

Computer Graphics

Illumination

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Recall

Irradiance H = Power per unit area

Radiance L = Irradiance per unit solid-angle

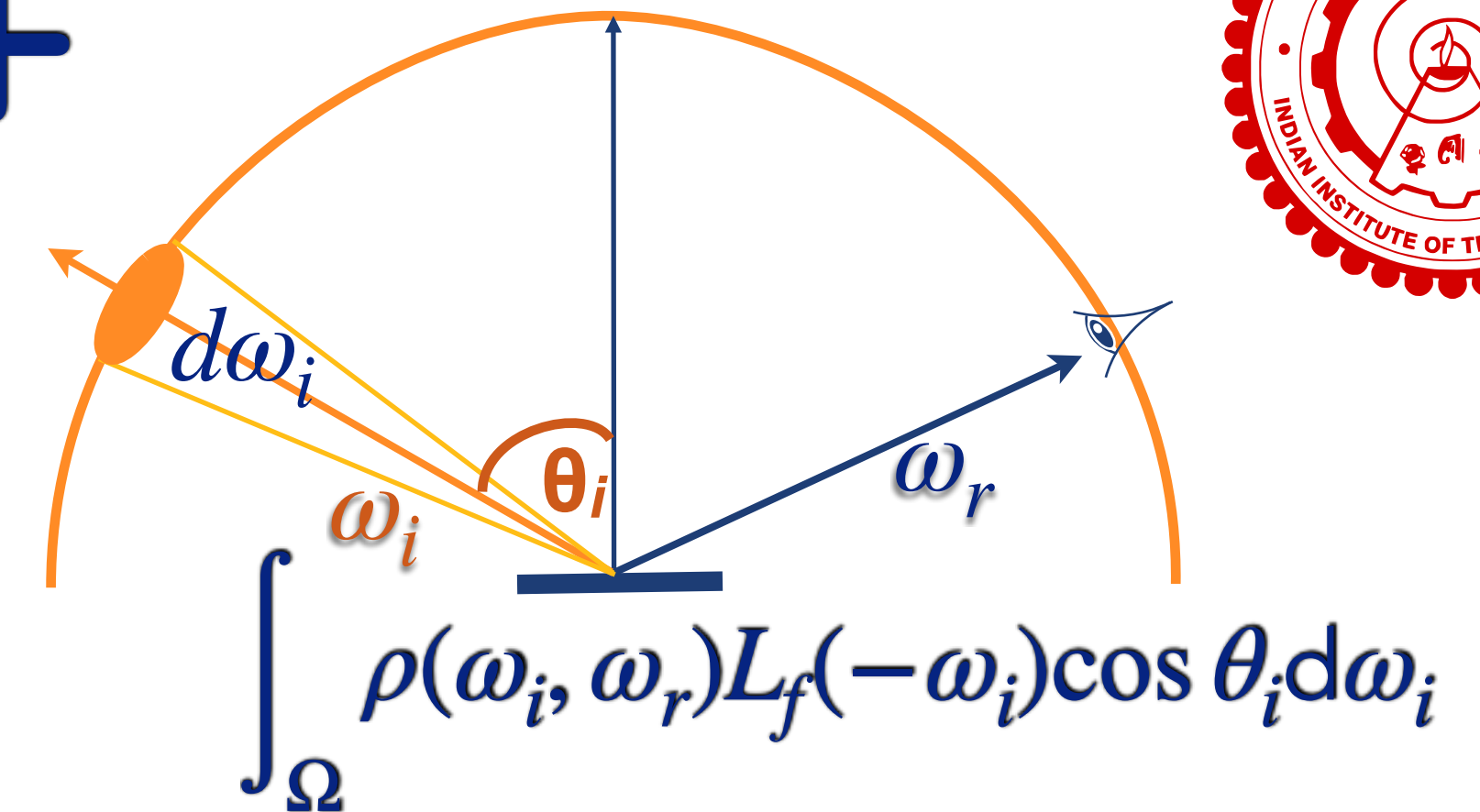
$$\text{BRDF } \rho = \frac{\text{Outgoing Radiance}}{\text{Incoming Irradiance}}$$

$$\begin{aligned} \text{Reflected Power from } \Delta A \text{ on surface due to } H_i &= \Delta A \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} H_i \rho \sin \theta d\theta d\phi \\ &= \Delta A \pi C, \text{ if } \rho = C \\ &\text{for all reflected directions} \end{aligned}$$

If 100% of incident power is reflected, $C = \frac{1}{\pi}$



Special BRDF



- Lambertian surface: Constant (spectral) BRDF, k_d

➔ Light reflects equally well in all directions

- For an emitter with uniform radiant exitance H , the the radiance in any direction is $\frac{H}{\pi}$

- If we consider a small uniform light in direction I ,

➔ For a small receiver, the irradiance is $\frac{H}{\pi}(-I \cdot N)\Delta\omega$ ➔ angle subtended by the light

- Hence, surface radiance = $k_d \frac{H}{\pi}(-I \cdot N)\Delta\omega = k_d \frac{H}{\pi r^2}(-I \cdot N)\Delta A$ ➔ projected light area

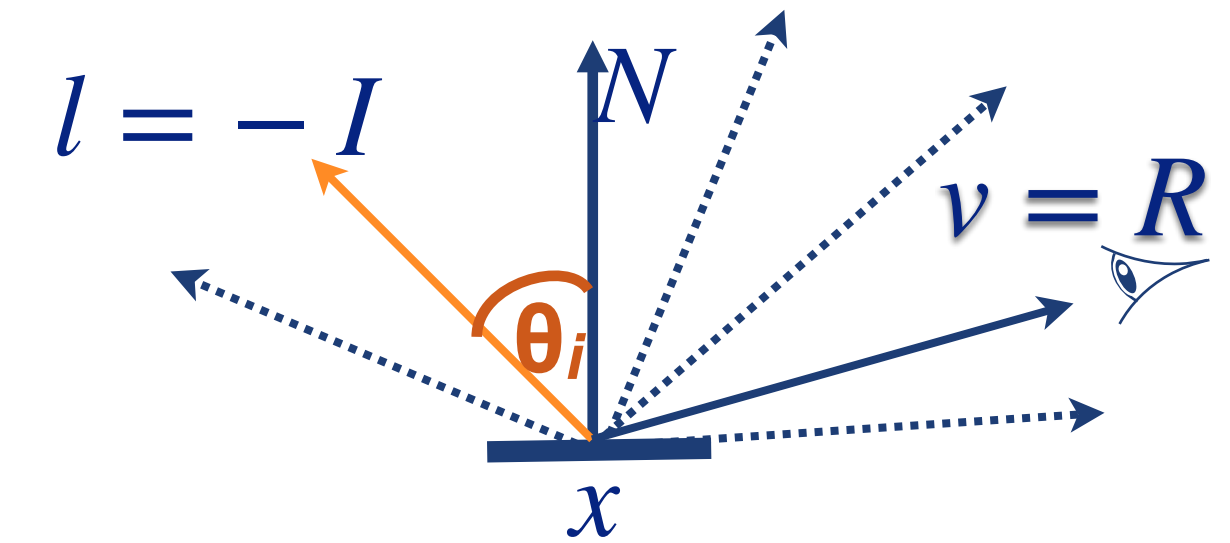
- To model light as purely directional (i.e., $\Delta A = 0$, $r = \infty$), we let $\frac{H}{\pi r^2}\Delta A = C$

$$k_d C (-I \cdot N)$$

🔔 Handle -ve cosine
Vectors are unit length



Diffuse Reflectance



- Diffuse reflectance: Constant BRDF

- ➡ Light reflects equally well in all directions

- Directional model of light in direction l with color C_i in spectral band i

- ➡ e.g., $C_0 C_1 C_2 = R G B$

Denote: k_{d_i}

Reflected Color $L_i = C_i * \text{Diffuse_Reflectance}_i * (l \cdot N)$ independent of R

- To model light as a point source, instead, at point p , $l = (p-x)/|(p-x)|$

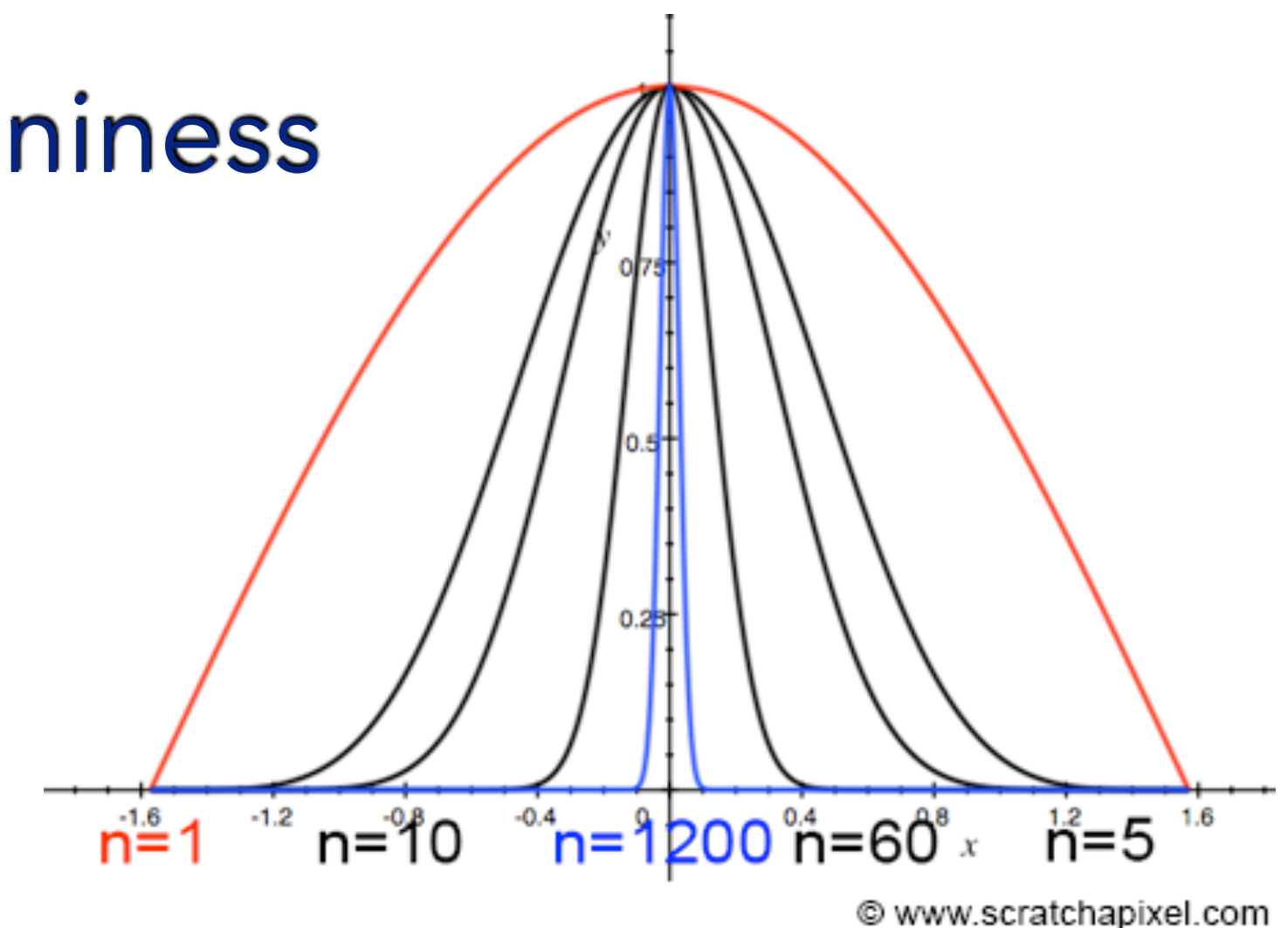
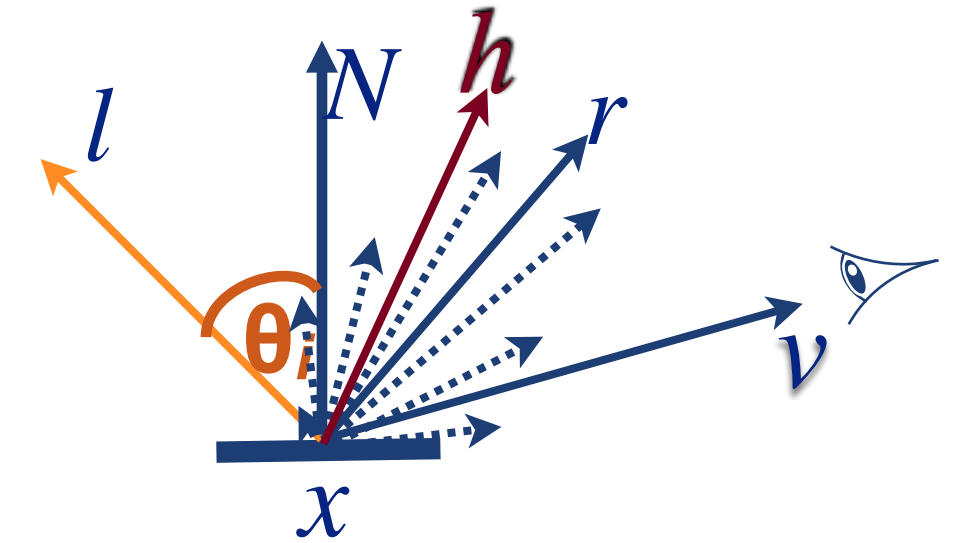
- ➡ Attenuation of light can also be modeled dividing by r^2 , where $r = |(p-x)|$

- ➡ This is sometimes artificially "controlled" by dividing by $k_2 r^2 + k_1 r + k_0$ instead



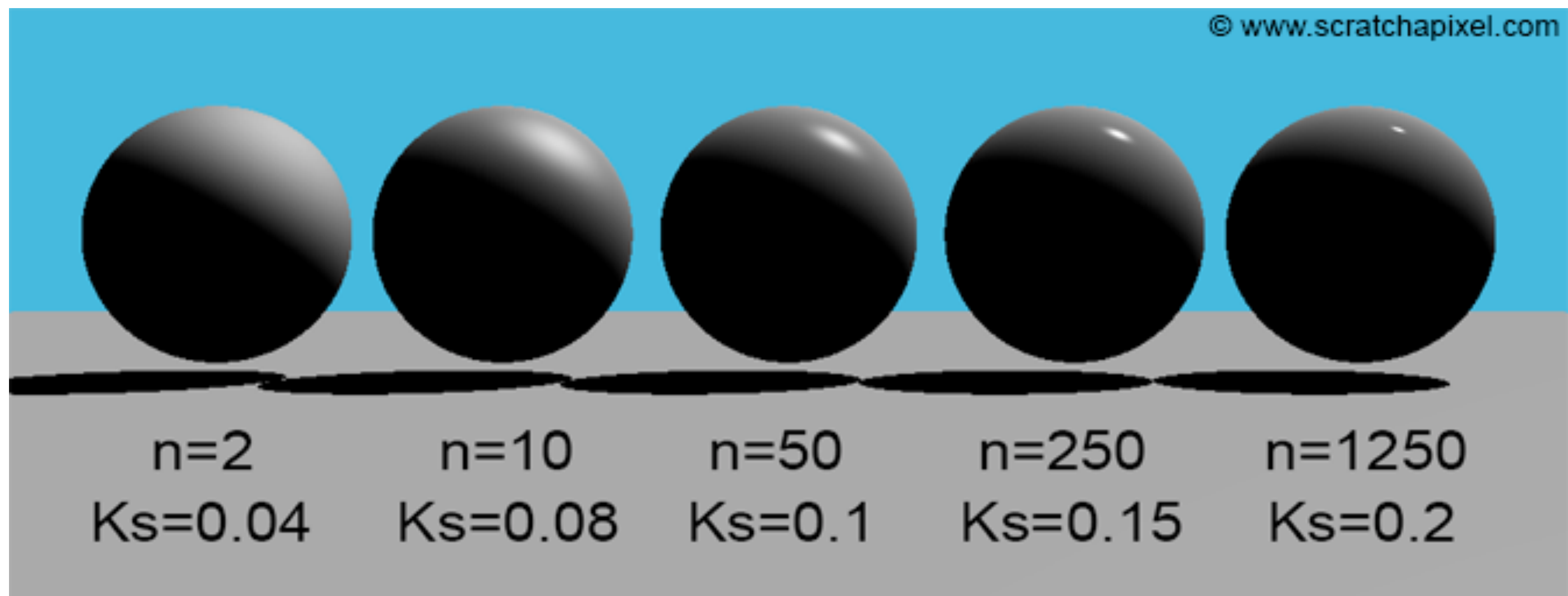
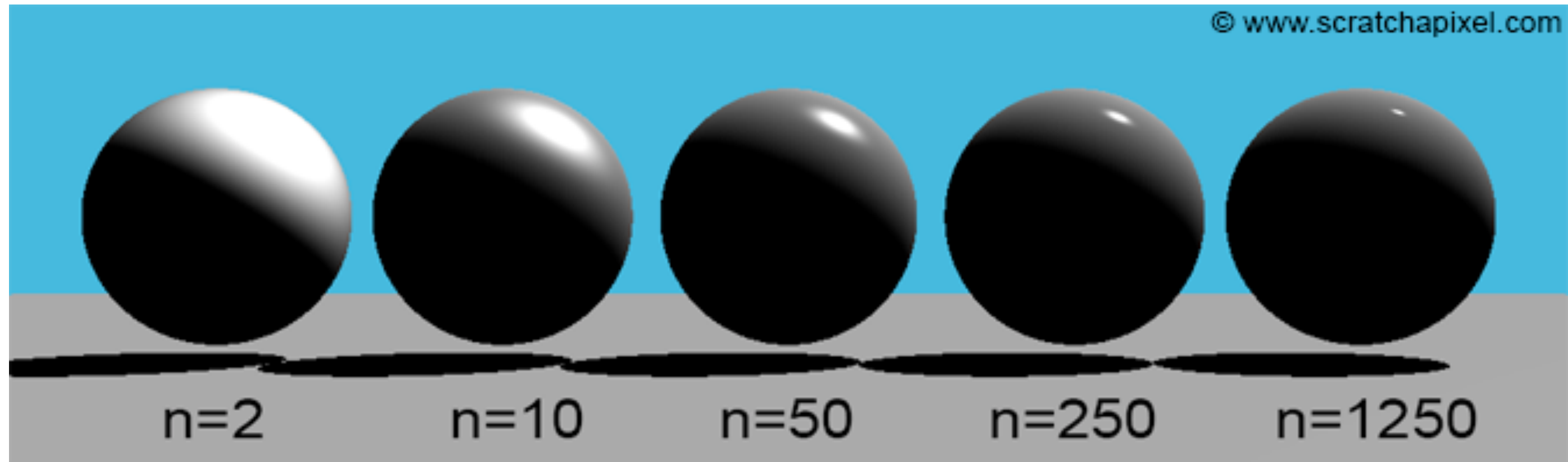
Specular Reflectance

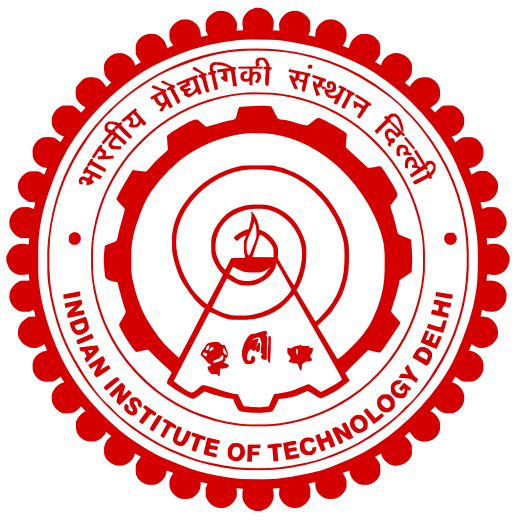
- Reflectance is a function of how much “away” the viewing direction is from the ideal reflected direction r
- Proportional to the cosine of angle between v and r : $v \cdot r$
- Or, a power of the angle $(v \cdot r)^n$ for some chosen n : shininess
 - ➔ Phong Model: Not based on any physical BRDF
- A simplification: replace $v \cdot r$ with $h \cdot N$,
where $h = (v + l) / |v + l|$



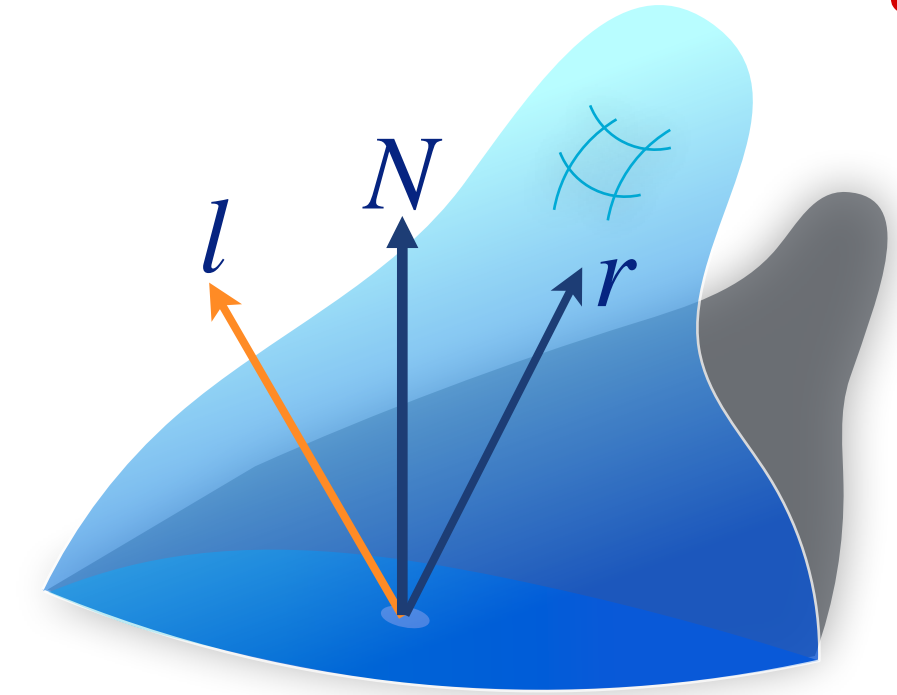
$$L = C * k_s * (h \cdot N)^n$$

(for each spectral band i)





An Illumination Model

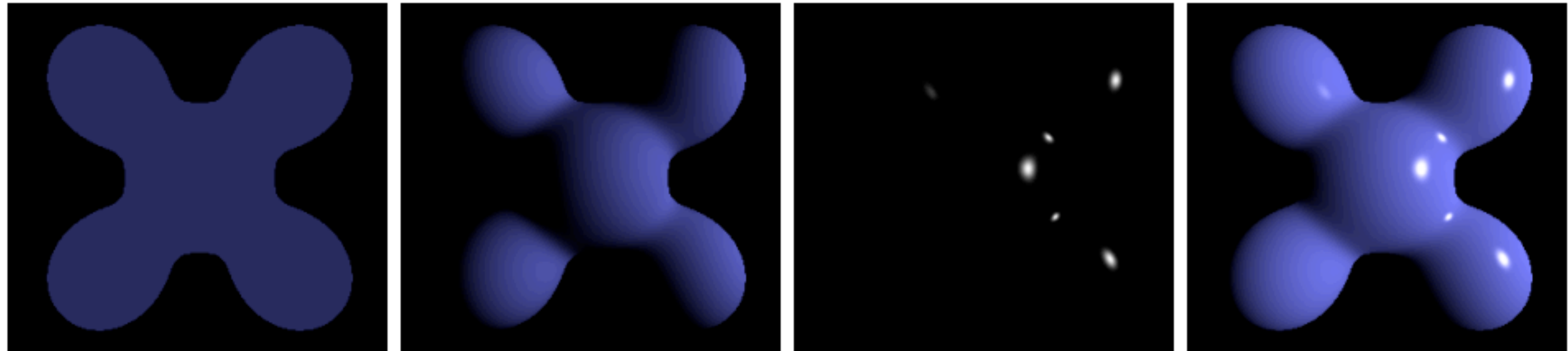


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$$L = C * k_s * (h \cdot N)^n \text{ (for each spectral band } i)$$

$$L = C_s * k_s * (h \cdot N)^n + C_d * k_d * (l \cdot N) + C_a \text{ Arbitrary constant ambient color}$$

(A part of C , i.e., C_s , reflects specularly, the remaining C_d is diffuse)



Ambient

+

Diffuse

+

Specular

=

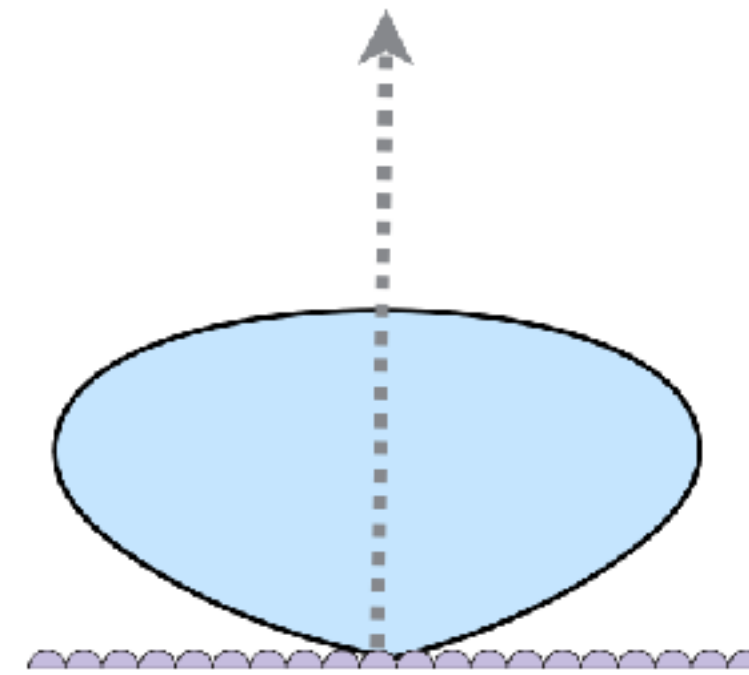
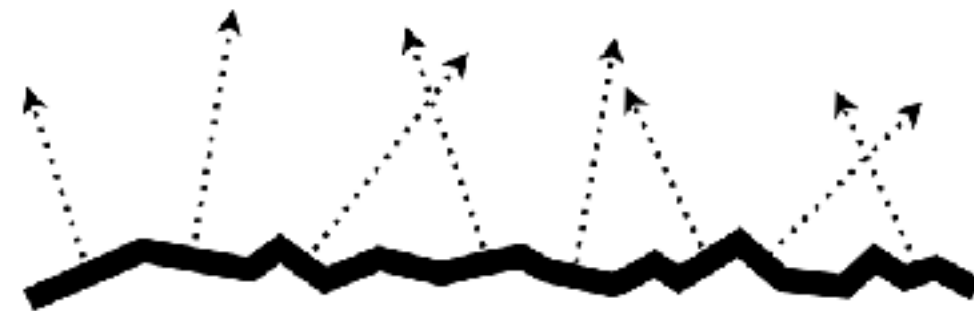
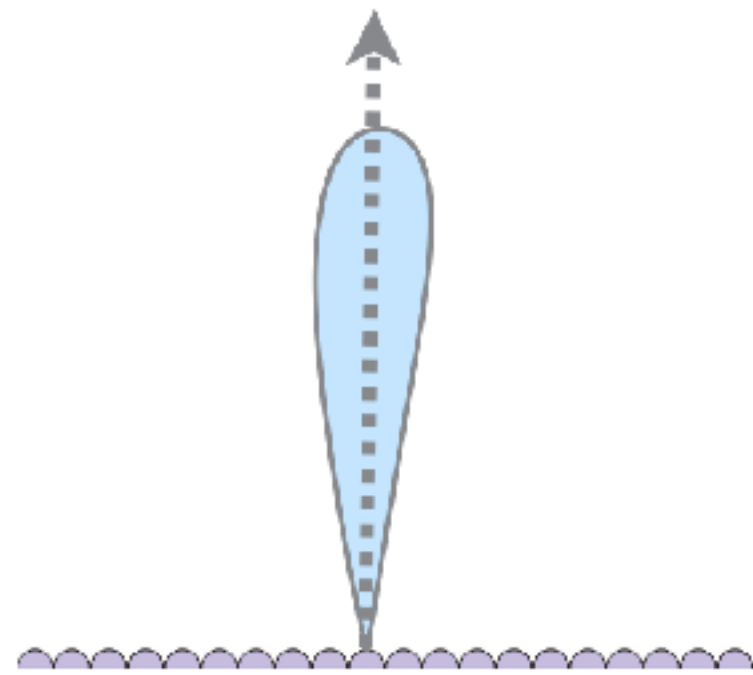
**Blinn-Phong
reflectance model**

⚠ Not Energy Conserving

$$L = k_a C_a + \sum_j \left(C_{s_j} k_s (N \cdot h_j)^n + C_{d_j} k_d (N \cdot l_j) \right)$$

Microfacet models

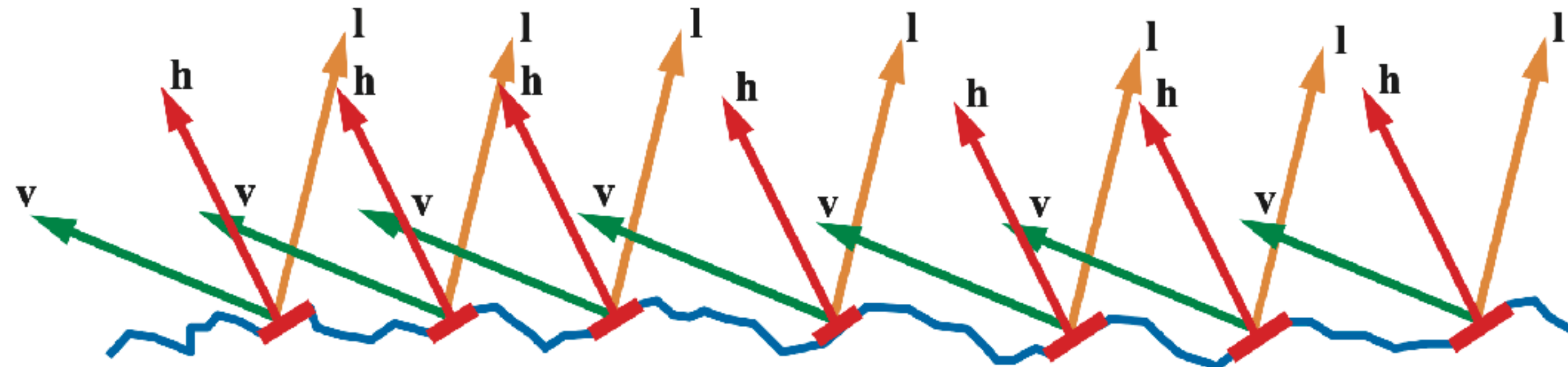
- Assume rough surface is actually perfectly specular but bumpy at microscale
- Made of small flat mirror-like "microfacets"



BRDF from Microfacet Distribution



- Light from direction l is reflected towards v *iff* microfacet normal m = halfway vector h
- $D(m)$ = relative area of points on surface that have microfacet normal m



$$\rho_{ir} = dL/dH = \frac{D(h)}{4 \cos \theta_i \cos \theta_r}$$

[Torrance & Sparrow 1965]



Full Microfacet Model

- Microfacets are not mirror like at all incidences

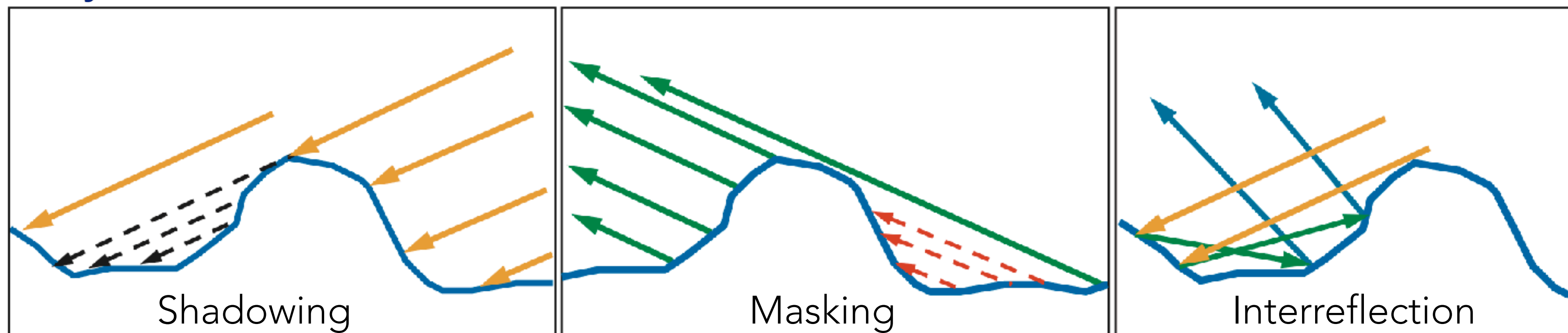
➔ $F(l, v) \leq 1$: Fresnel factor

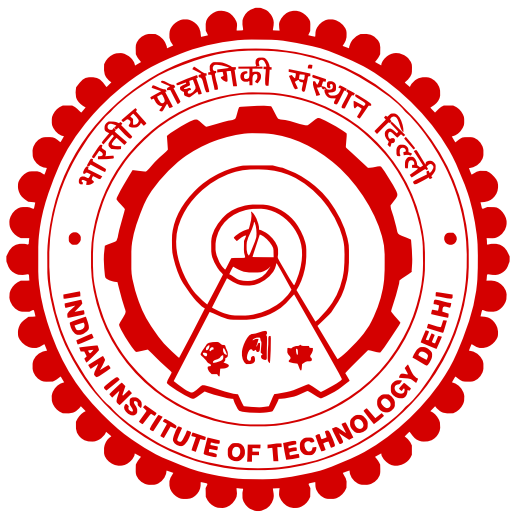
$$\rho = \frac{F(l, v)G(l, v, h)D(h)}{4(N \cdot l)(N \cdot v)}$$

- Microfacets occlude each other

➔ $G(l, v, h) \leq 1$: Geometric attenuation

➔ Probability that microfacet with normal = h is visible from both l and v





Distribution/Roughness

- **Beckmann:**

$$D = \frac{e^{-\frac{\tan^2 \alpha}{m^2}}}{\pi m^2 \cos^4 \alpha}$$

α : angle between N and h (angular bisector of v and l)

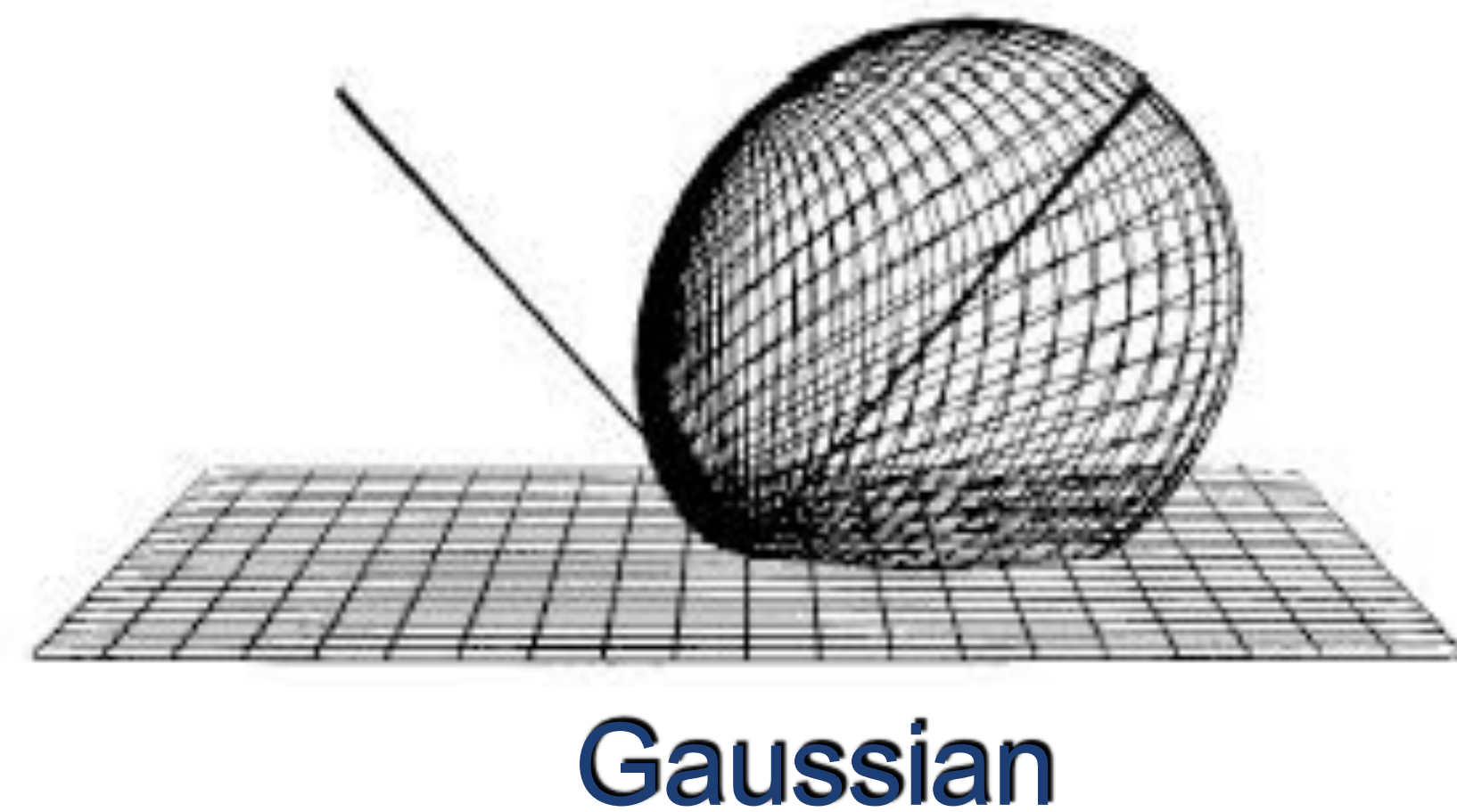
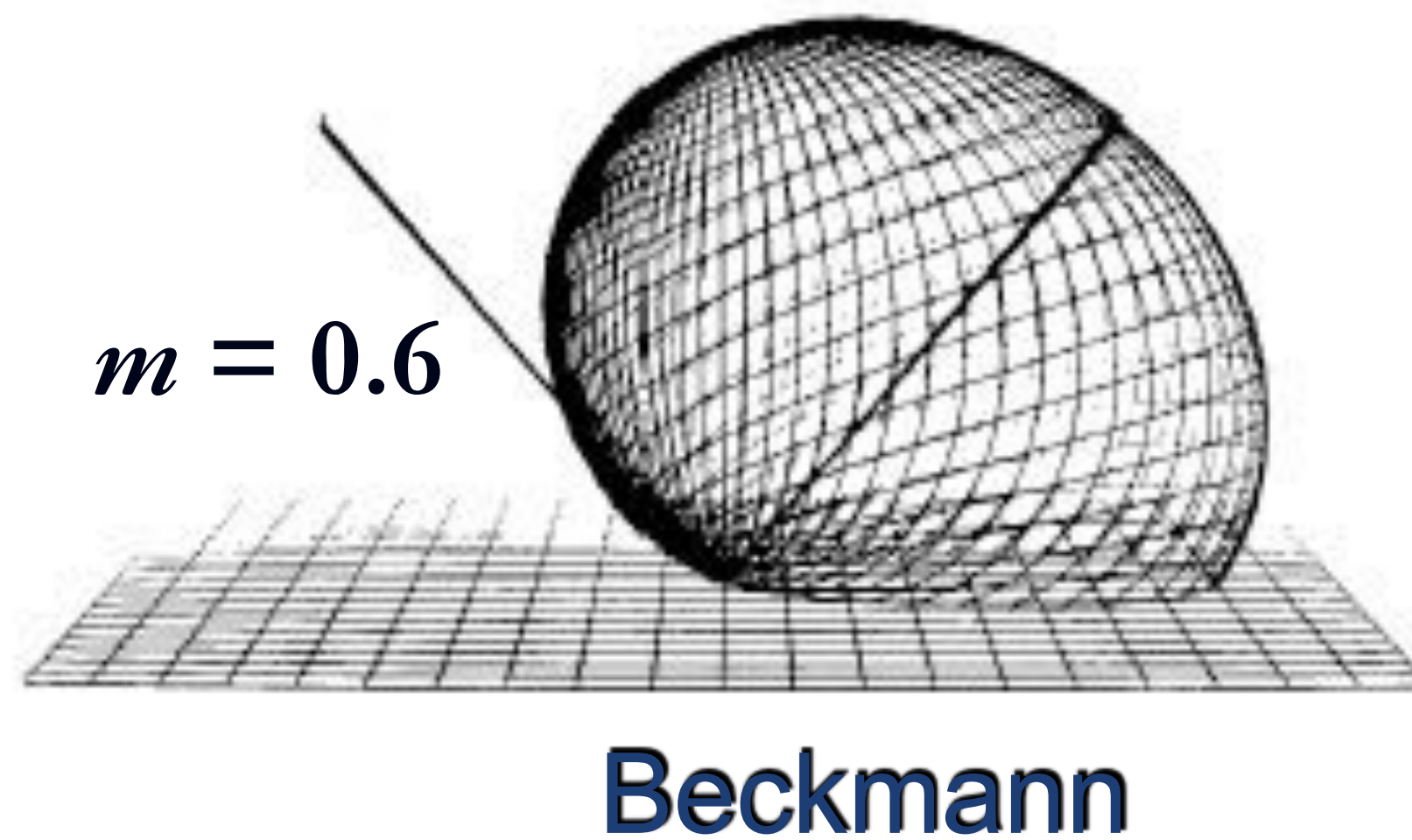
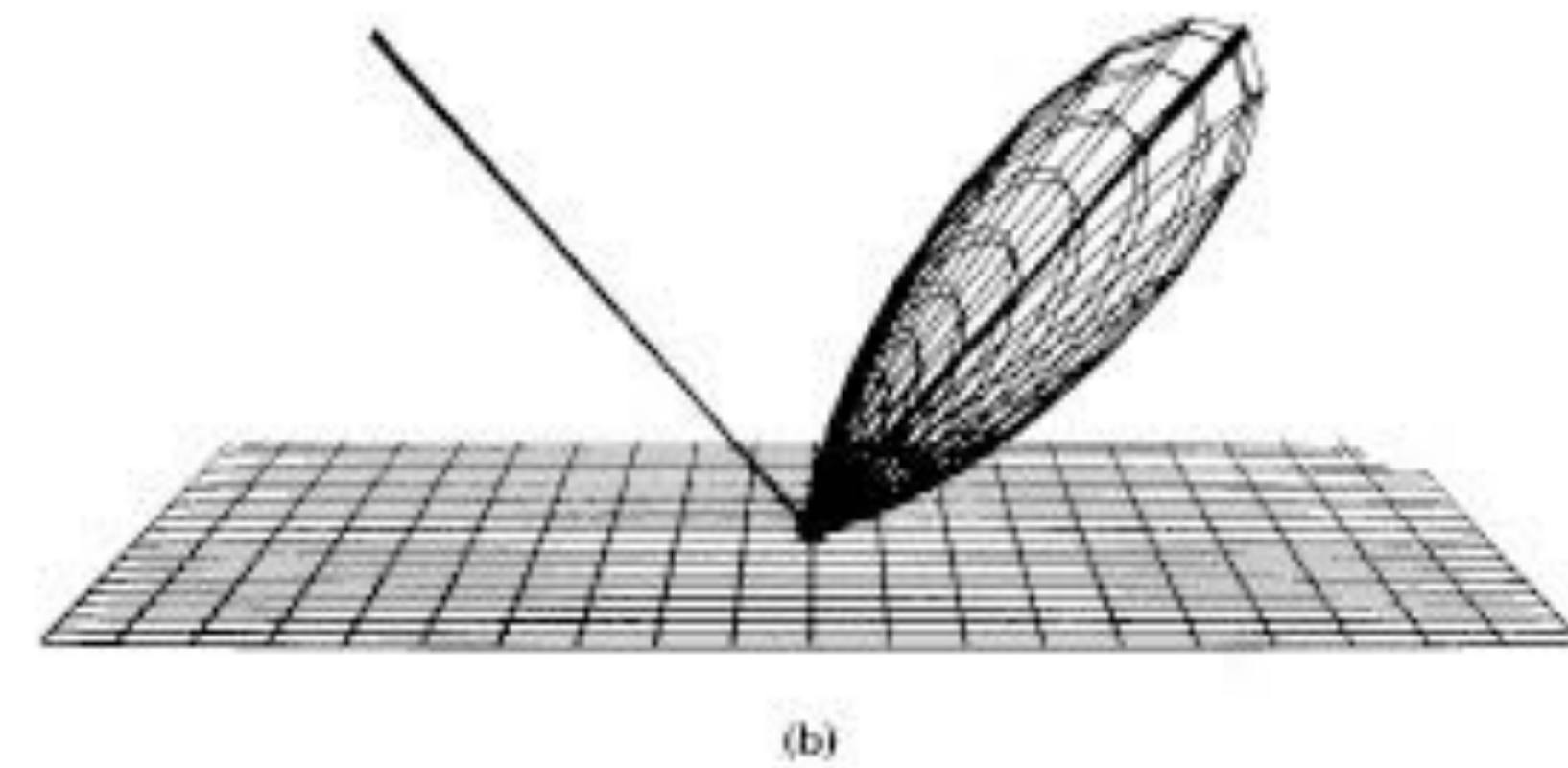
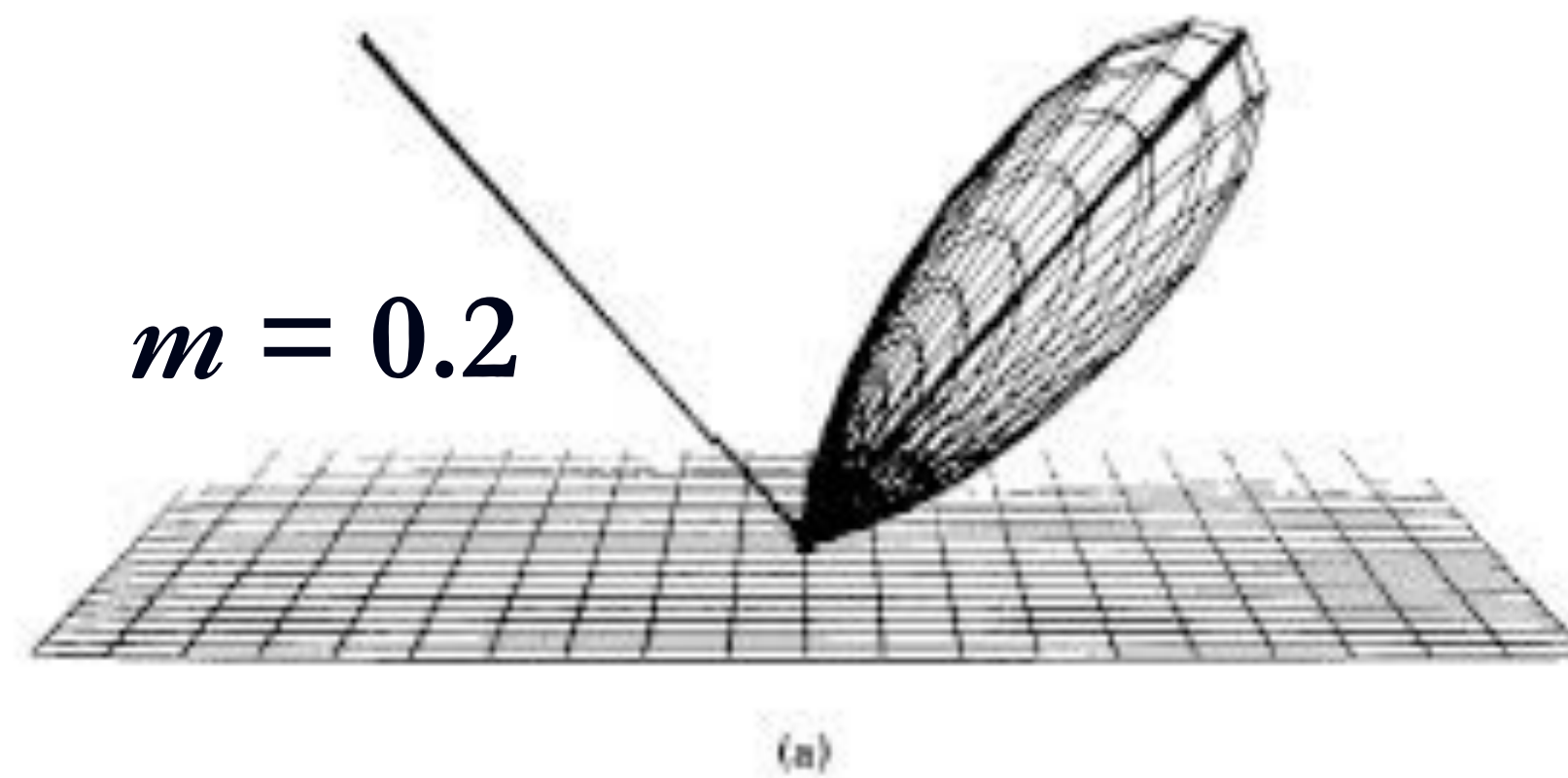
m : root mean square (rms) slope of the facets (roughness)

- **Gaussian:** $D = Ke^{-\left(\frac{\alpha}{m}\right)^2}$

- **Blinn:** $D = (\cos \alpha)^\beta$

Beckmann vs Gaussian

[Cook, Torrance '82]





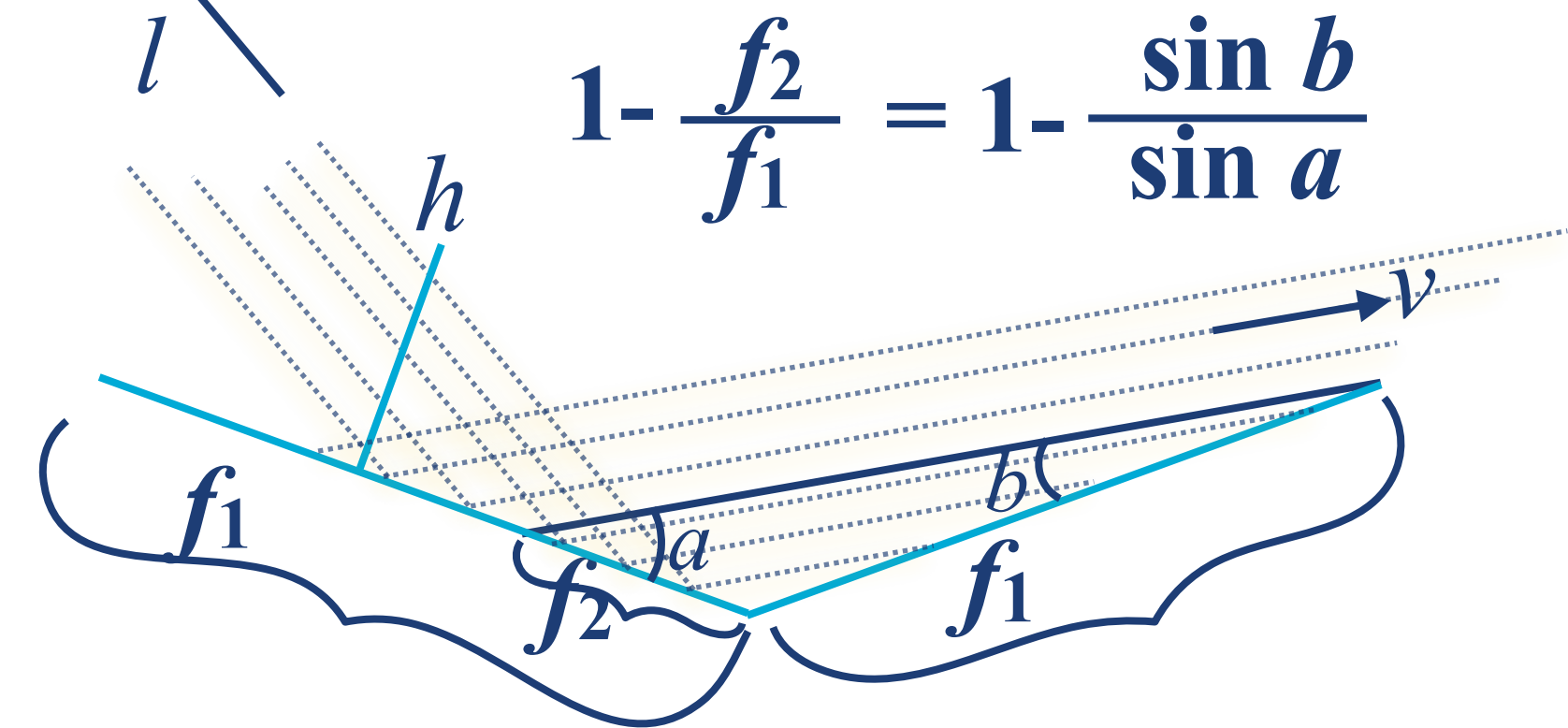
Geometric Attenuation

[Blinn'77] assumes symmetrical v-shaped grooves

$$G_a = 1.0$$

$$G_b = \frac{2(N \cdot h)(N \cdot v)}{(v \cdot h)} \quad (\text{Reflected fraction occluded})$$

$$G_c = \frac{2(N \cdot h)(N \cdot l)}{(v \cdot h)} \quad (\text{Incoming fraction occluded})$$



$$1 - \frac{f_2}{f_1} = 1 - \frac{\sin b}{\sin a}$$

Analogous if l and v are reversed

$$G = \min (G_a, G_b, G_c)$$