

Modeling Radial Distortion

Non-linear, Final (u, v)

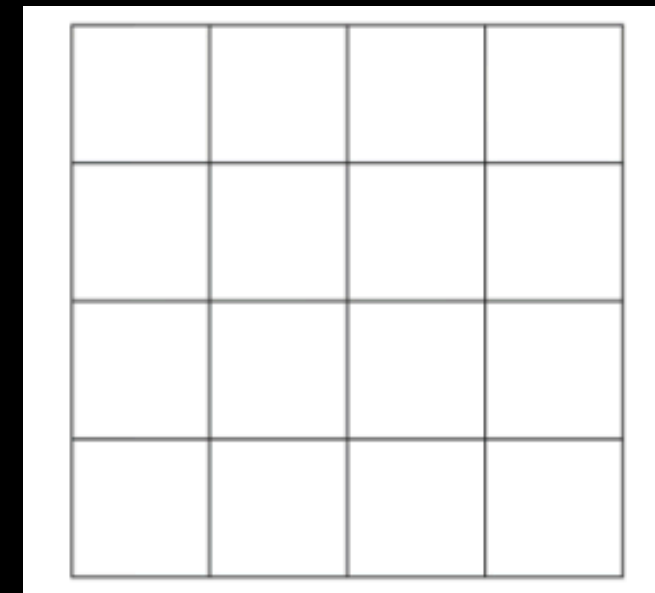
$$\begin{bmatrix} u_D \\ v_D \end{bmatrix} = D(r) \begin{bmatrix} u - O_u \\ v - O_v \end{bmatrix} + \begin{bmatrix} O_u \\ O_v \end{bmatrix},$$

where $r^2 = (u - O_u)^2 + (v - O_v)^2$

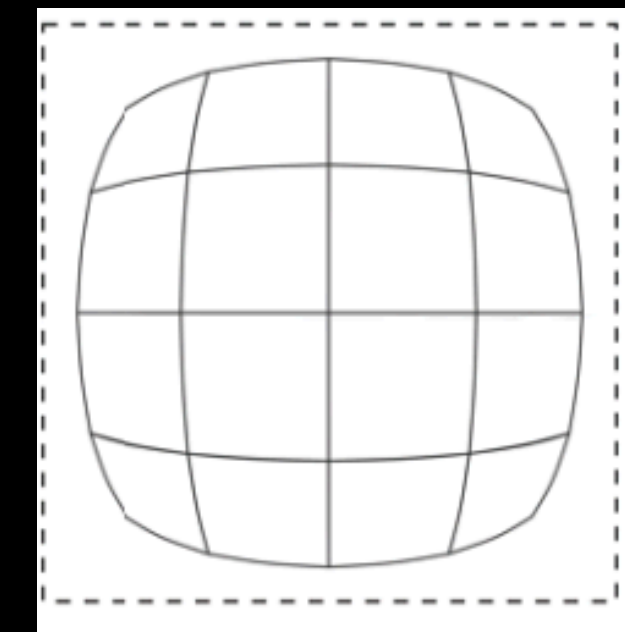
$$D(r) = 1 + k_1 r^2 + k_2 r^4 + k_3 r^6$$

$$D(r) \rightarrow 1 + k_1 r^2 \quad \begin{matrix} \downarrow & \downarrow \\ \approx 0 & \approx 0 \end{matrix}$$

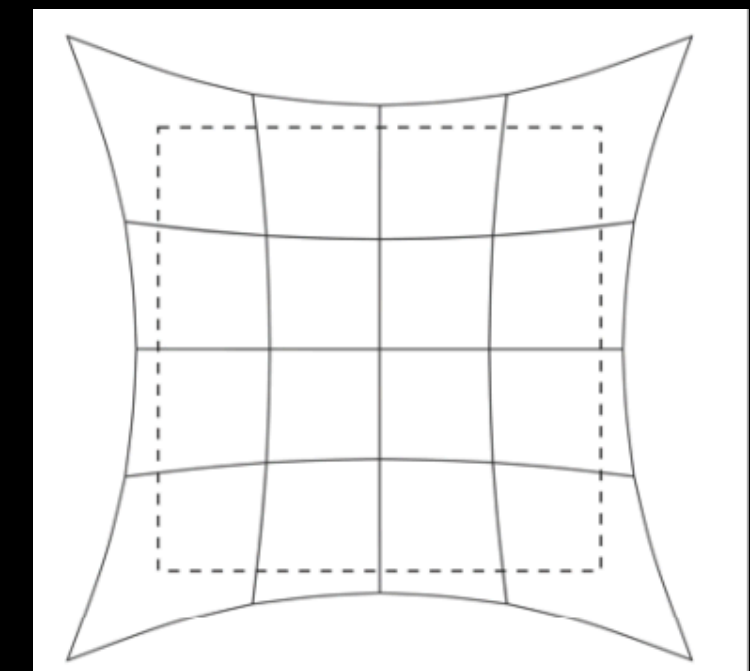
Modern cameras use computational photography techniques to undistort images. However, that can result in non uniform pixel density.



No Distortion



Barrel
Distortion



Pincushion
Distortion

Perspective Projection Matrix

2D homogeneous

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

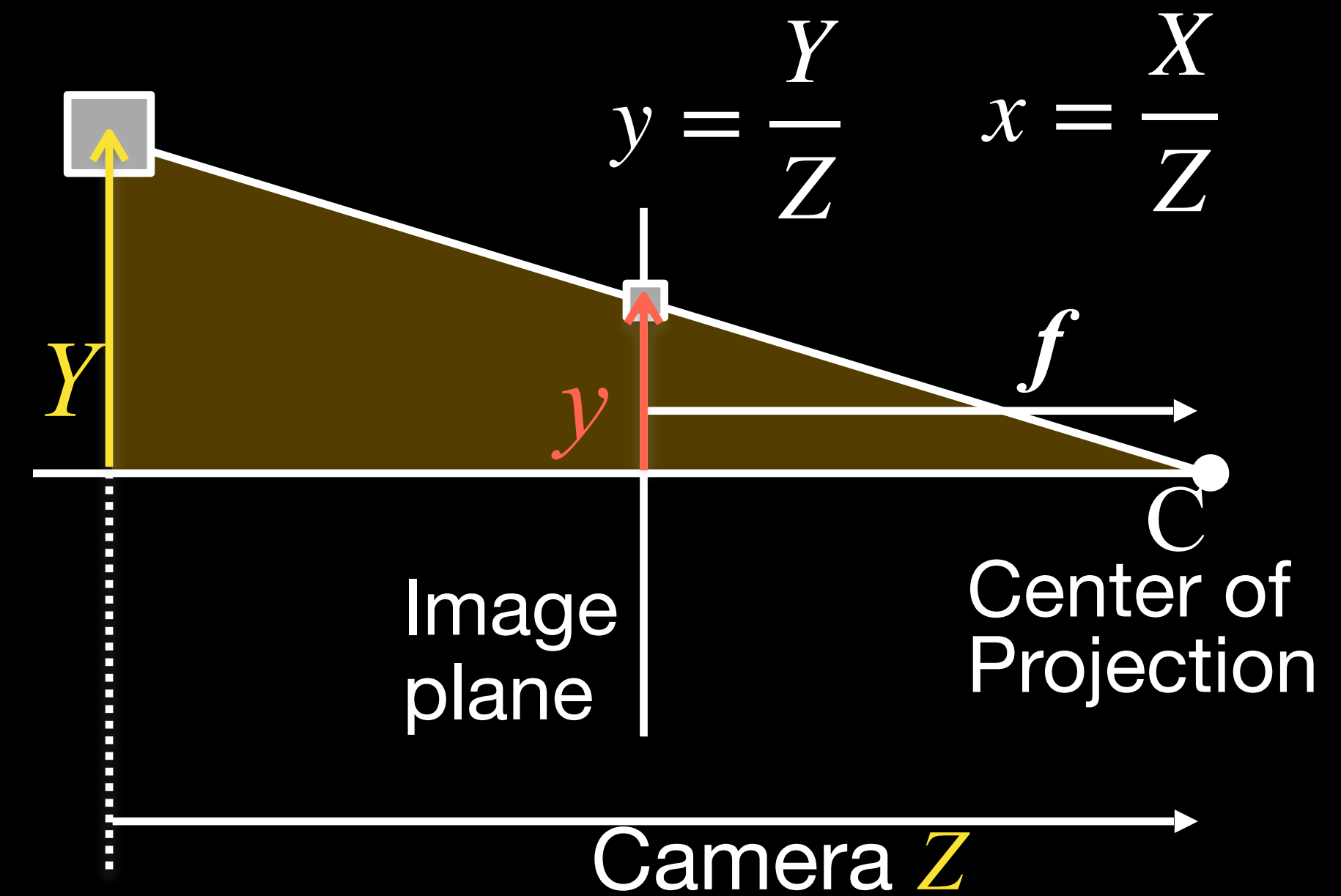
3D homogeneous

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Dehomogenize:

$$\begin{bmatrix} \frac{X}{Z} \\ \frac{Y}{Z} \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{X}{Z} \\ \frac{Y}{Z} \end{bmatrix}$$

2D Cartesian

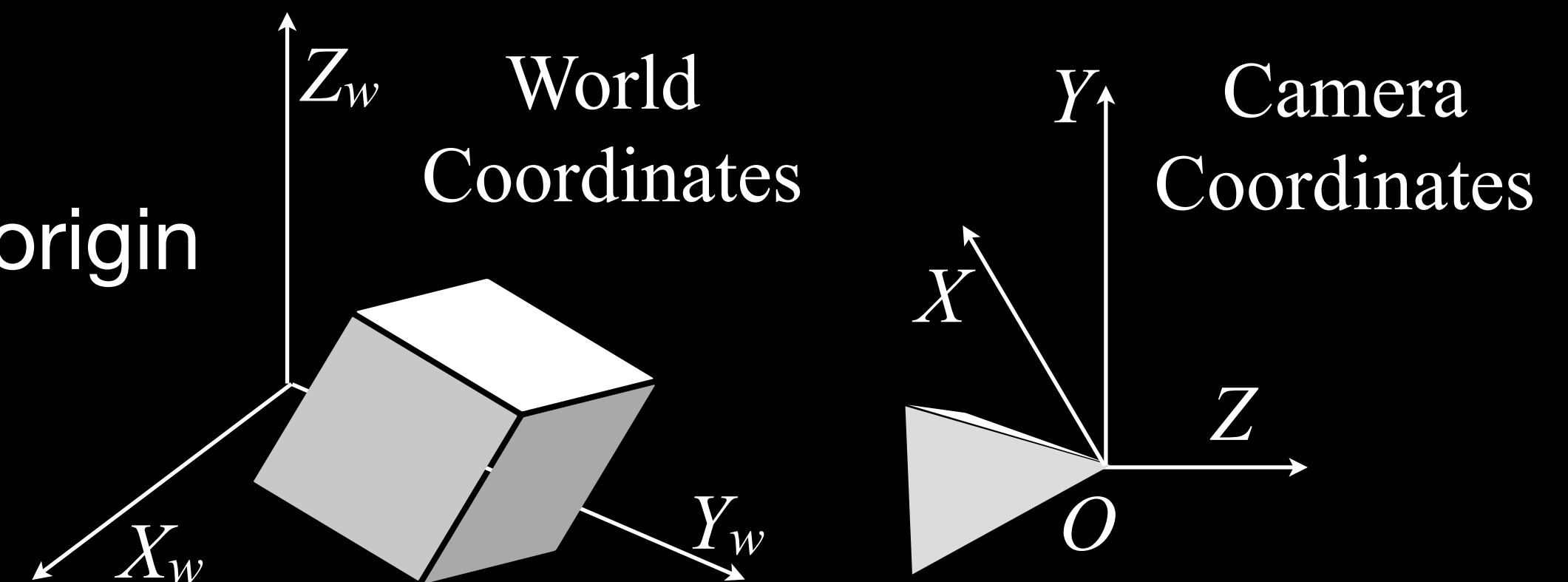


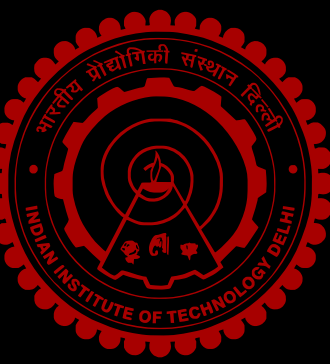
We keep projection as normalized coordinates independent of f .
Akin to keeping the image plane at unit distance from C,
and later adjust for f by linear scaling.

Camera Matrix

aka viewing matrix

- Suppose basis vectors and the origin of camera coordinates: X , Y , Z and O are known in the $[X_w, Y_w, Z_w]$ world coordinate system
 - How to change other points from X_w, Y_w, Z_w coordinates to X, Y, Z coordinates
 - Given X, Y, Z vectors are each of unit length and perpendicular to each other
- Suppose $X = X_w, Y = Y_w, Z = Z_w$
 - *i.e.*, camera Origin is translated by O from XYZ origin
 - Then the camera coords of any world point P is $P - O$: a translation
- Suppose O is $(0,0,0)$, the two axes are simply rotated relative each other then





Translation

By vector V

$$\mathcal{T}_v(P) = P + V$$

If $P = [X \ Y \ W]^T$ and $V = [U \ V \ W]^T$

$$\mathcal{T}_v(P) = [(X + U) \ (Y + V) \ (Z + W)]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & U \\ 0 & 1 & 0 & V \\ 0 & 0 & 1 & W \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Homogenized

To keep the result in homogeneous space

$$\begin{bmatrix} 1 & 0 & 0 & U \\ 0 & 1 & 0 & V \\ 0 & 0 & 1 & W \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Can you translate a vector?

Vector: $\begin{bmatrix} X \\ Y \\ Z \\ 0 \end{bmatrix}$

In 2D $\begin{bmatrix} 1 & 0 & U \\ 0 & 1 & V \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$

Rotation

As change of basis vectors

- Rotation is an orthonormal matrix: three degrees of freedom
 - Direction of rotational axis and angle; or Euler rotation angles against each axis
 - Also a change of basis with new basis vectors given
 - Orthonormal basis

$$X^T X = Y^T Y = Z^T Z = 1,$$

$$X^T Y = 0, X^T Z = 0, Y^T Z = 0$$

Unknowns:

Given

$$\begin{bmatrix} r_{00} & r_{01} & r_{02} \\ r_{10} & r_{11} & r_{12} \\ r_{20} & r_{21} & r_{22} \end{bmatrix}$$

$$[X \ Y \ Z]$$

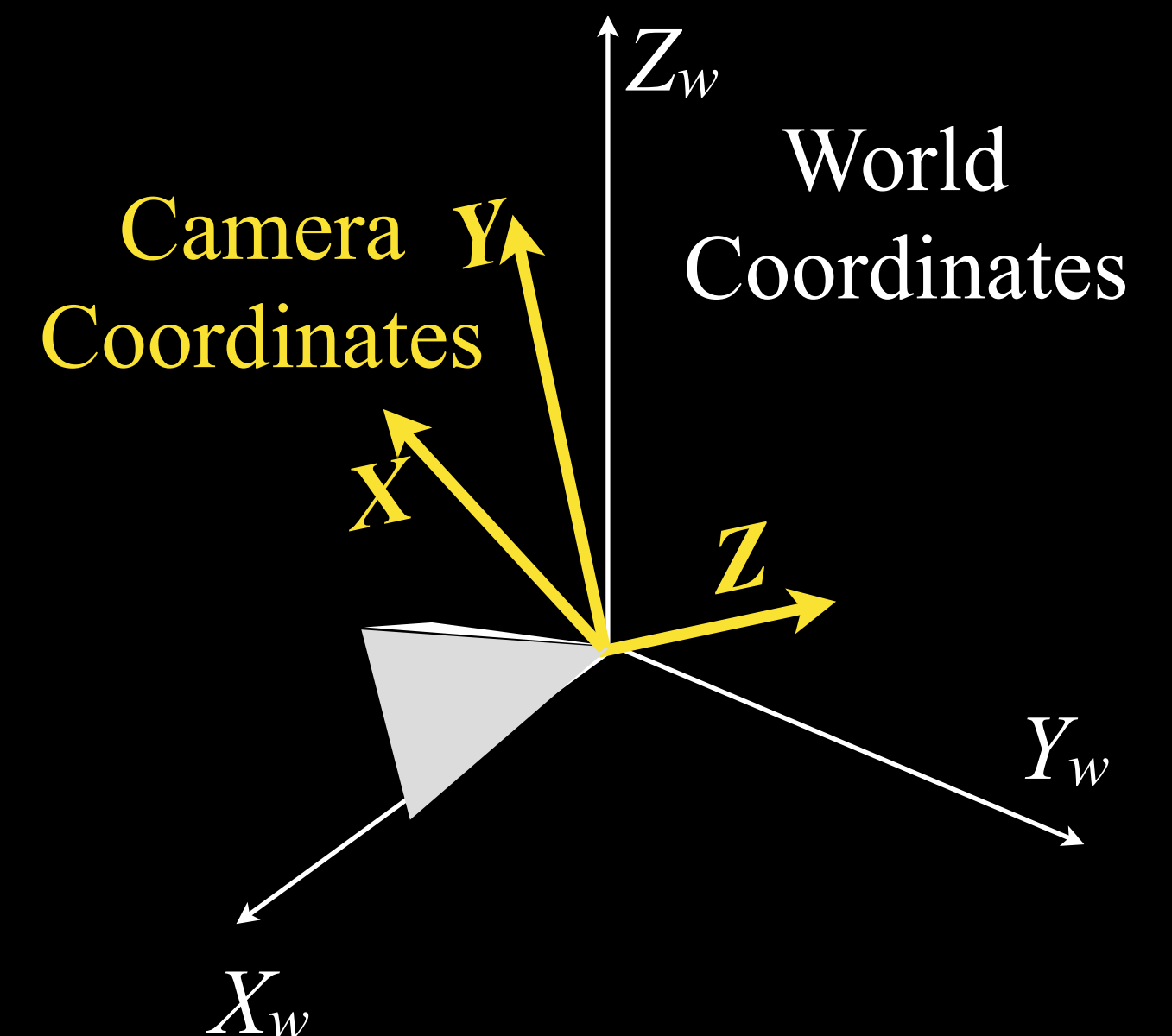
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution R =

$$\begin{bmatrix} X^T \\ Y^T \\ Z^T \end{bmatrix}$$

$$[X \ Y \ Z]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



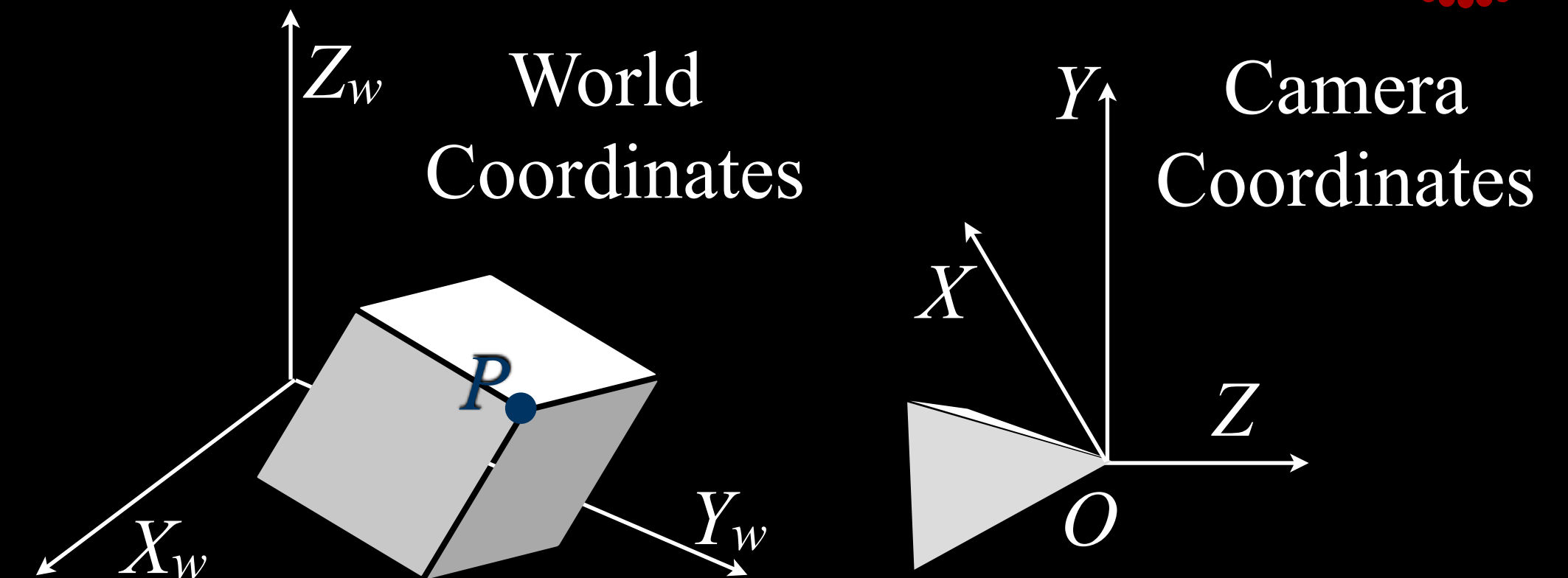
Translation and Rotation

Change to new coordinate system

- Combine rotation and Translation:

$$\begin{bmatrix} X_x & X_y & X_z & 0 \\ Y_x & Y_y & Y_z & 0 \\ Z_x & Z_y & Z_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -O_x \\ 0 & 1 & 0 & -O_y \\ 0 & 0 & 1 & -O_z \\ 0 & 0 & 0 & 1 \end{bmatrix} ?$$

~~$$\begin{bmatrix} 1 & 0 & 0 & -O_x \\ 0 & 1 & 0 & -O_y \\ 0 & 0 & 1 & -O_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_x & X_y & X_z & 0 \\ Y_x & Y_y & Y_z & 0 \\ Z_x & Z_y & Z_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ? \begin{bmatrix} X_x & X_y & X_z & O_x \\ Y_x & Y_y & Y_z & O_y \\ Z_x & Z_y & Z_z & O_z \\ 0 & 0 & 0 & 1 \end{bmatrix} ?$$~~



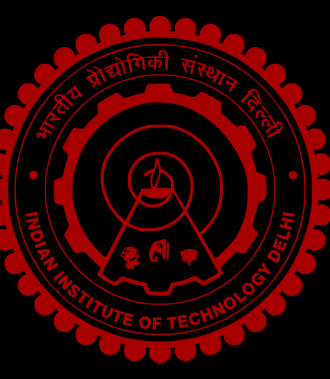
Rotate $(P - O)$

Or Rotate P and then translate by $-O$?

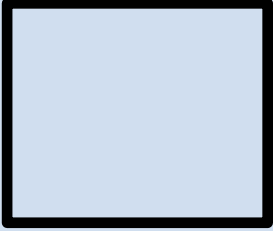
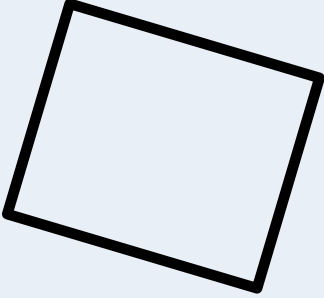
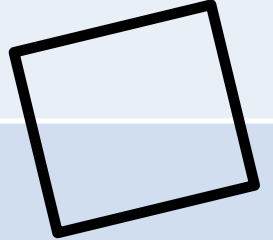
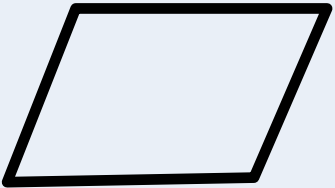
Alternatively,

Find coord with origin at O with XYZ_w ,
then with XYZ at O

Or Find coord in XYZ at the $(0,0,0)$
then shift origin to O ?



A Summary of Geometric Transformations

Transformation	Degree of Freedom 2D	Degree of Freedom 3D	Preserves	Example (2D)
Translation	2	3	Orientation	
Rigid (Euclidean)	3	6	Lengths	
Similarity	4	7	Angles	
Affine	6	12	Parallelism	
Projective	8	15	Straight Lines	