarrow p(t) = p_1 + (p_2 - p_1)t, 0 \leq t \leq 1$$

- Screen space interpolation:

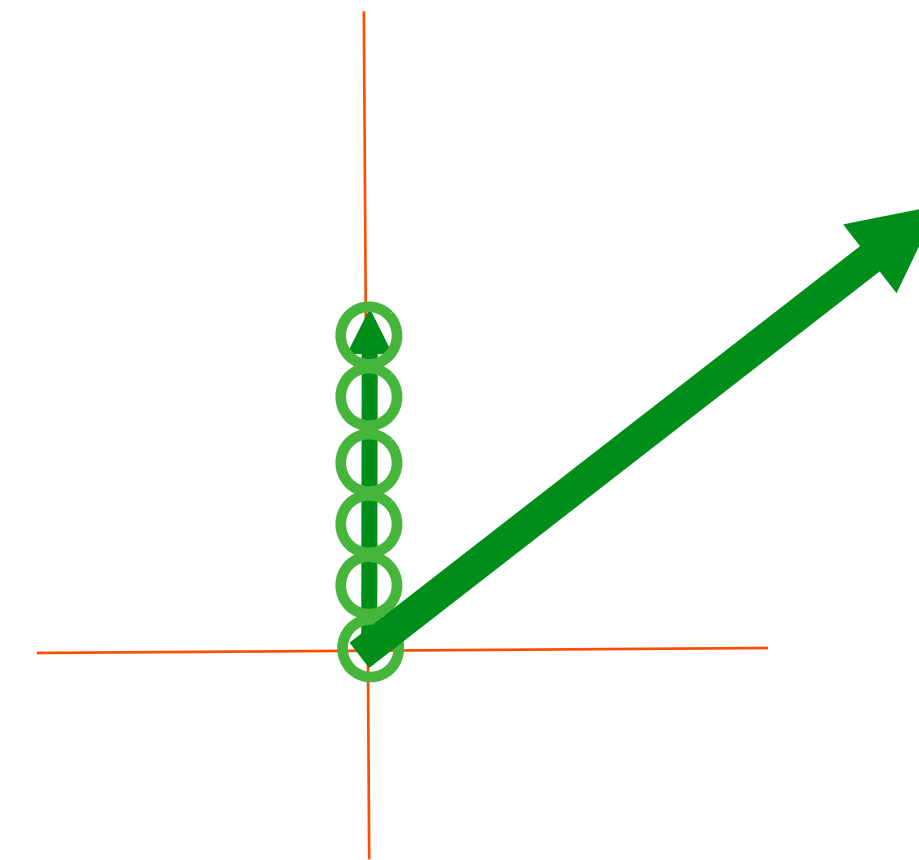
$$y^s = y_1^s + t(y_2^s - y_1^s)$$

$$y/z = y_1/z_1 + t(y_2/z_2 - y_1/z_1)$$

- But we know:

$$\frac{y = y_1 + u(y_2 - y_1)}{z = z_1 + u(z_2 - z_1)} = \frac{y_1}{z_1} + t \left(\frac{y_2}{z_2} - \frac{y_1}{z_1} \right)$$

u is the linear parameter in object space,
 t is the required parameter in screen space





Perspective Correction

$$\frac{y_1}{z_1} + t \left(\frac{y_2}{z_2} - \frac{y_1}{z_1} \right) = \frac{y_1 + u(y_2 - y_1)}{z_1 + u(z_2 - z_1)}$$

$$\Rightarrow u = t \frac{z_1}{z_2 + t(z_1 - z_2)}$$

Since $p = p_1 + u(p_2 - p_1)$, $p = p_1 + t \left(\frac{z_1}{z_2 + t(z_1 - z_2)} \right) (p_2 - p_1)$

$$= \frac{p_1 / z_1 + t(p_2 / z_2 - p_1 / z_1)}{1/z_1 + t(1/z_2 - 1/z_1)}$$



Using Perspective Correct Z

- Compute z^{-1} for each vertex in clip-space
- Form a plane of z^{-1}
 - ⇒ such that $z^{-1}(x, y) = a_z x + b_z y + c_z$
- Similarly, for any attribute p
 - ⇒ compute pz^{-1} for each vertex
 - ⇒ Form plane for attribute p
 - such that $pz^{-1}(x, y) = a_p x + b_p y + c_p$
- Perspective correct p at $(x, y) = pz^{-1}(x, y)/z^{-1}(x, y)$

$$z = z_1 + u(z_2 - z_1)$$

$$= z_1 + t \frac{z_1}{z_2 + t(z_1 - z_2)} (z_2 - z_1)$$

$$\frac{1}{z} = \frac{1}{z_1} + t \left(\frac{1}{z_2} - \frac{1}{z_1} \right)$$

Perspective Corrected

