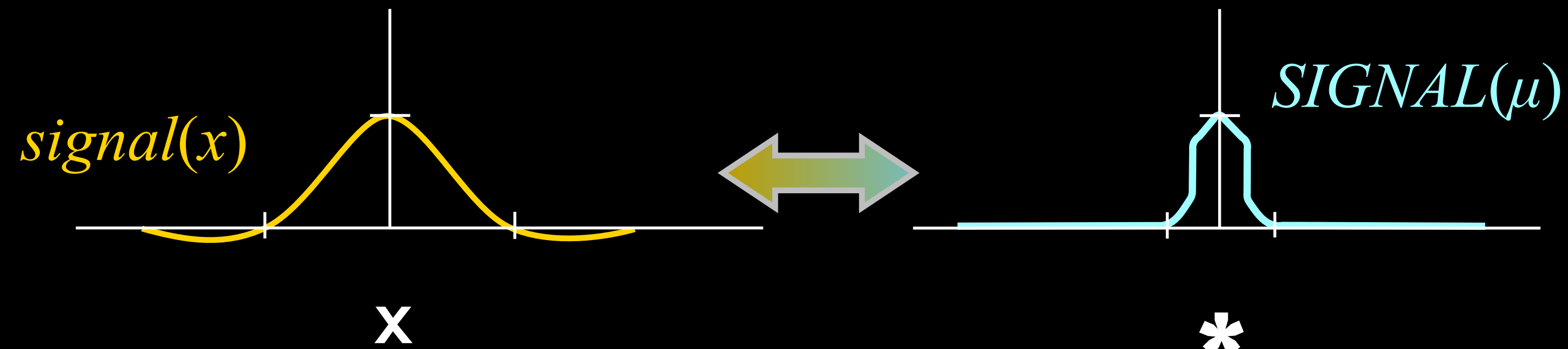
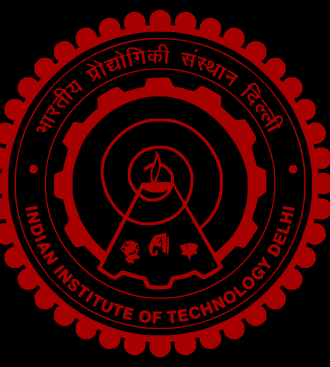
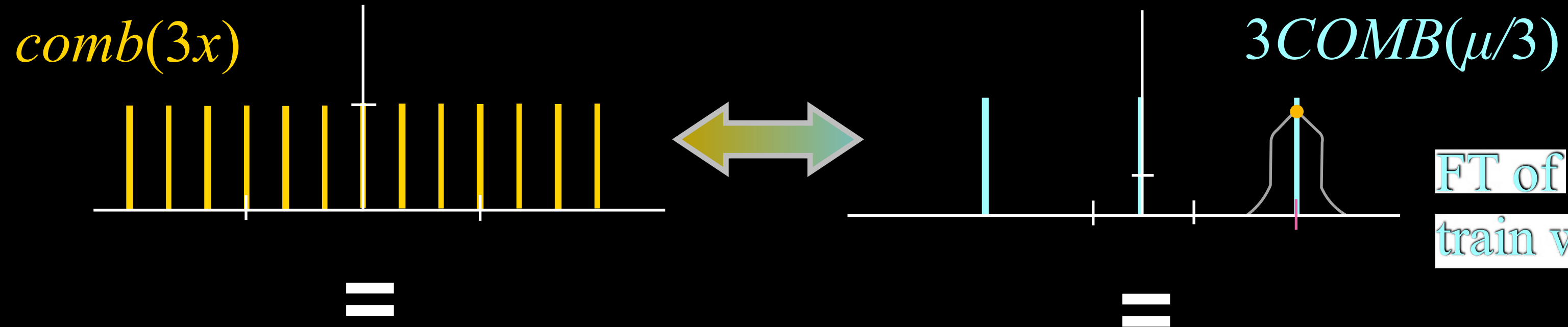


Sampling



Impulse
Train:



FT of $comb$ is an impulse train with inverse period

$$\sum_{n=-\infty}^{\infty} f_n \delta[k - n]$$

Sampled
 $signal(x)$

$$\sum_{n=-\infty}^{\infty} f(n\Delta x) \delta(x - n\Delta x)$$

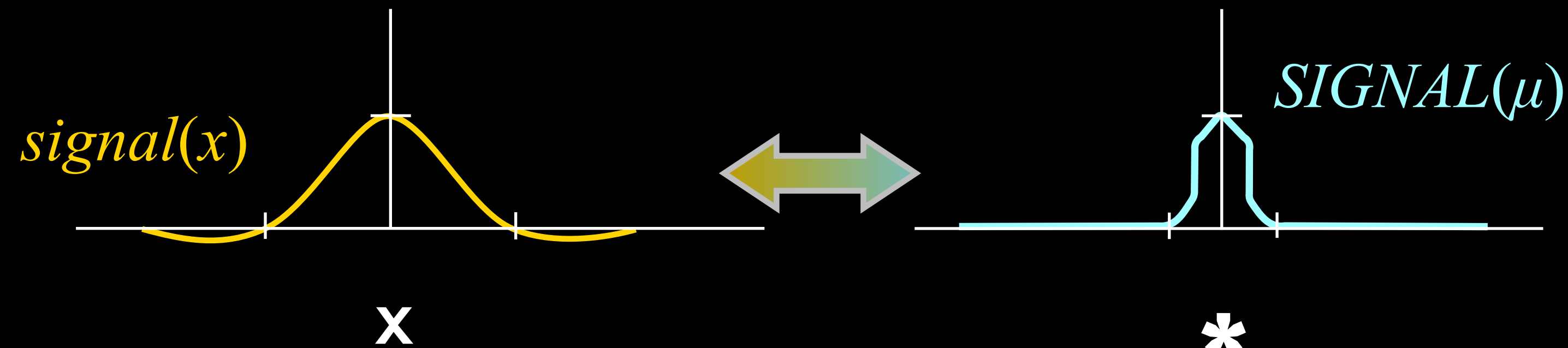
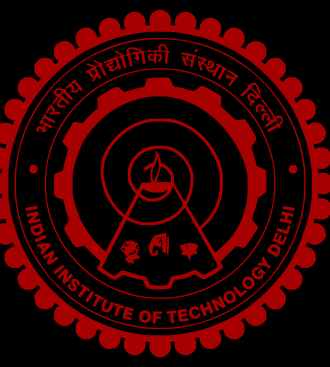
$f(x)$

$F(\mu)$

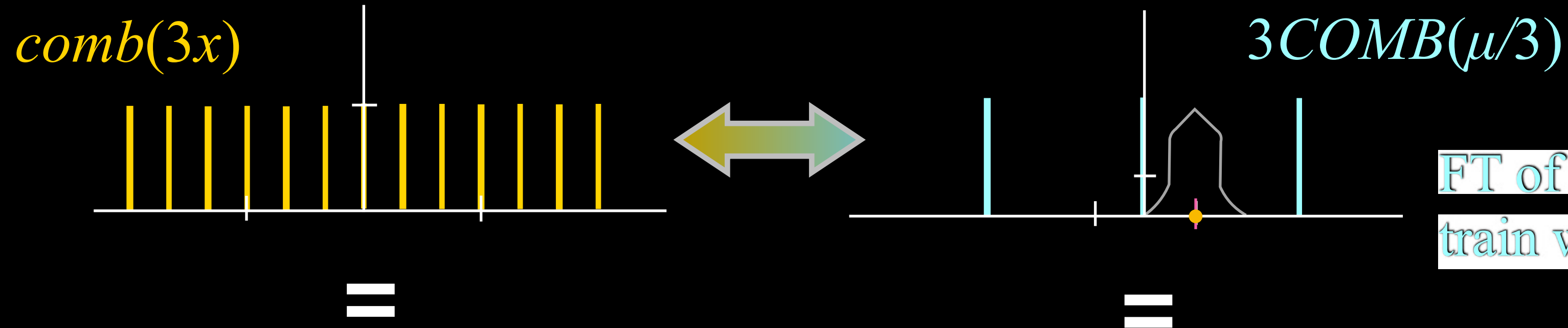
$$\delta(x) = \begin{cases} \infty & \text{at } x = 0 \\ 0, & \text{elsewhere} \end{cases}$$

$$\delta[x] = \begin{cases} 1 & \text{at } x = 0 \\ 0, & \text{elsewhere} \end{cases}$$

Sampling



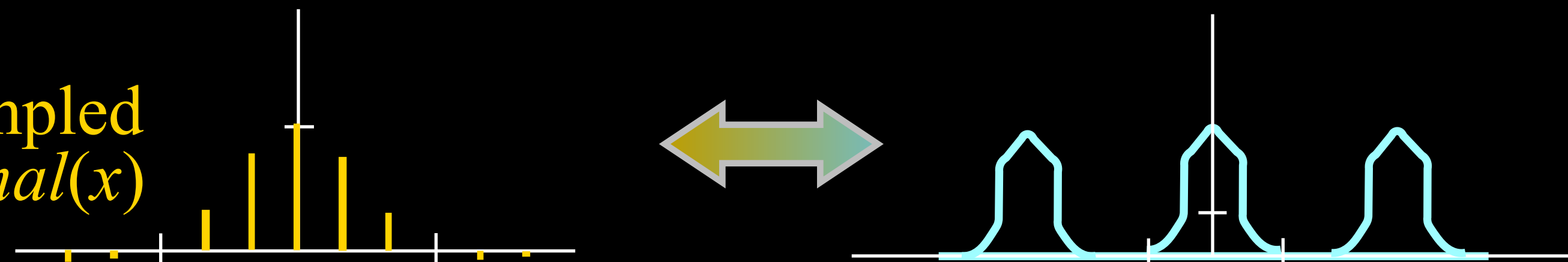
Impulse
Train:



FT of $comb$ is an impulse train with inverse period

$$\sum_{n=-\infty}^{\infty} f_n \delta[k - n]$$

Sampled
 $signal(x)$



$$\delta(x) = \begin{cases} \infty & \text{at } x = 0 \\ 0, & \text{elsewhere} \end{cases}$$

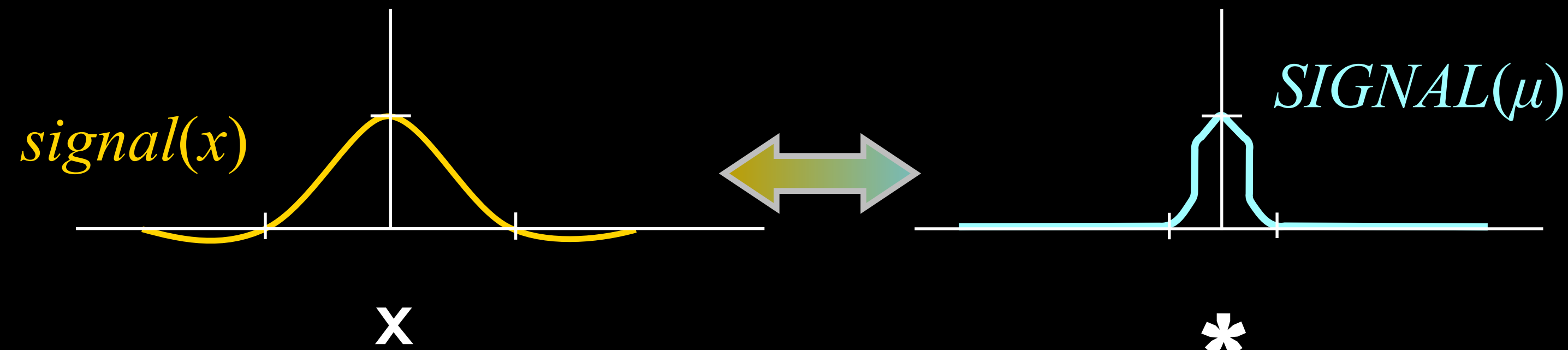
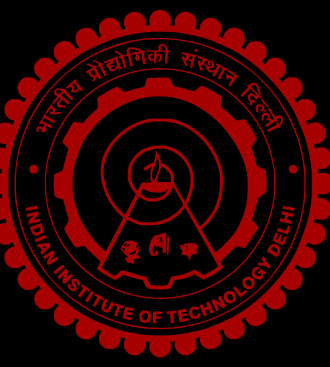
$$\sum_{n=-\infty}^{\infty} f(n\Delta x) \delta(x - n\Delta x)$$

$f(x)$

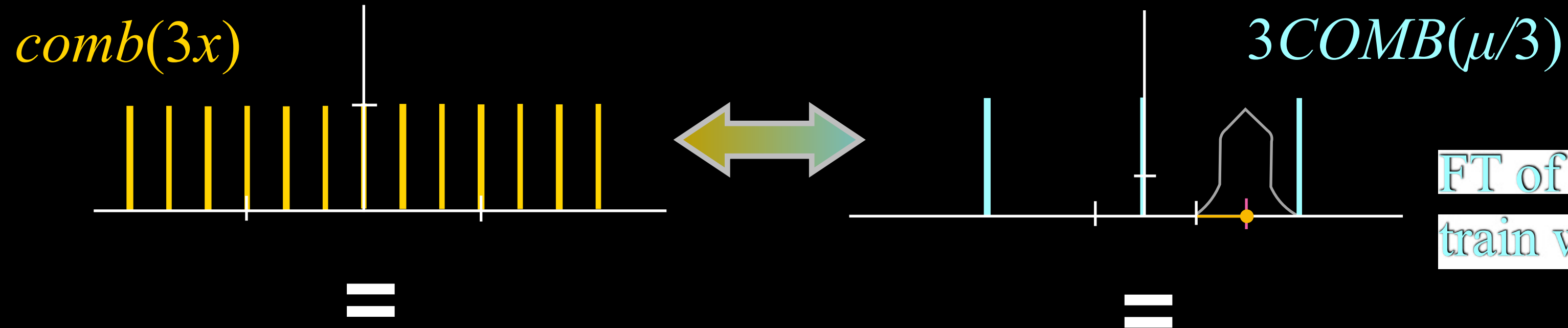
$F(\mu)$

$$\delta[x] = \begin{cases} 1 & \text{at } x = 0 \\ 0, & \text{elsewhere} \end{cases}$$

Sampling



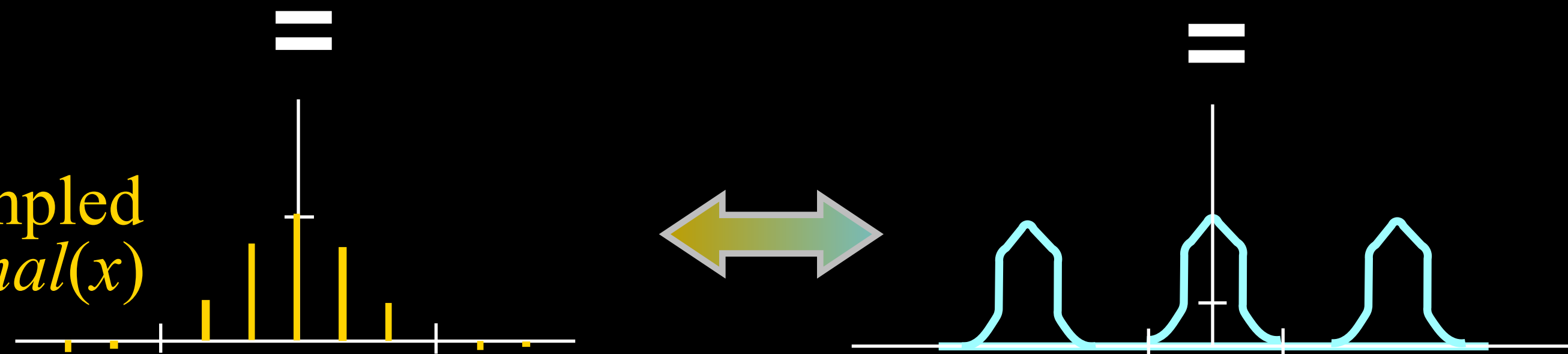
Impulse
Train:



FT of $comb$ is an impulse train with inverse period

$$\sum_{n=-\infty}^{\infty} f_n \delta[k - n]$$

Sampled
 $signal(x)$



$$\delta(x) = \begin{cases} \infty & \text{at } x = 0 \\ 0, & \text{elsewhere} \end{cases}$$

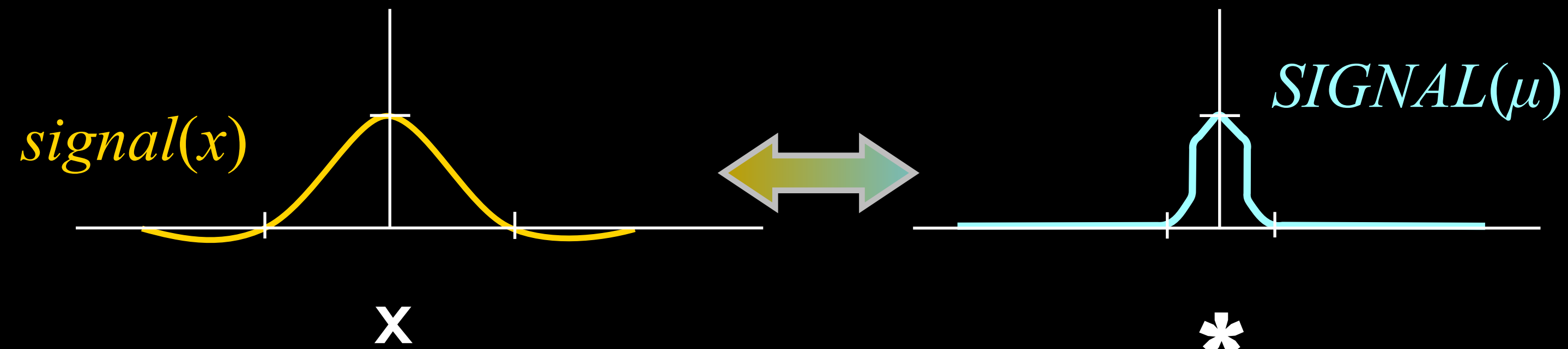
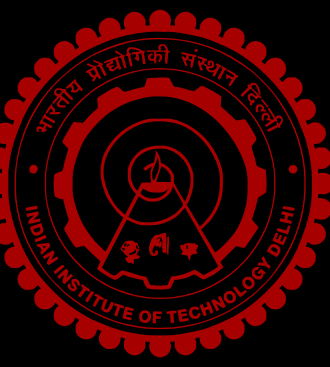
$$\sum_{n=-\infty}^{\infty} f(n\Delta x) \delta(x - n\Delta x)$$

$f(x)$

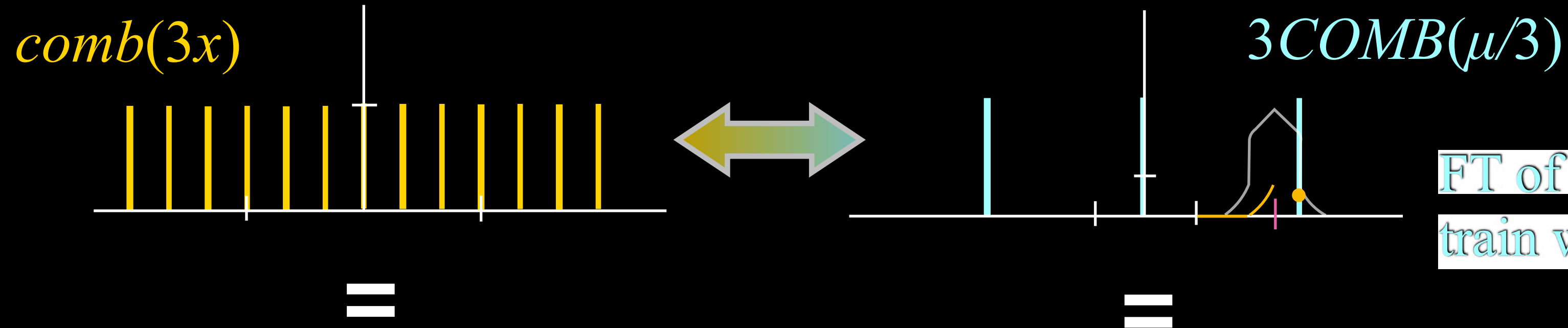
$F(\mu)$

$$\delta[x] = \begin{cases} 1 & \text{at } x = 0 \\ 0, & \text{elsewhere} \end{cases}$$

Sampling



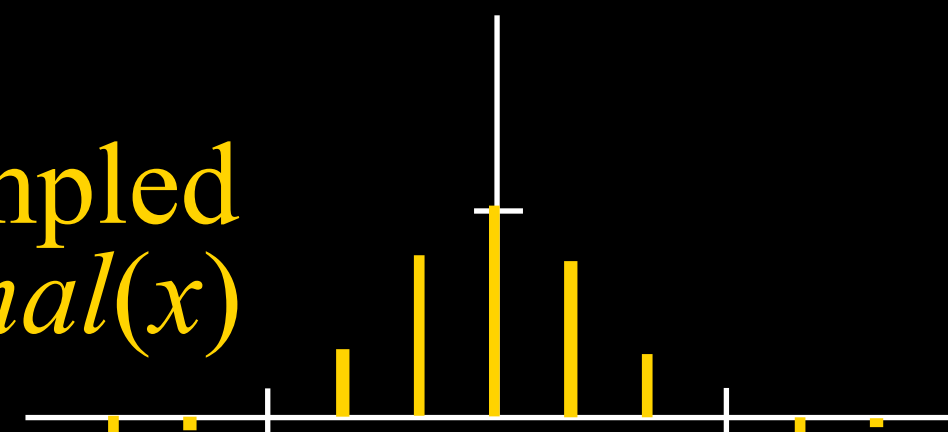
Impulse
Train:



FT of *comb* is an impulse train with inverse period

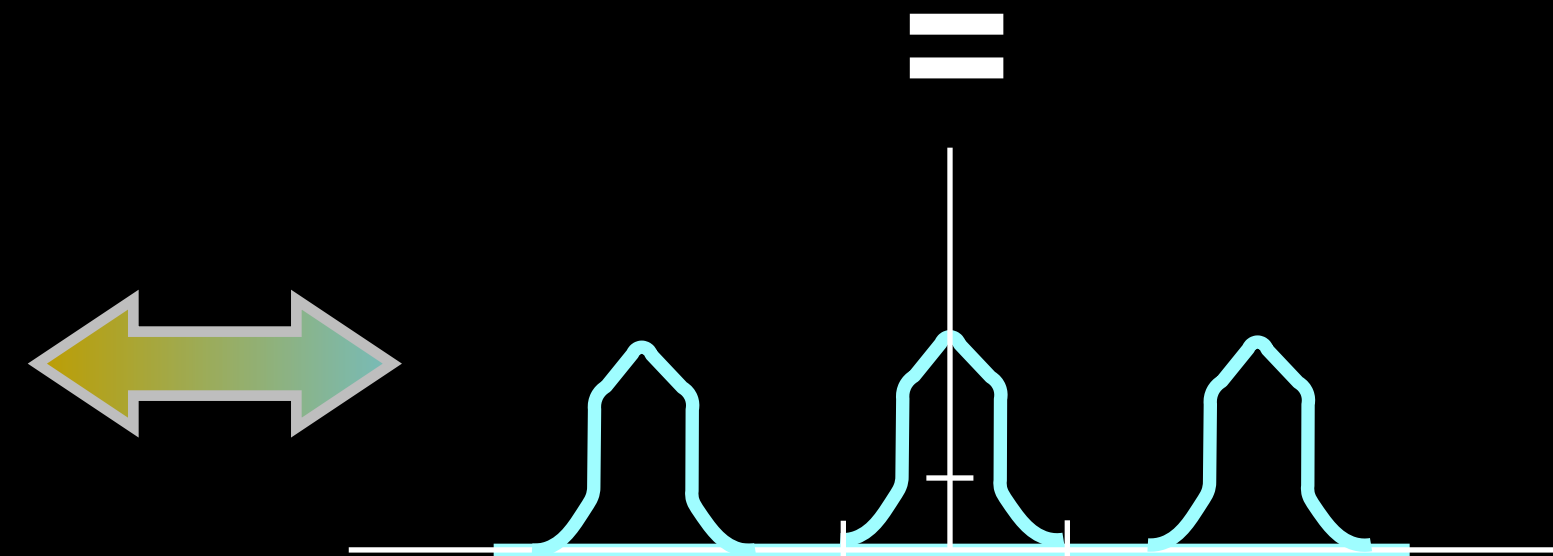
$$\sum_{n=-\infty}^{\infty} f_n \delta[k - n]$$

Sampled
signal(x)



$$\sum_{n=-\infty}^{\infty} f(n\Delta x) \delta(x - n\Delta x)$$

f(x)

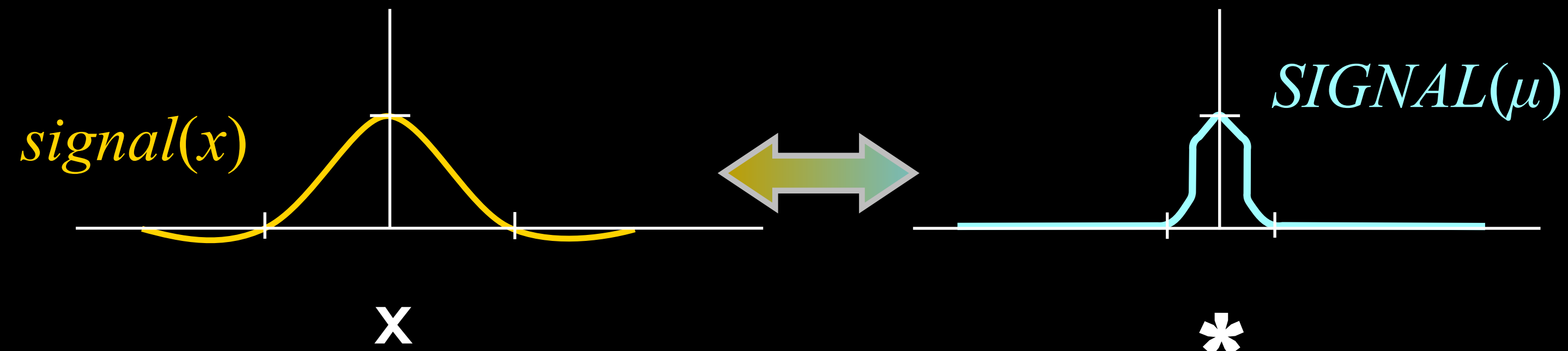
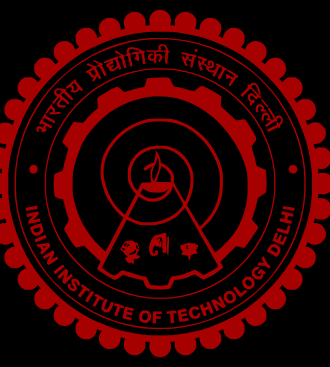


F(μ)

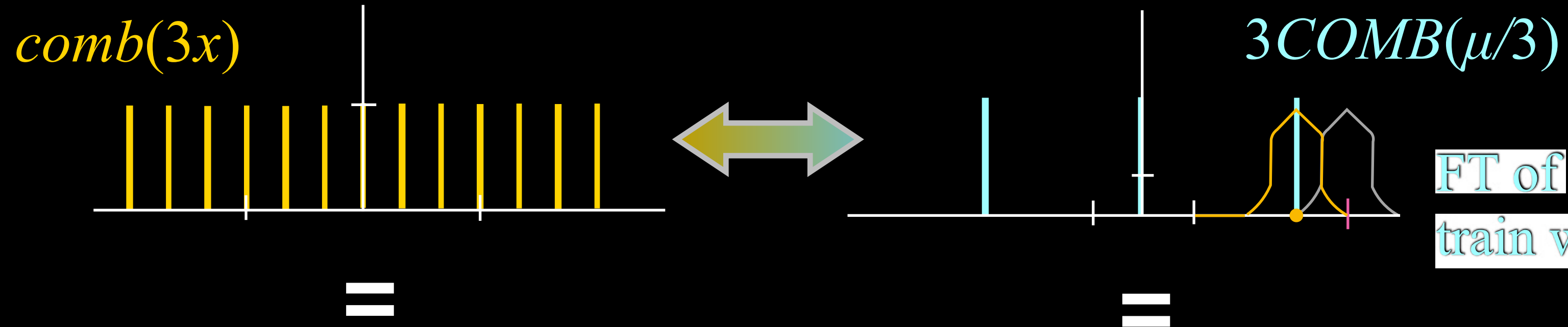
$$\delta(x) = \begin{cases} \infty & \text{at } x = 0 \\ 0, & \text{elsewhere} \end{cases}$$

$$\delta[x] = \begin{cases} 1 & \text{at } x = 0 \\ 0, & \text{elsewhere} \end{cases}$$

Sampling



Impulse
Train:



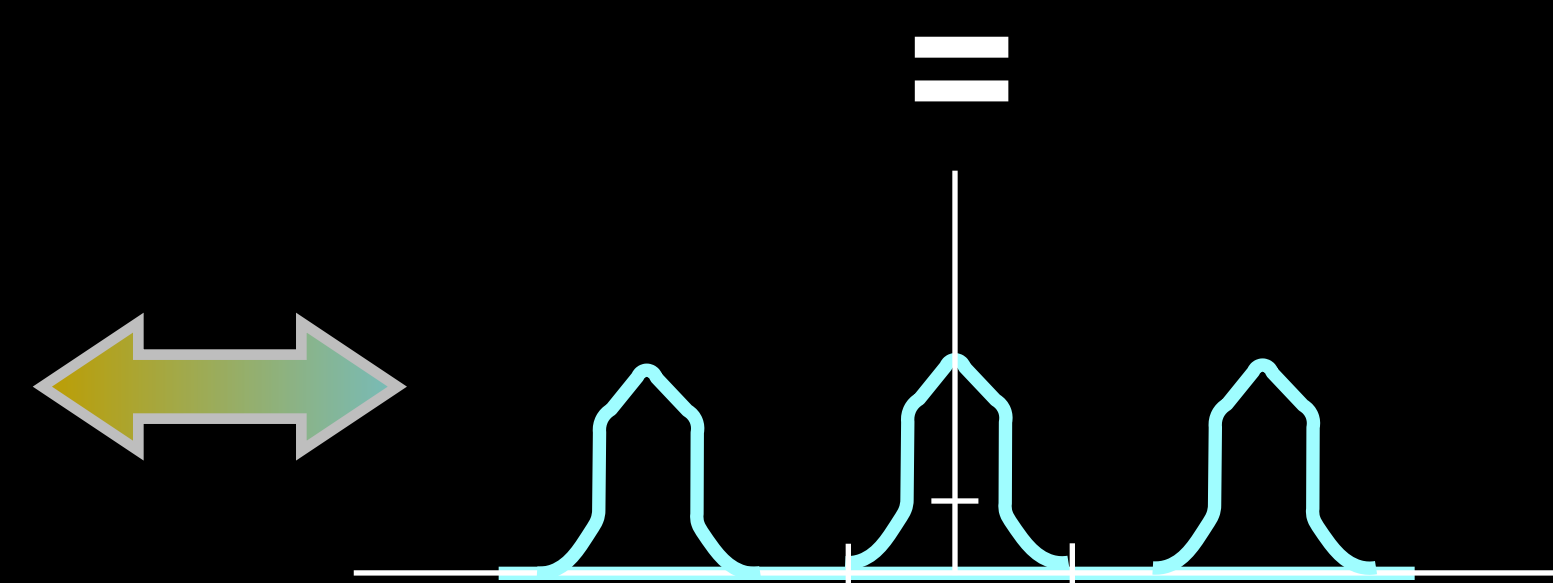
FT of $comb$ is an impulse train with inverse period

$$\sum_{n=-\infty}^{\infty} f_n \delta[k - n]$$

Sampled
 $signal(x)$

$$\sum_{n=-\infty}^{\infty} f(n\Delta x) \delta(x - n\Delta x)$$

$f(x)$



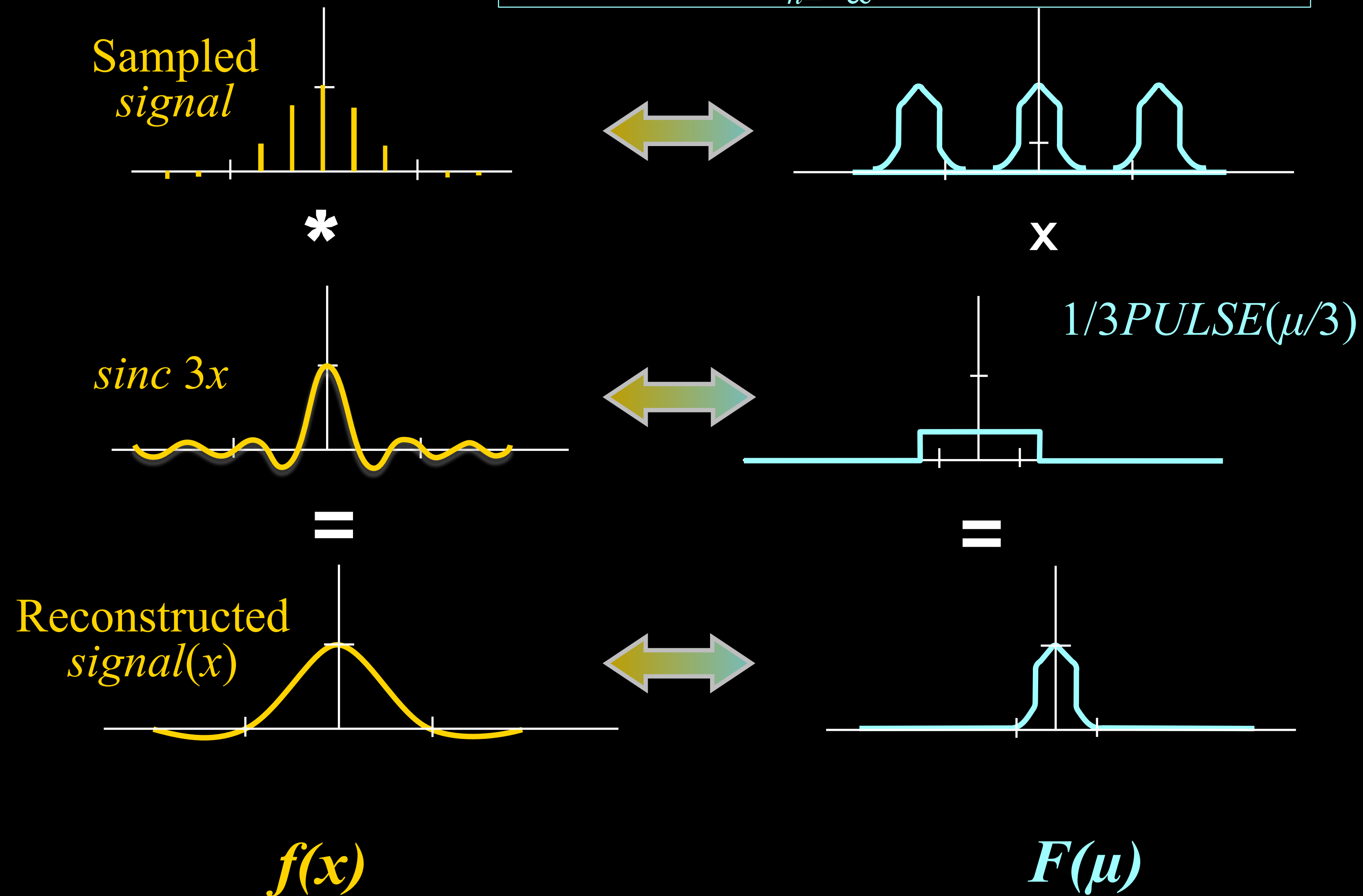
$F(\mu)$

$$\delta(x) = \begin{cases} \infty & \text{at } x = 0 \\ 0, & \text{elsewhere} \end{cases}$$

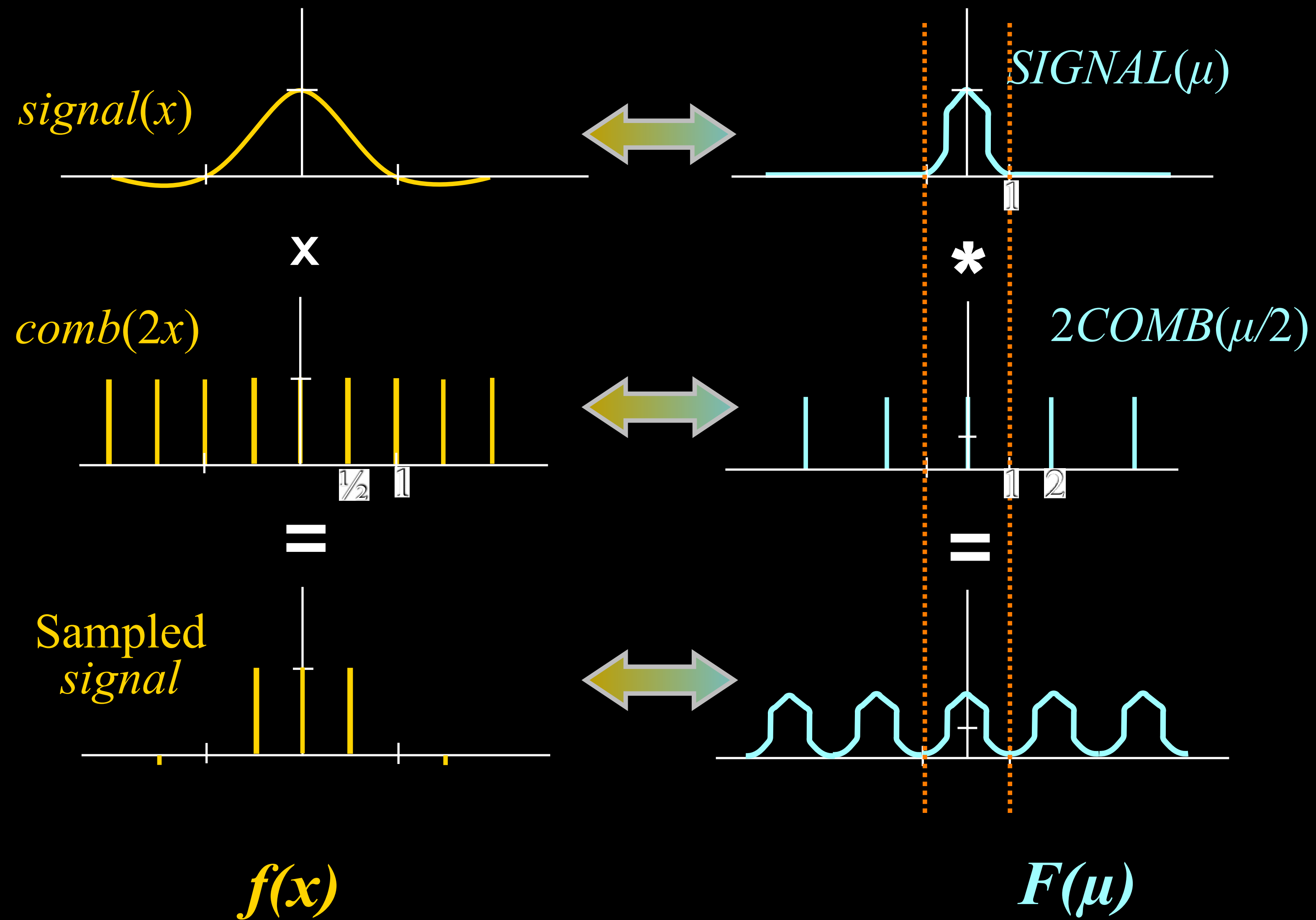
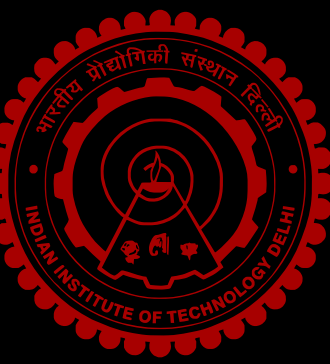
$$\delta[x] = \begin{cases} 1 & \text{at } x = 0 \\ 0, & \text{elsewhere} \end{cases}$$

Reconstruction

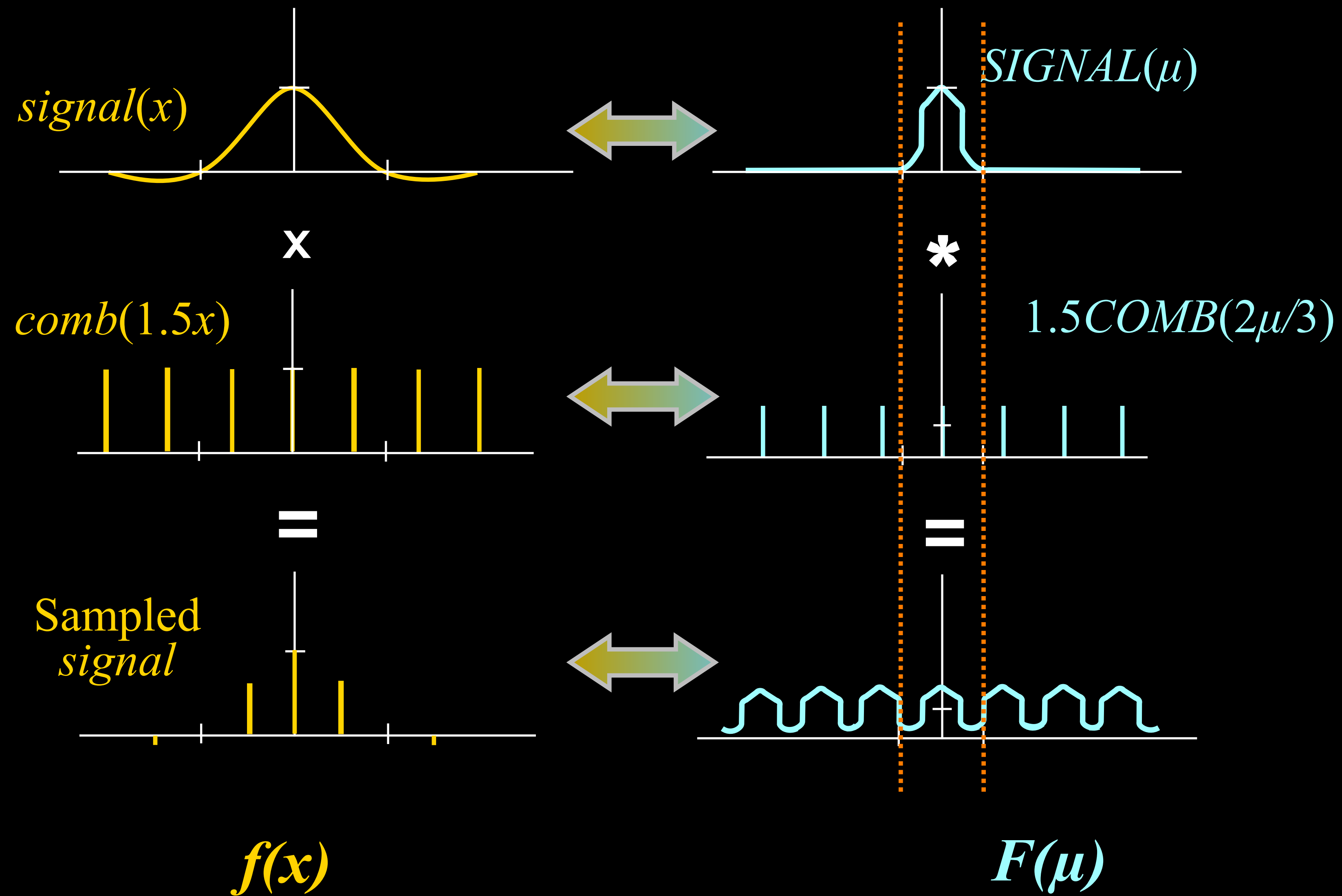
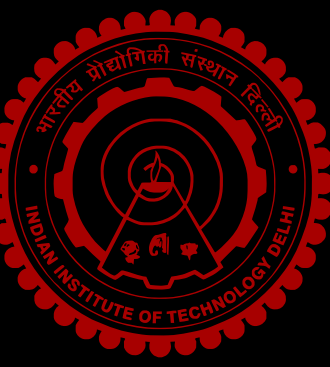
$$f(x) * \text{sinc}(x) = \sum_{n=-\infty}^{\infty} f(n\Delta x) \text{sinc}\left(\frac{x - n\Delta x}{\Delta x}\right)$$



Sufficient Samples



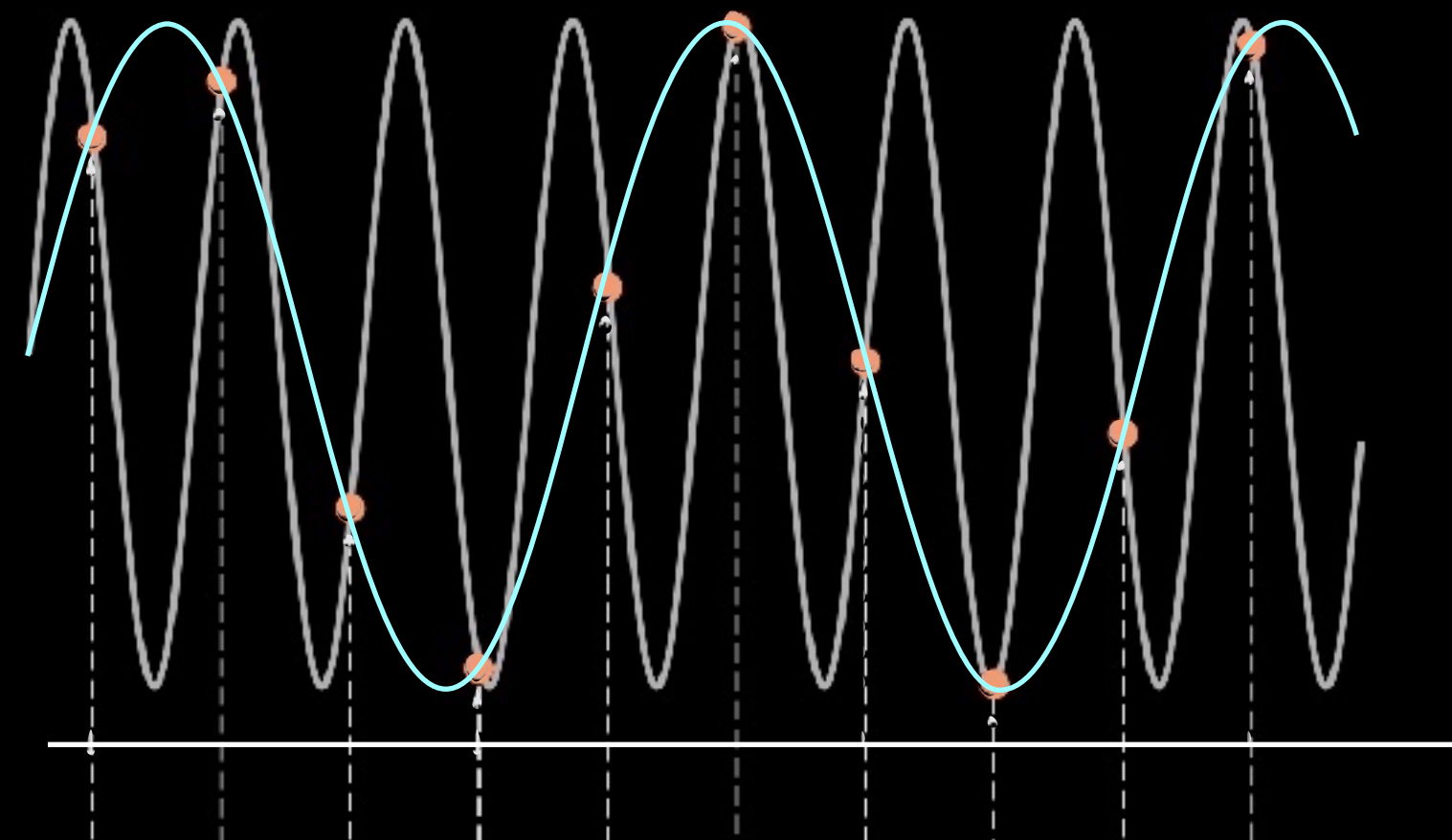
Insufficient Samples



Aliasing

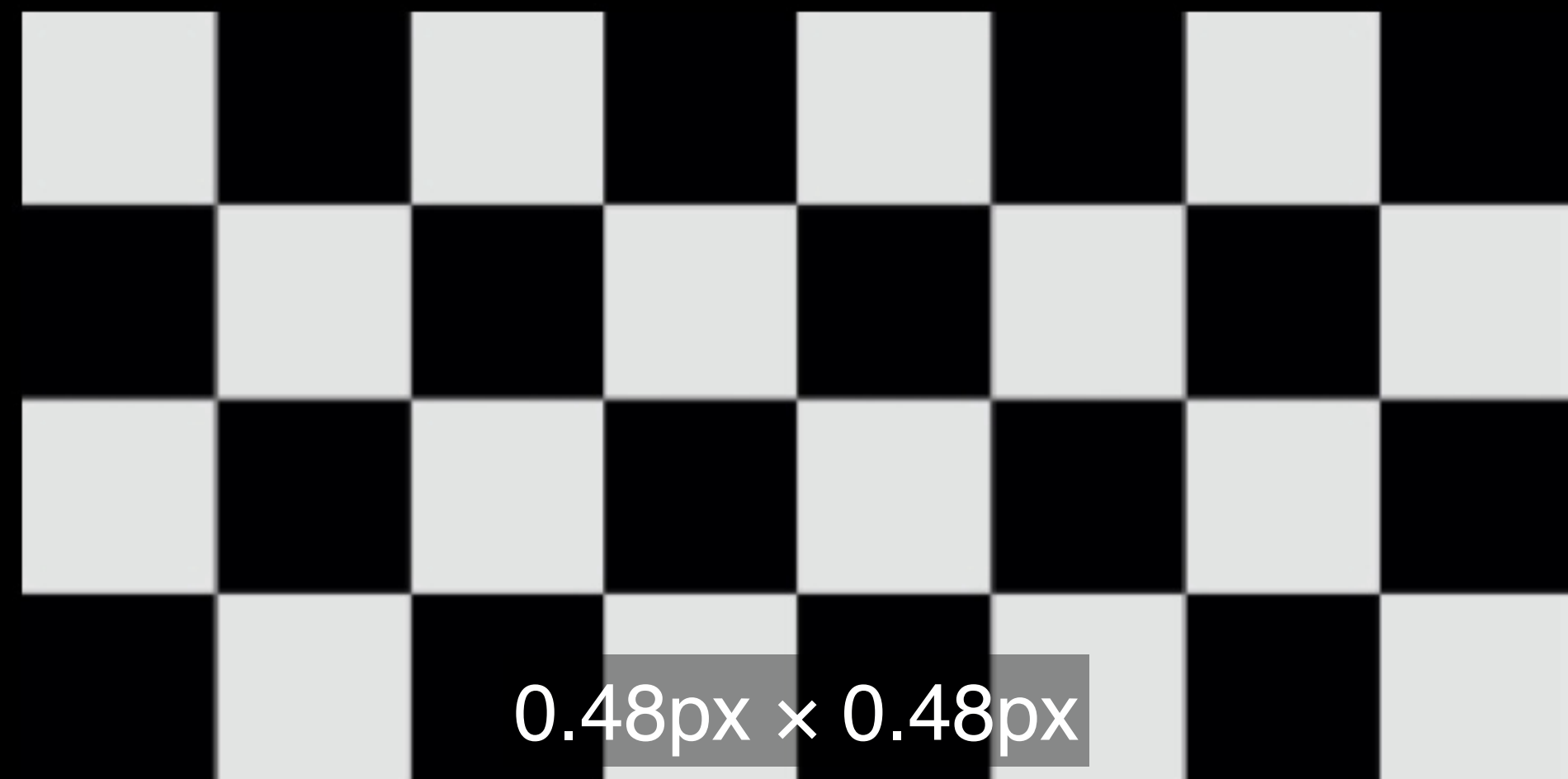
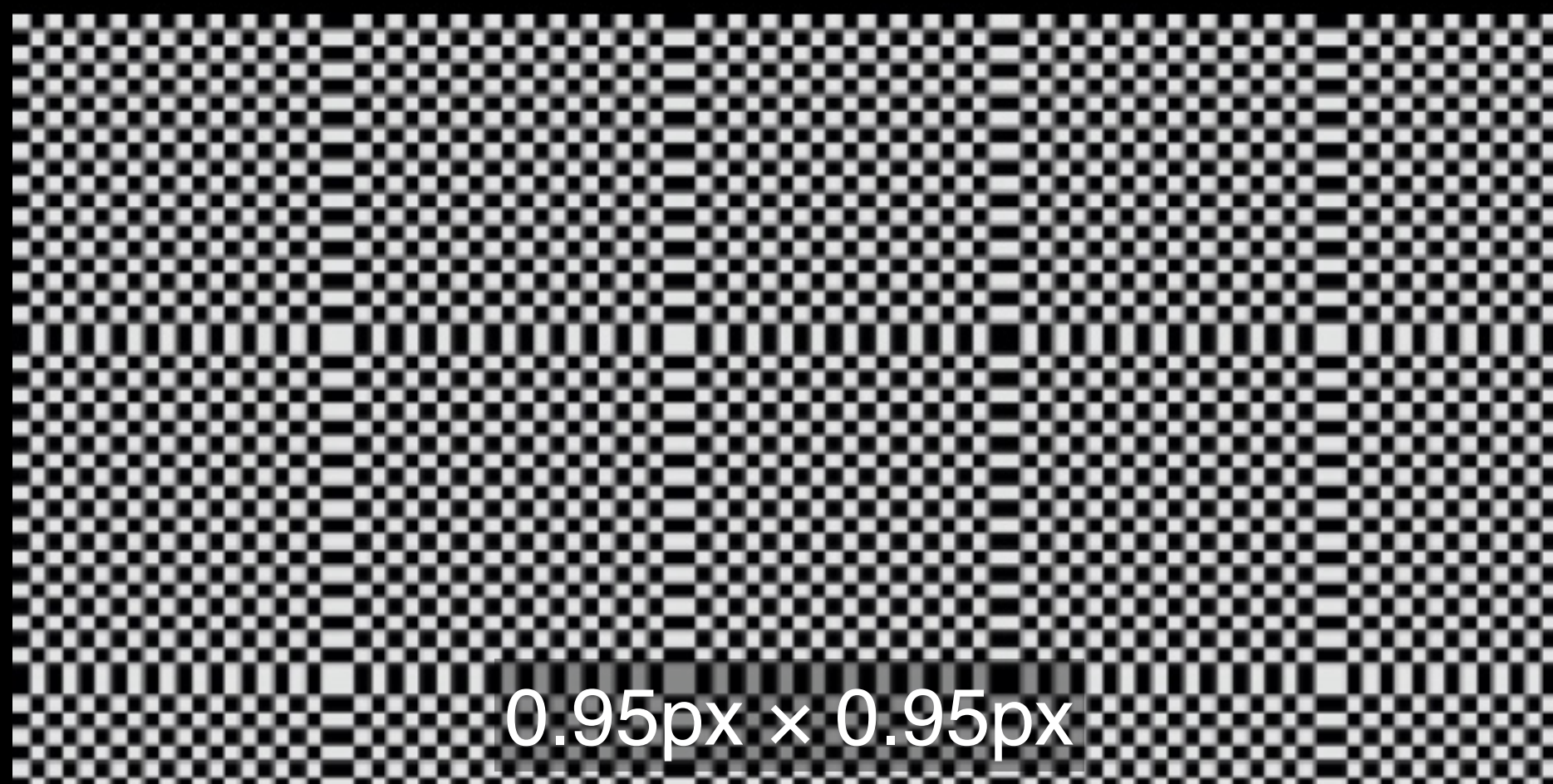
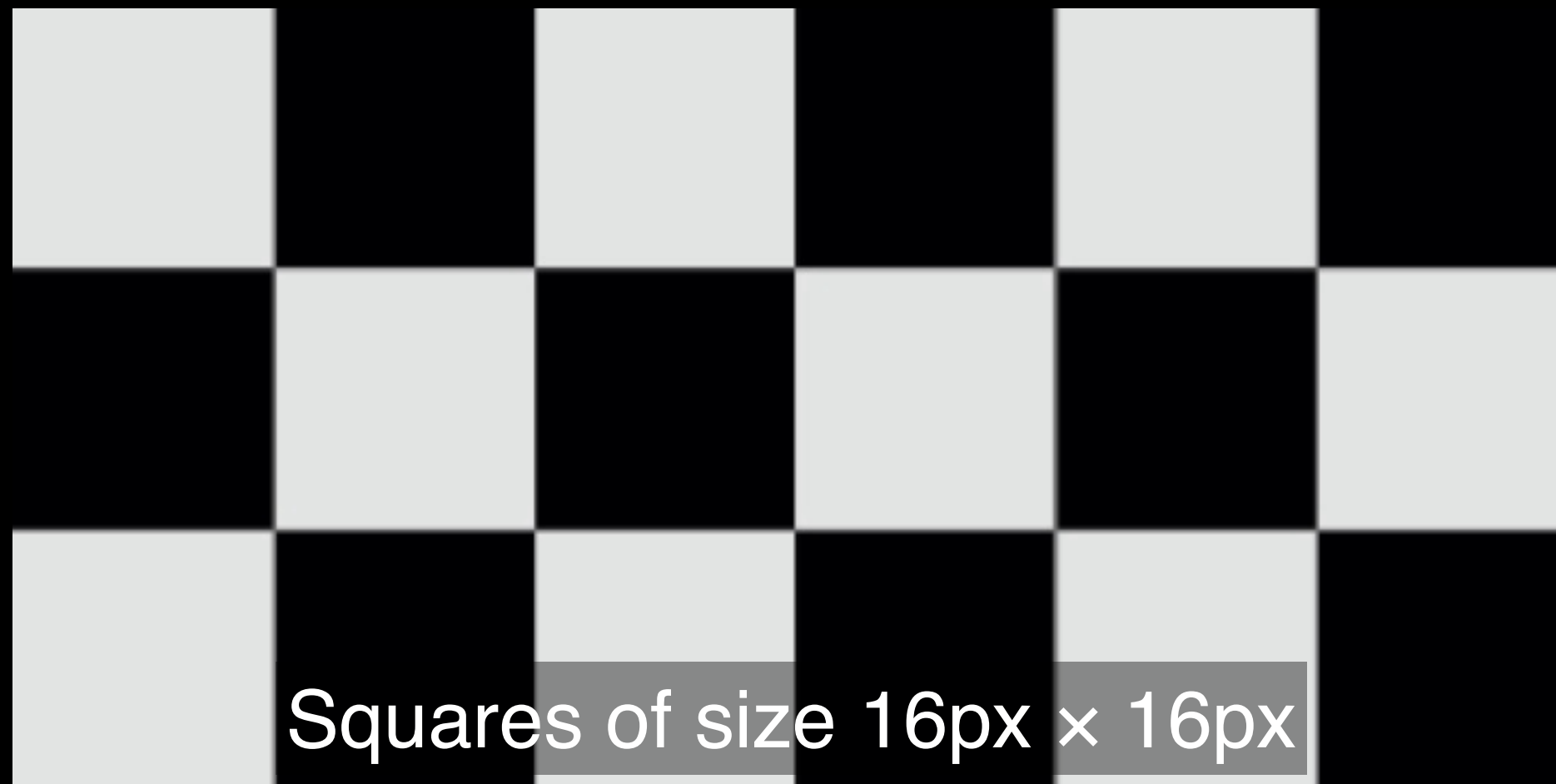
Alias: A different identity

- What happens if sampling rate is too low, $1/\Delta T < 2\mu_{\max}$?
- Different frequencies become indistinguishable!

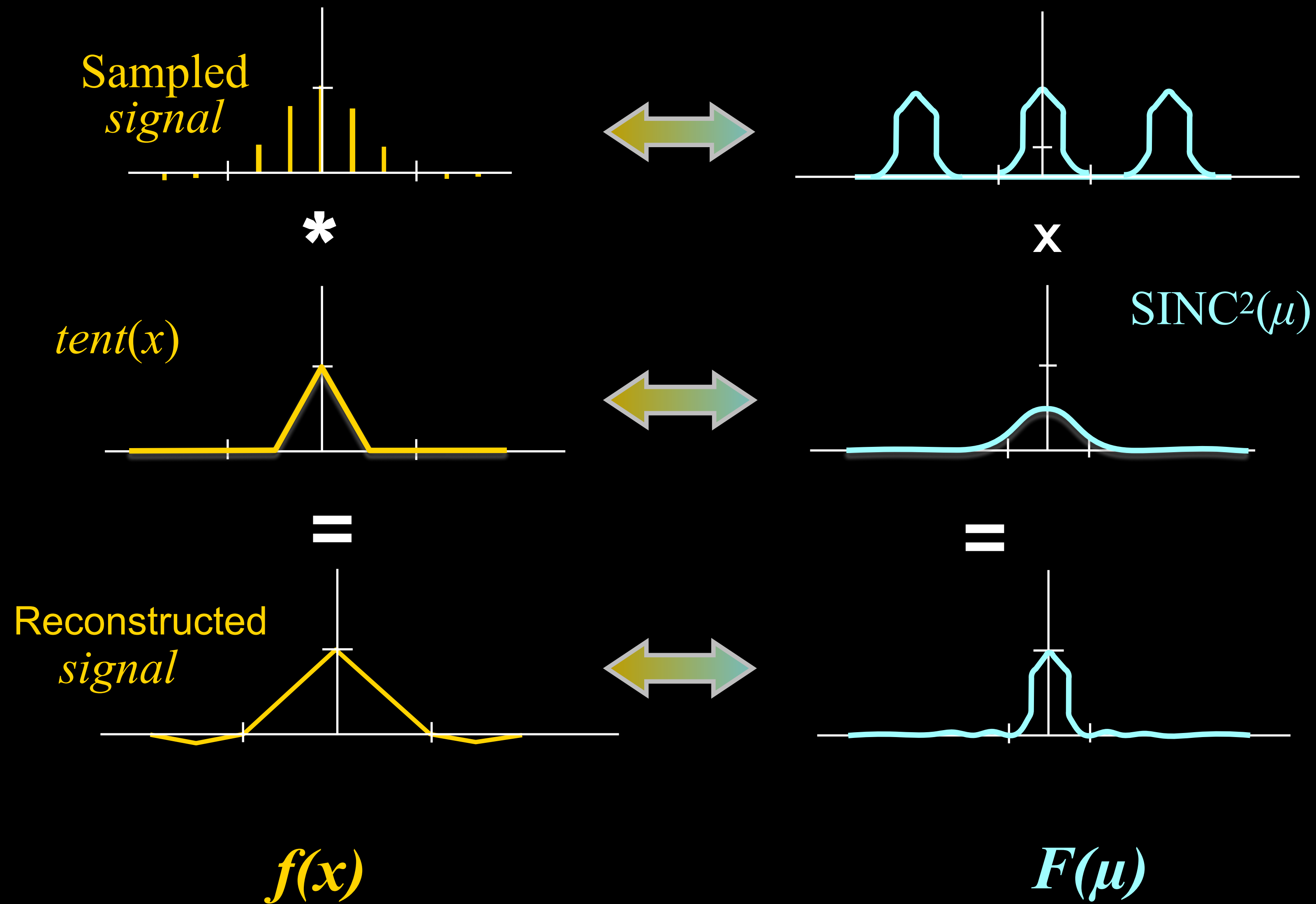


Aliasing in images

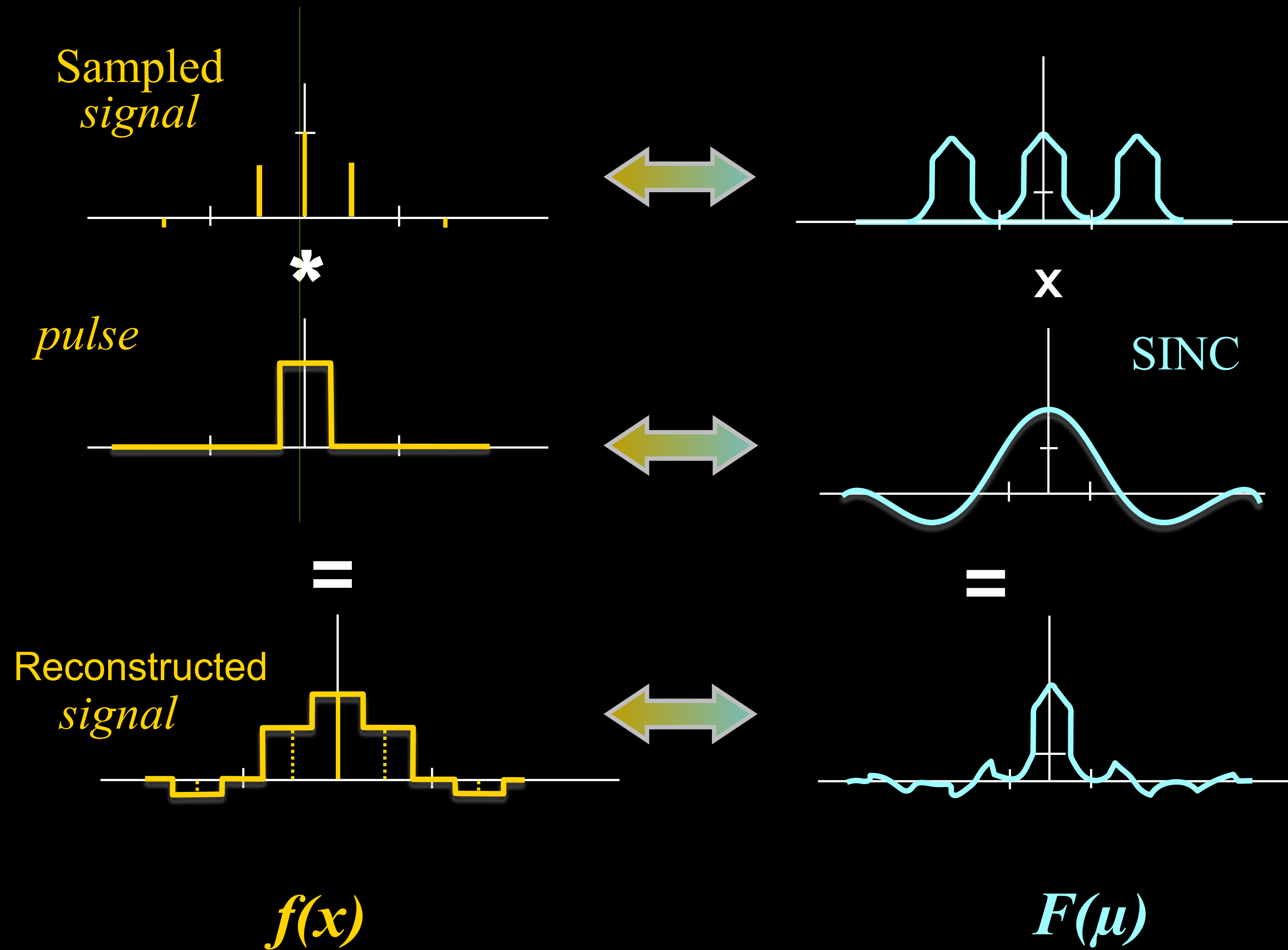
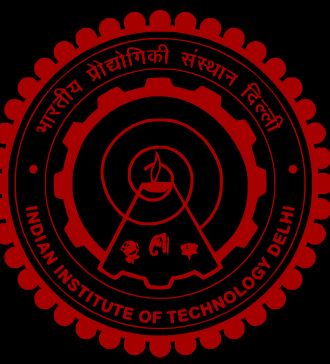
Frequency-limit signal BEFORE sampling



Triangle/Tent Filter



Box Filter

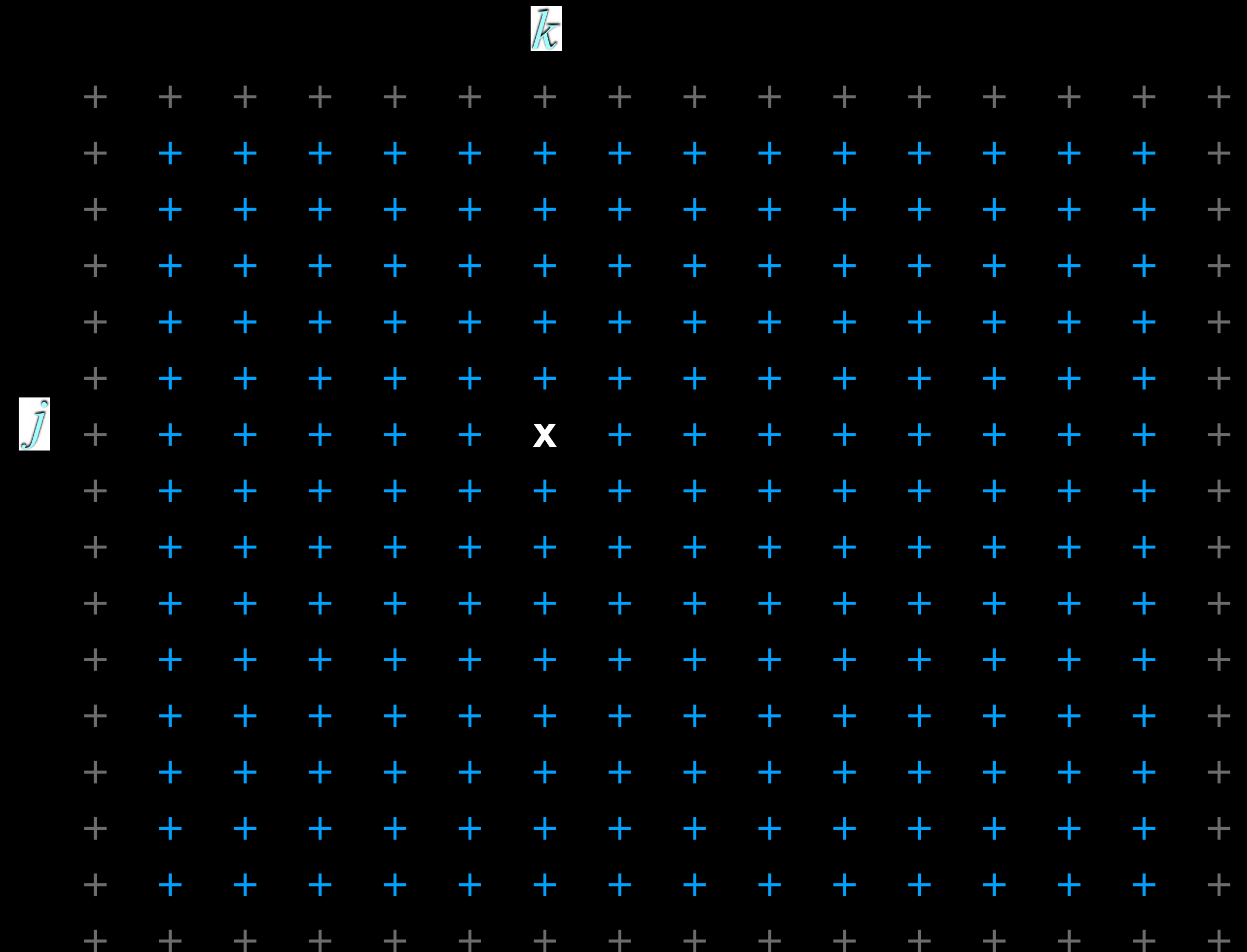


2D Box Filter

Recall convolution $(c * f)[j, k] = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} C[m, n] f[j - m, k - n]$

- $f(m, n)$ is 2D pulse/box, Height = 1?

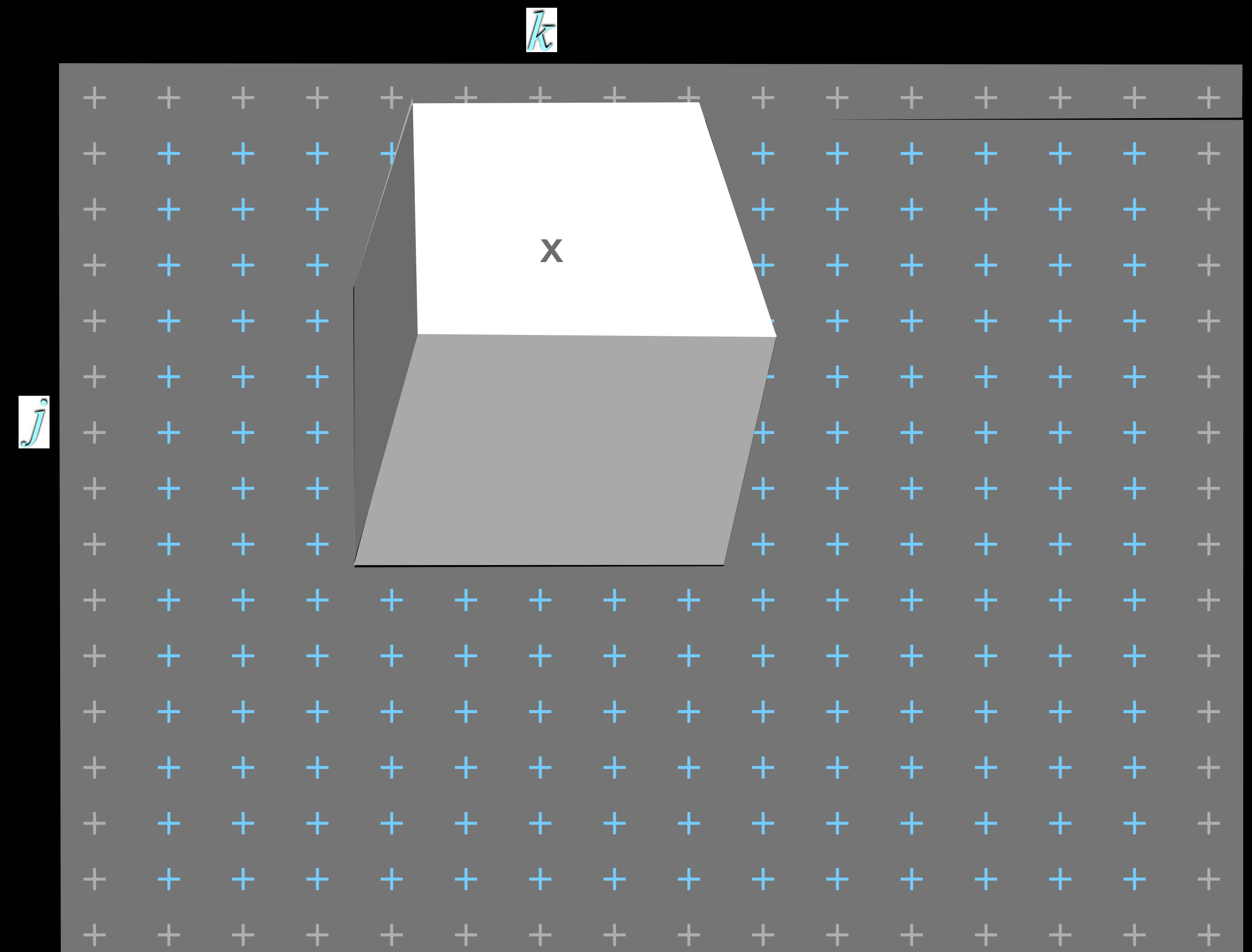
- Symmetric about x and y axes



2D Box Filter

Recall convolution $(c * f)[j, k] = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} C[m, n] f[j - m, k - n]$

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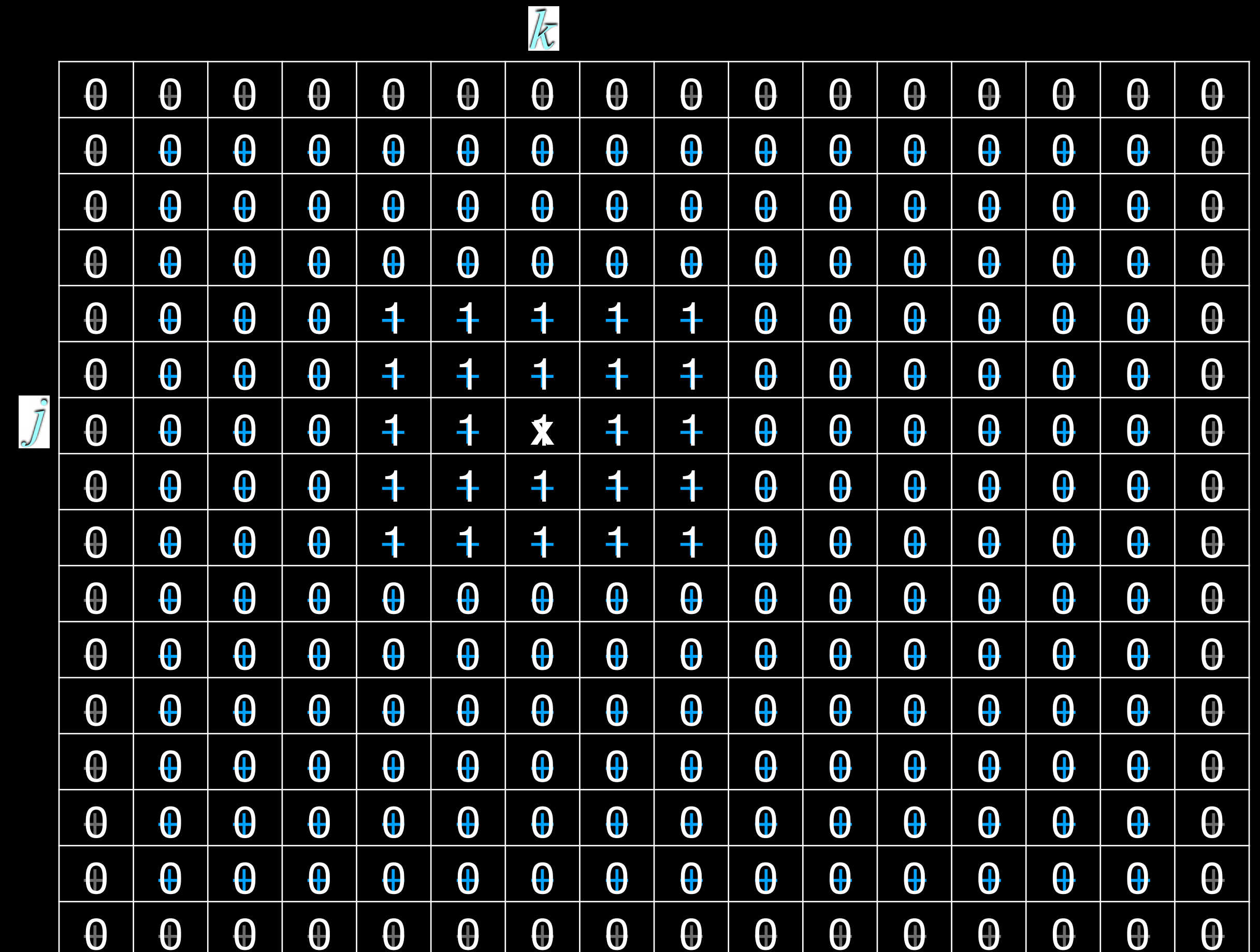


2D Box Filter

Recall convolution

$$(c * f)[j, k] = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} C[m, n] f[j - m, k - n]$$

- $f(m,n)$ is 2D pulse/box, Height = 1?
 - Symmetric about x and y axes



2D Box Filter

Recall convolution $(c * f)[j, k] = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} C[m, n] f[j - m, k - n]$

- $f(m, n)$ is 2D pulse/box, Height = 1?
- Symmetric about x and y axes
- Normalize: Divide by sum of filter values

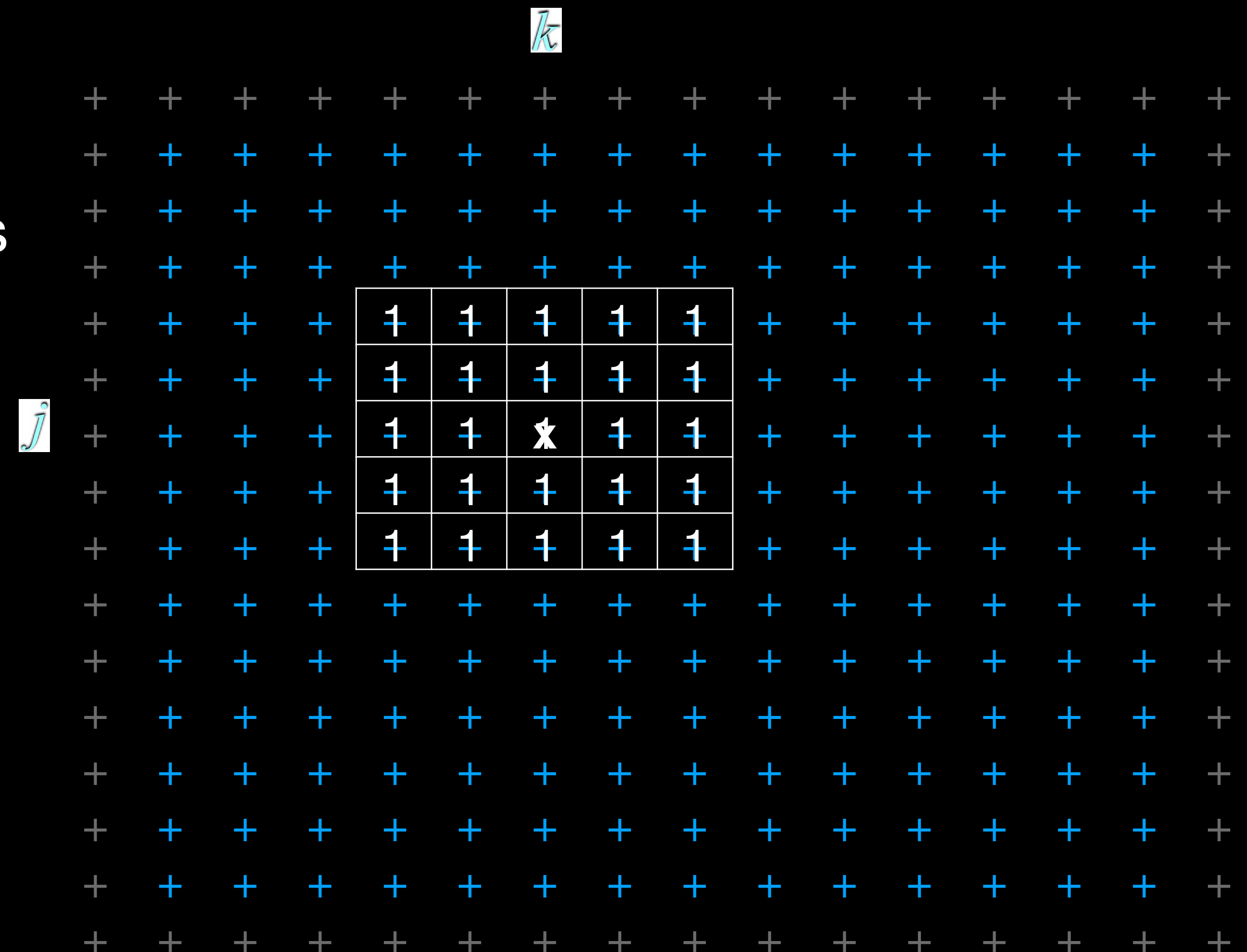
$$F[\mu, \nu] = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f[m, n] e^{-i2\pi \frac{\mu m + \nu n}{N}}$$

$$\Rightarrow F[0, 0] = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f[m, n]$$

(DC component)

Normalizing $\Rightarrow C[0, 0]$ is scaled by 1 in Fourier domain

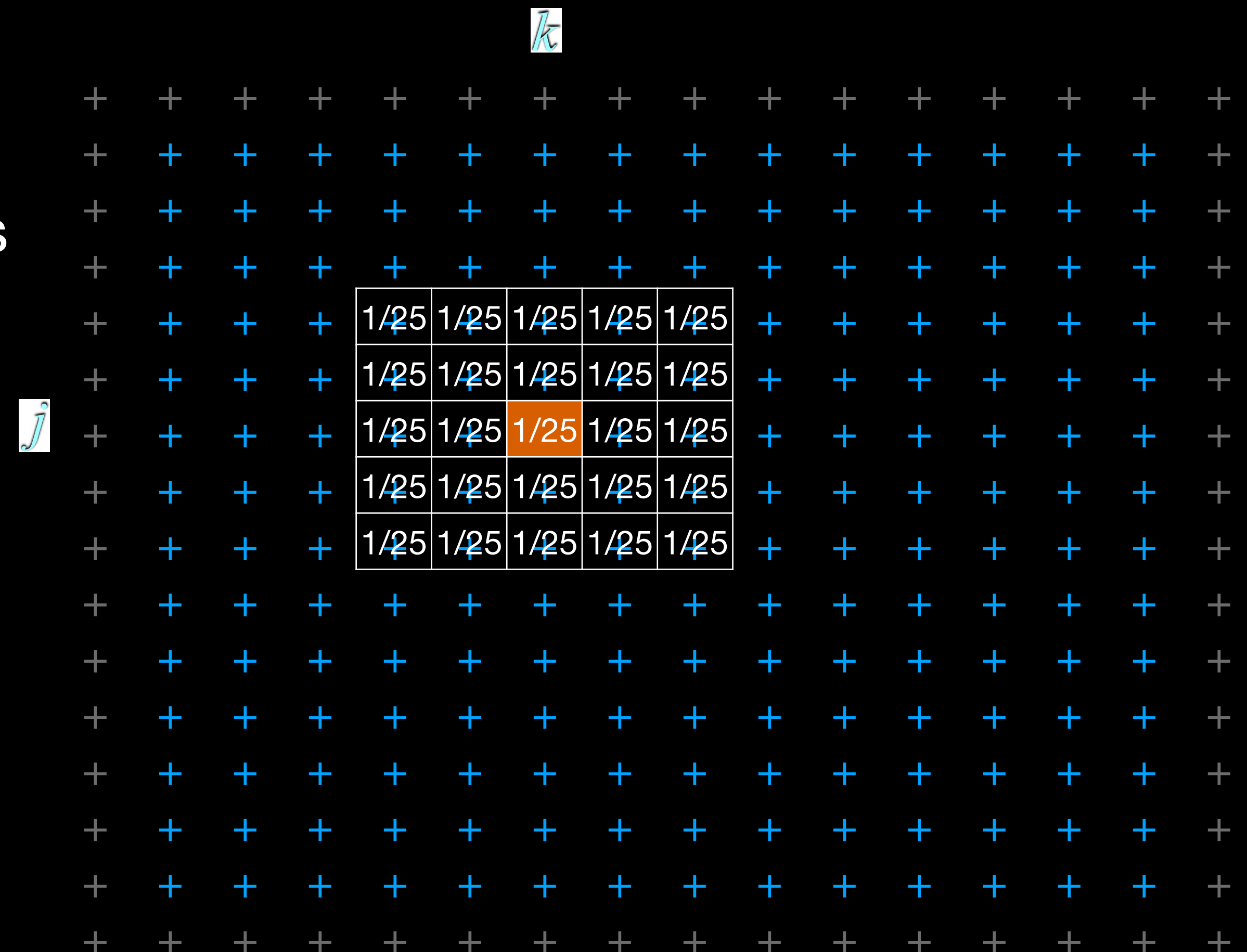
i.e., DC component of $c^* f$ and c are the same



2D Box Filter

Recall convolution $(c * f)[j, k] = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} C[m, n] f[j - m, k - n]$

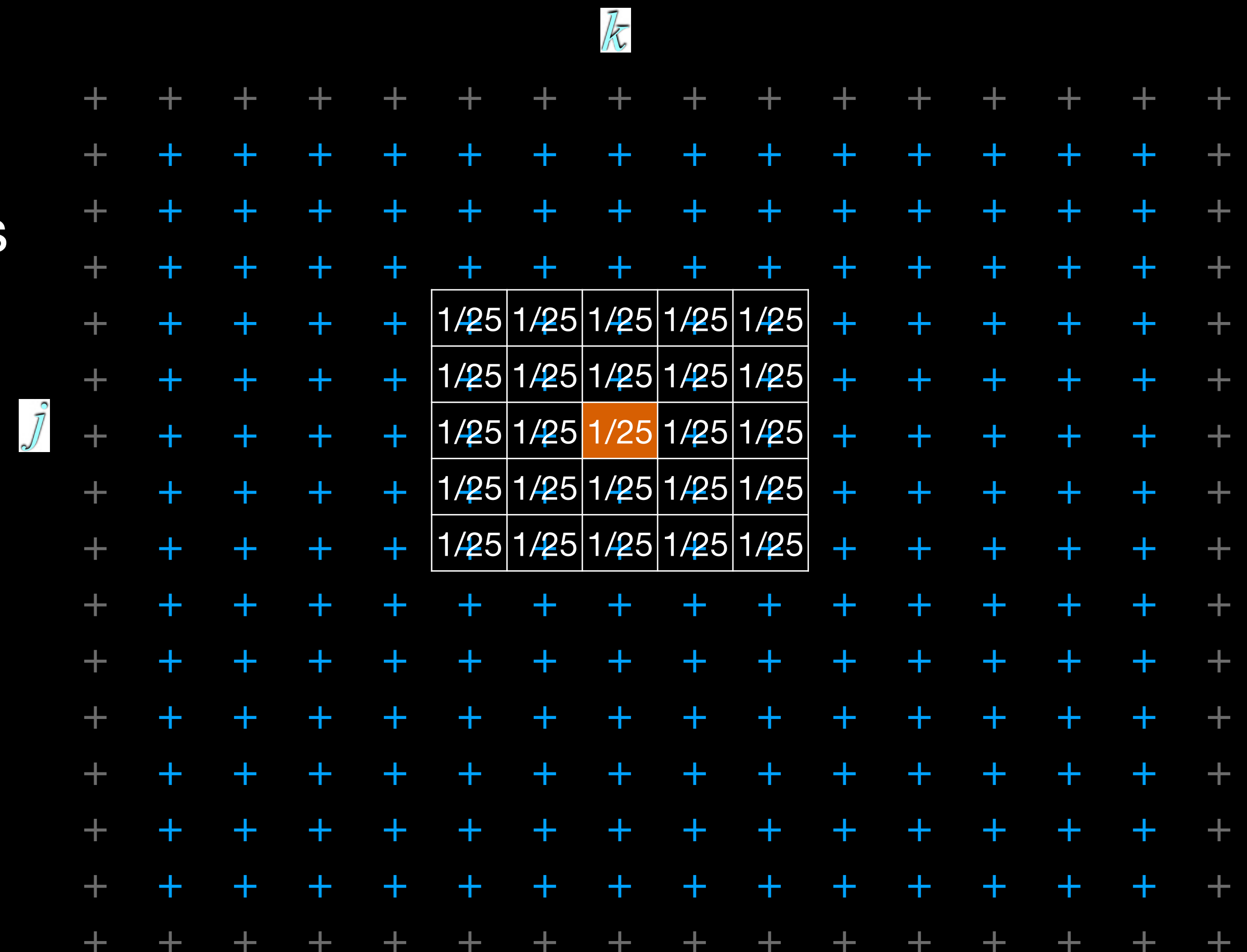
- $f(m, n)$ is 2D pulse/box, Height = 1?
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2D Box Filter

Recall convolution $(c * f)[j, k] = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} C[m, n] f[j - m, k - n]$

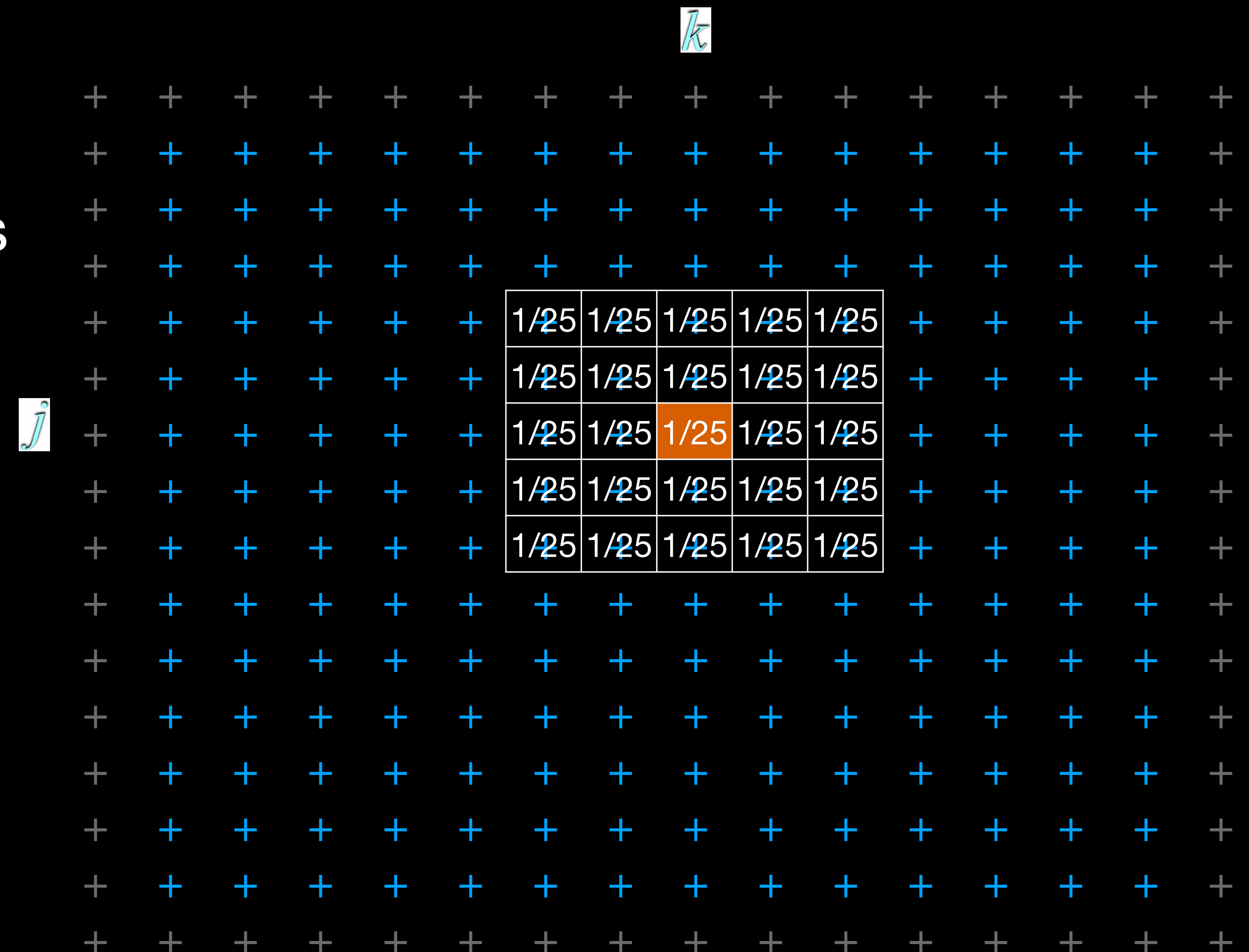
- $f(m, n)$ is 2D pulse/box, Height = 1?
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2D Box Filter

Recall convolution $(c * f)[j, k] = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} C[m, n] f[j - m, k - n]$

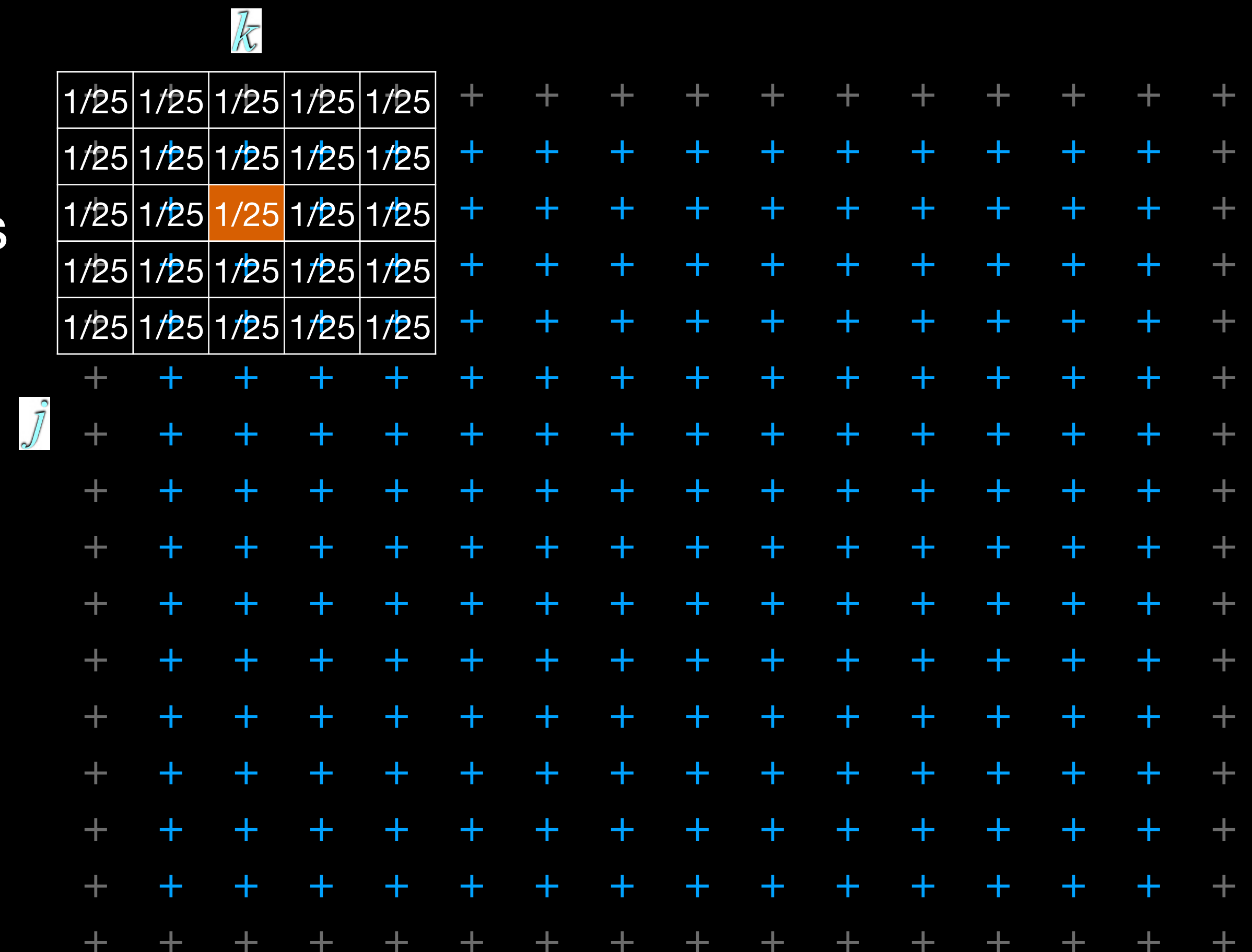
- $f(m, n)$ is 2D pulse/box, Height = 1?
- Symmetric about x and y axes
- Normalize: Divide by sum of filter values

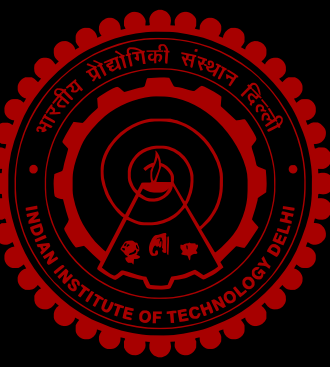


2D Box Filter

Recall convolution $(c * f)[j, k] = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} C[m, n] f[j - m, k - n]$

- $f(m, n)$ is 2D pulse/box, Height = 1?
 - Symmetric about x and y axes
 - Normalize: Divide by sum of filter values
- Options for boundary color
 - All 0
 - Repeat the edge colors
 - Repeat (periodically)

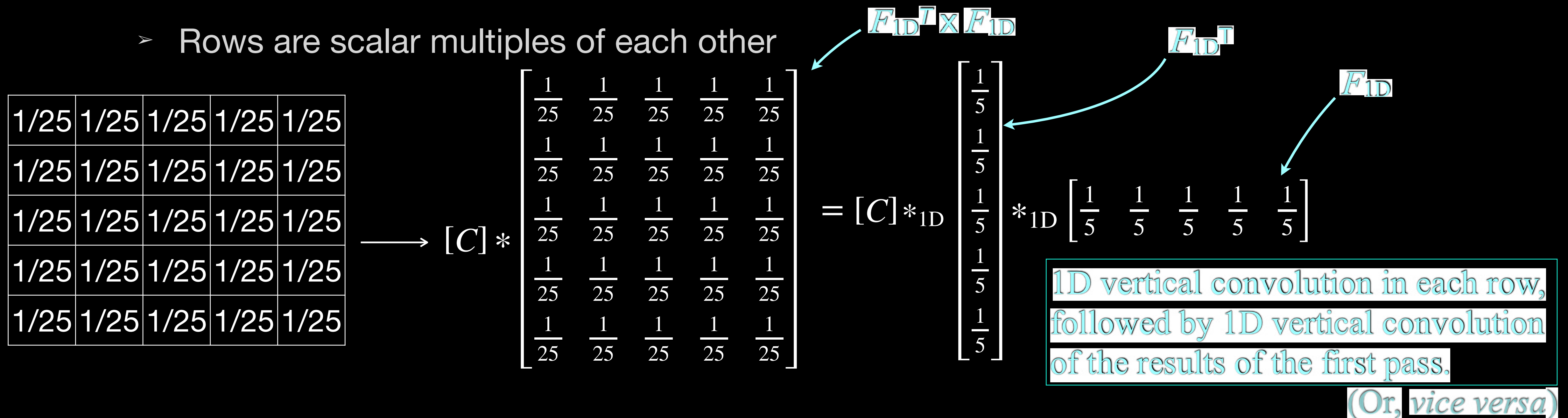




2D Box Filter is Separable

* Box $\Rightarrow$ x SINC (Fourier domain)

- $n \times n$ (Odd valued n)
 - $O(nMN)$ for an $M \times N$ image, as it is separable instead of $O(n^2MN)$
 - > $\Rightarrow$ Rank 1: rows are linearly dependent on each other
 - > Rows are scalar multiples of each other


$$\begin{bmatrix} 1/25 & 1/25 & 1/25 & 1/25 & 1/25 \\ 1/25 & 1/25 & 1/25 & 1/25 & 1/25 \\ 1/25 & 1/25 & 1/25 & 1/25 & 1/25 \\ 1/25 & 1/25 & 1/25 & 1/25 & 1/25 \\ 1/25 & 1/25 & 1/25 & 1/25 & 1/25 \end{bmatrix} \longrightarrow [C] * \begin{bmatrix} \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} \end{bmatrix} = [C] *_{1D} \begin{bmatrix} \frac{1}{5} \\ \frac{1}{5} \\ \frac{1}{5} \\ \frac{1}{5} \\ \frac{1}{5} \end{bmatrix} *_{1D} \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

1D vertical convolution in each row,
followed by 1D vertical convolution
of the results of the first pass.
(Or, vice versa)

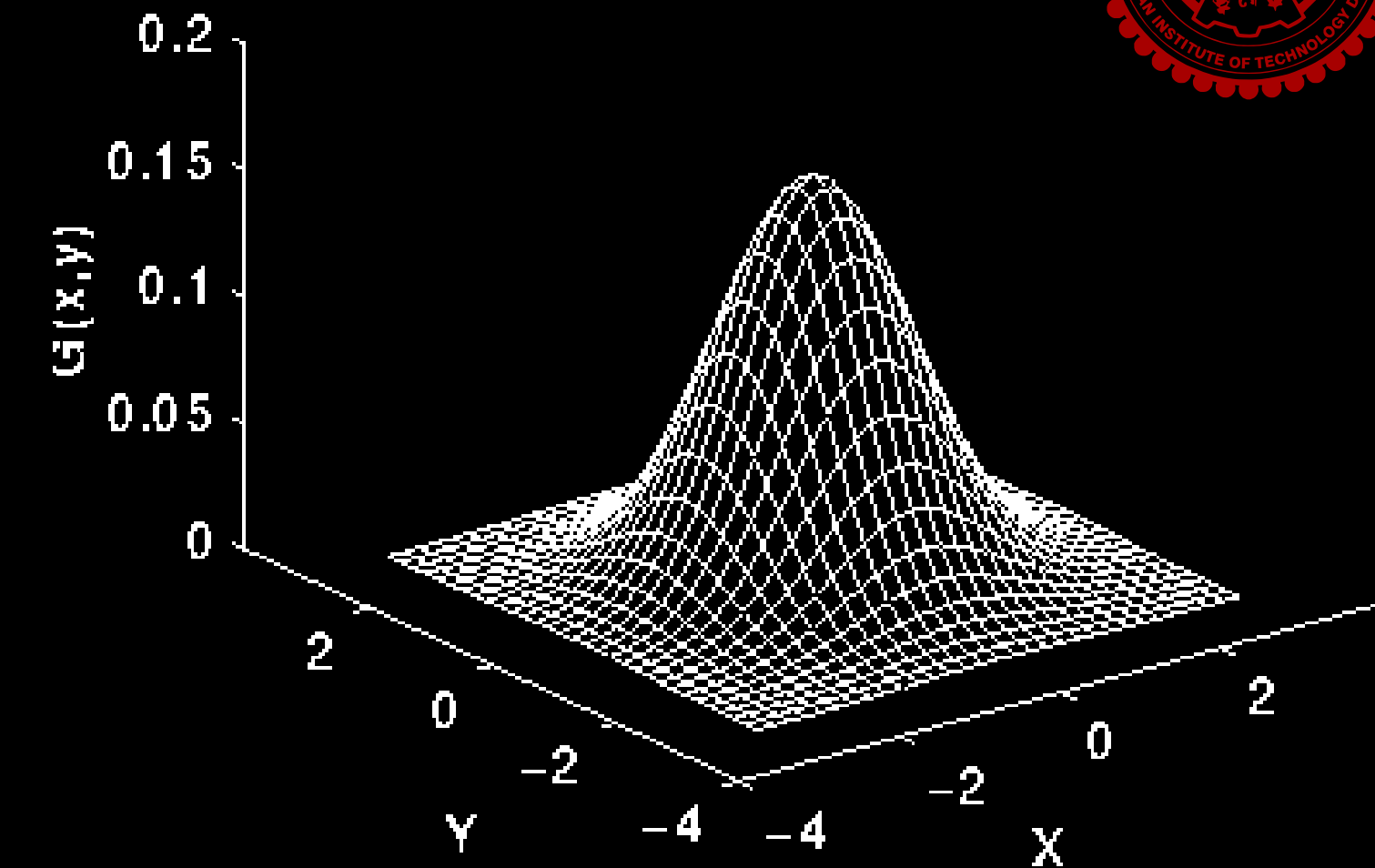
Gaussian Filter

* Gaussian $\Rightarrow$ x GAUSSIAN

$$g(x, y) = Ke^{-\frac{x^2}{2\sigma_x^2}} e^{-\frac{y^2}{2\sigma_y^2}} \sim Ke^{-\frac{x^2 + y^2}{2\sigma^2}}$$

- Separable and isotropic
- Gaussian is small for $|x|, |y| > 3\sigma$ (Even 2σ)
- Given σ , fix kernel radius $r = \lceil 3\sigma \rceil \Rightarrow M = N = 2r + 1$

“Blur size”



0.0030	0.0133	0.0219	0.0133	0.0030
0.0133	0.0596	0.0983	0.0596	0.0133
0.0219	0.0983	0.1621	0.0983	0.0219
0.0133	0.0596	0.0983	0.0596	0.0133
0.0030	0.0133	0.0219	0.0133	0.0030

5x5, $\sigma = 1$



Cameraman



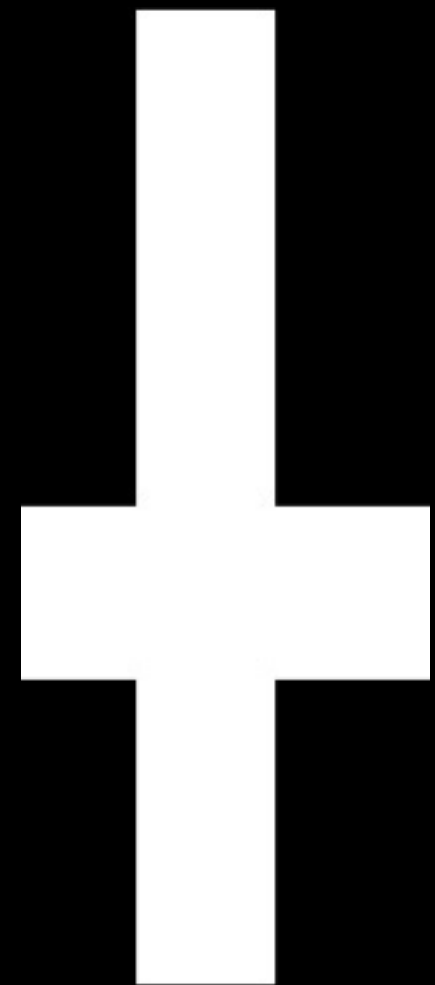
25x25 Box



25x25 Gaussian

The middle 5x5 of a 7x7 Gaussian with $\sigma = 1$ has 98.23% of the weight

Box vs Gaussian Filter



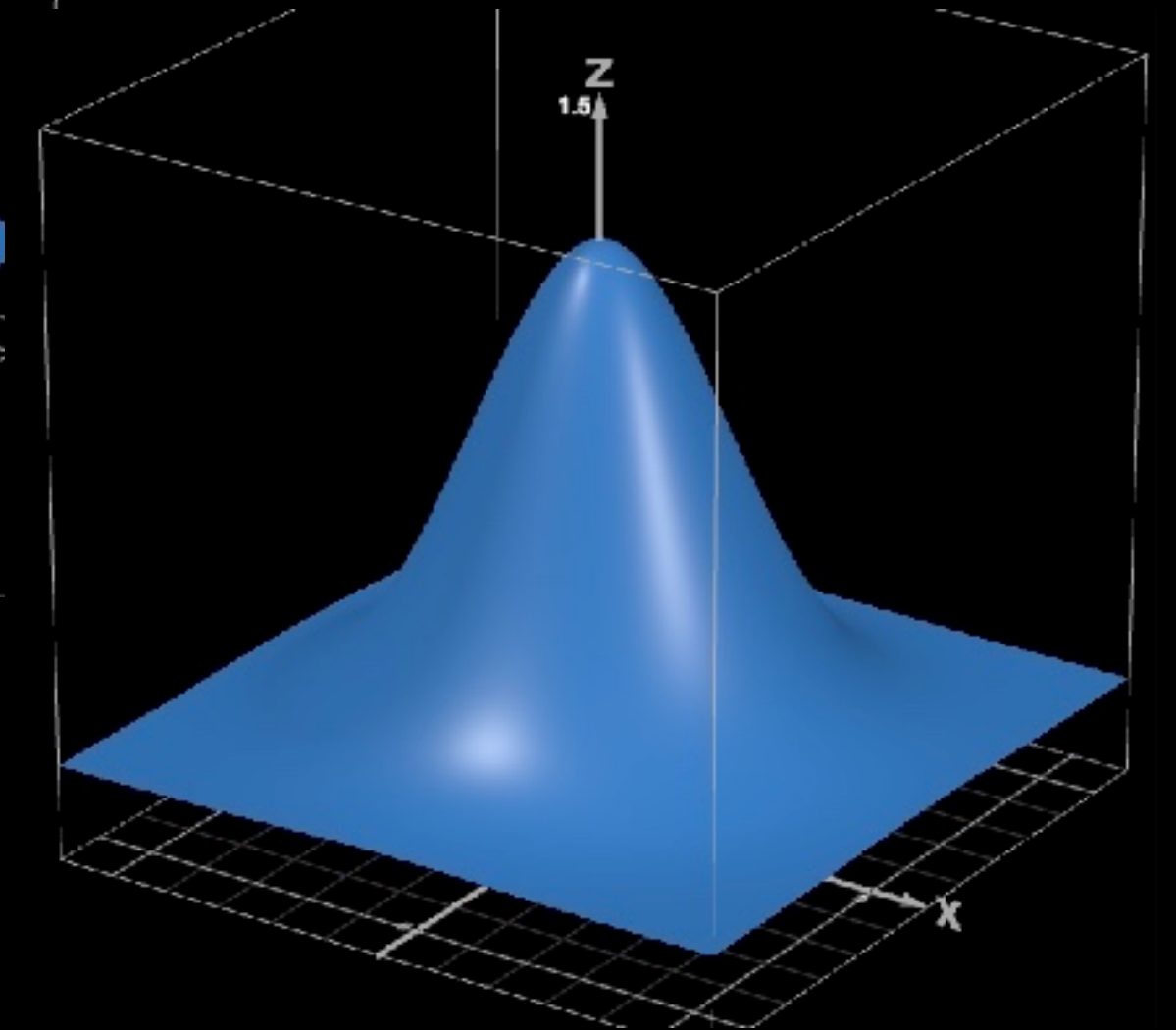
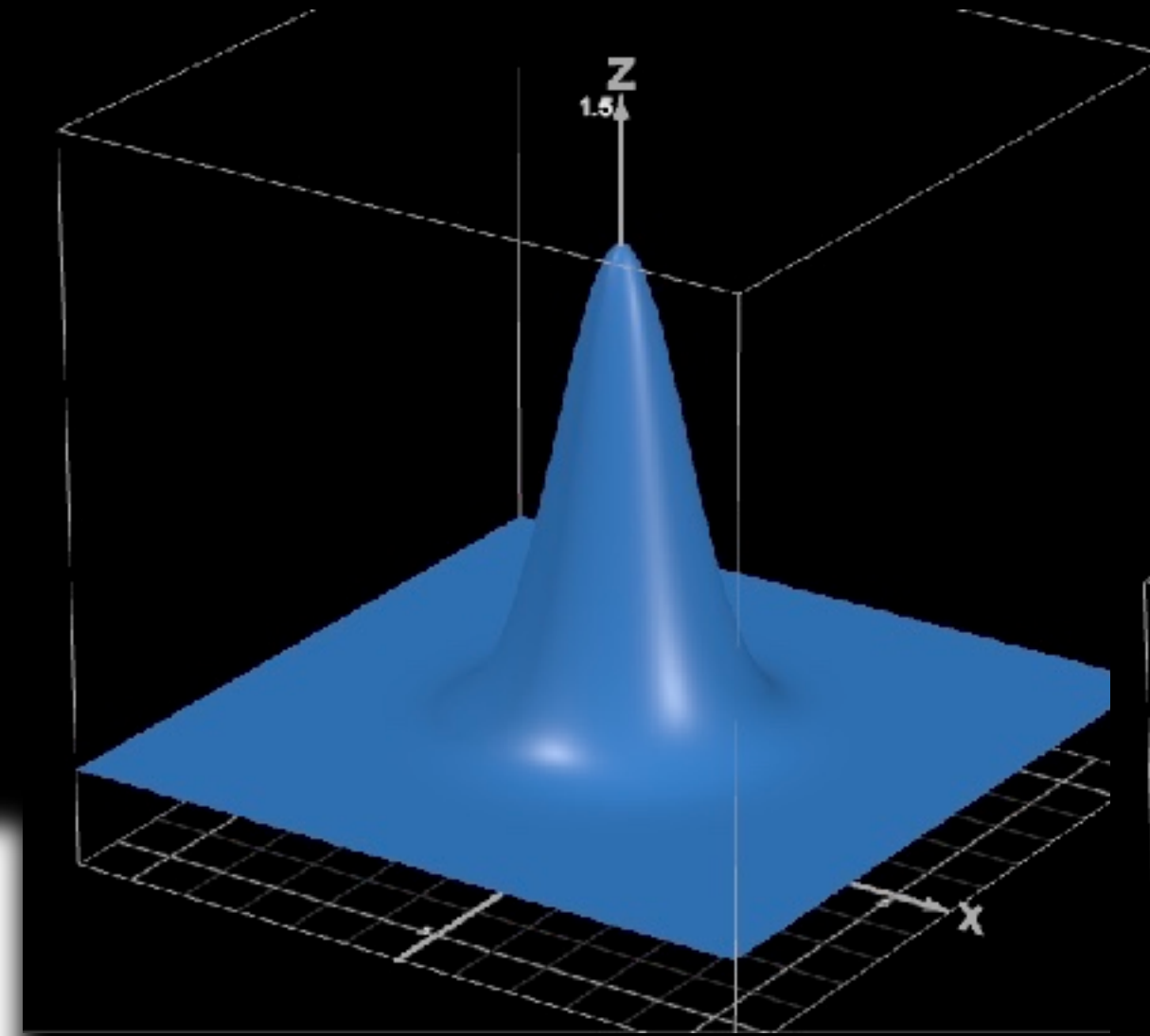
Pulse-like



Convolved with Box
(Tent-like)



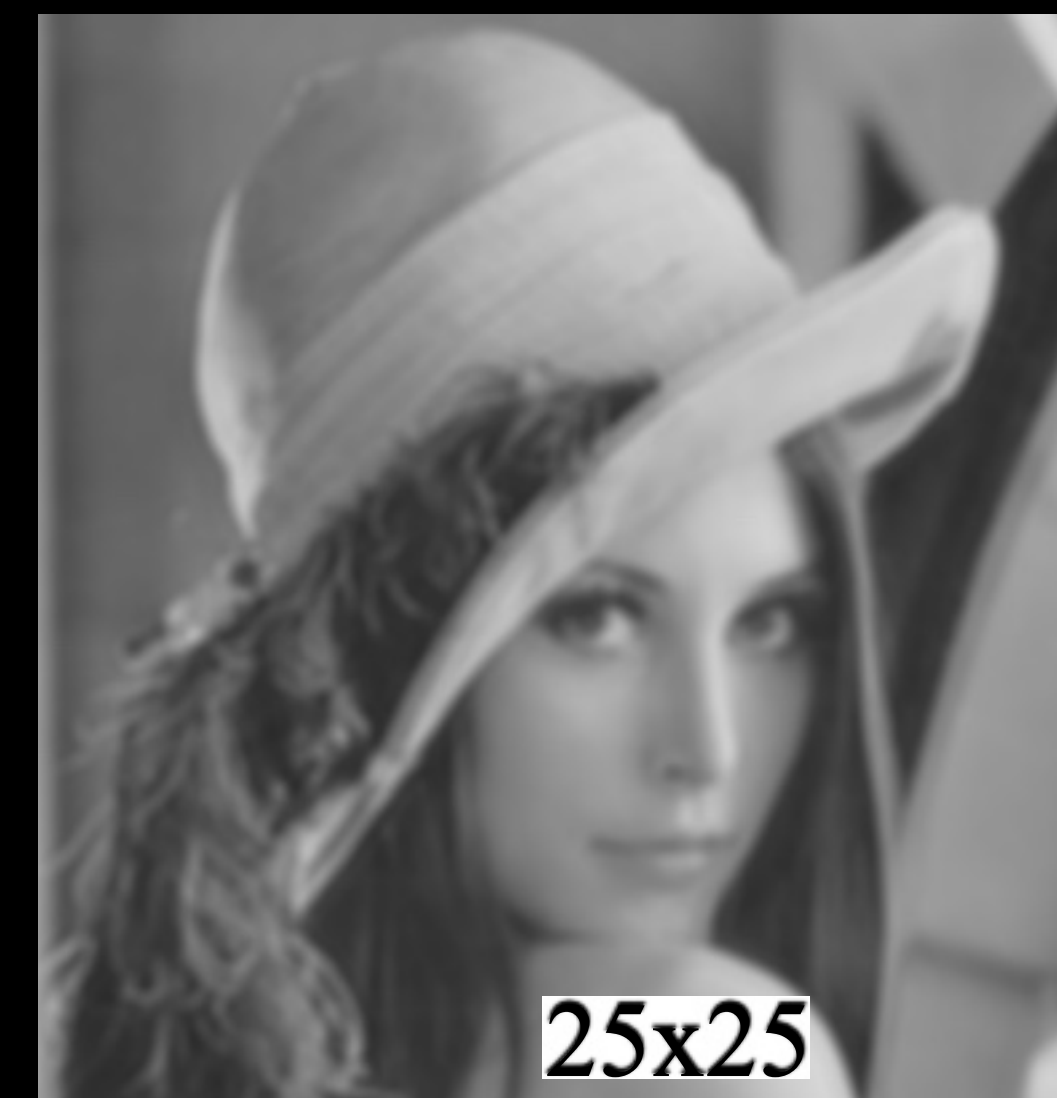
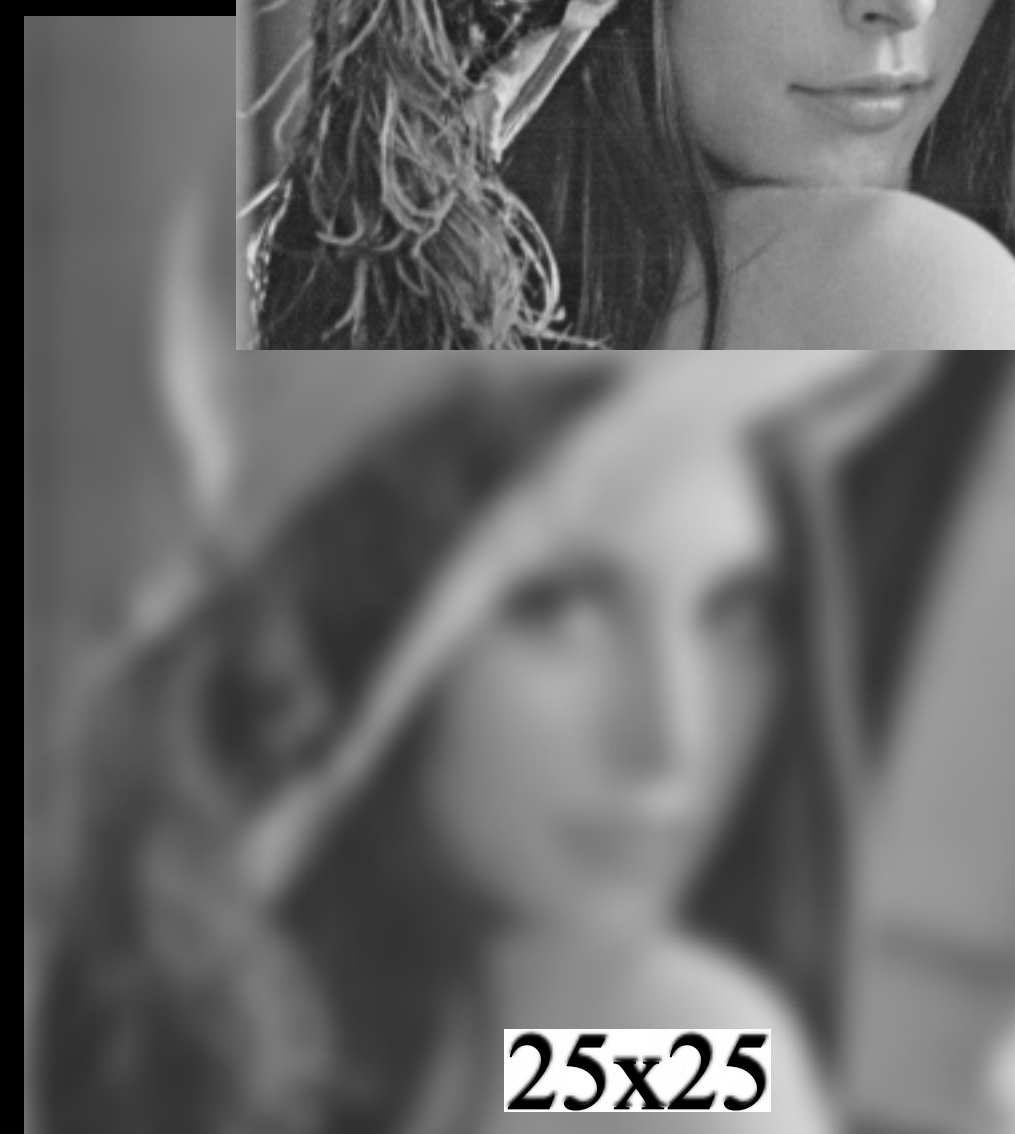
Convolved with Gaussian
(Flattened Gaussian)



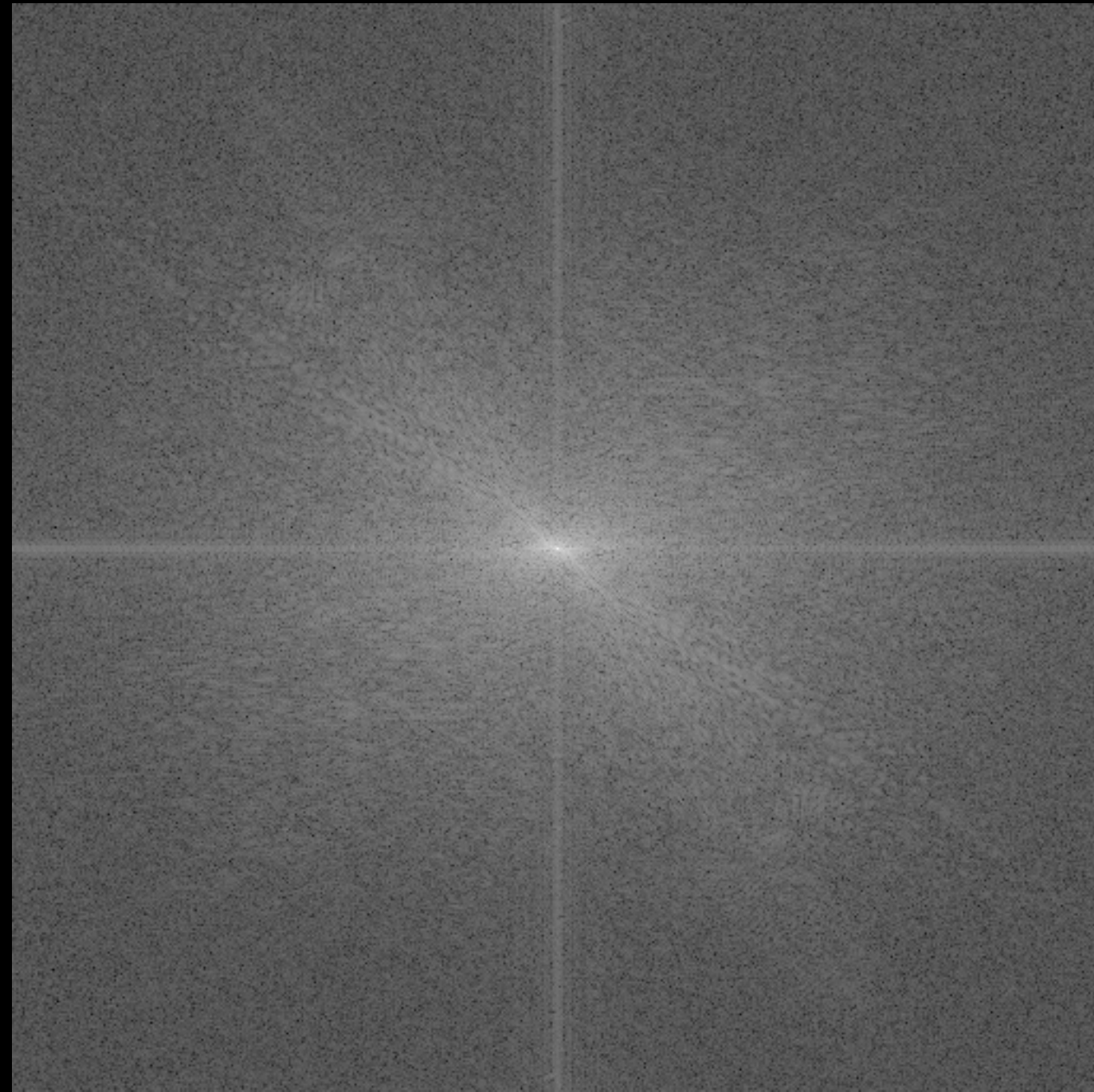
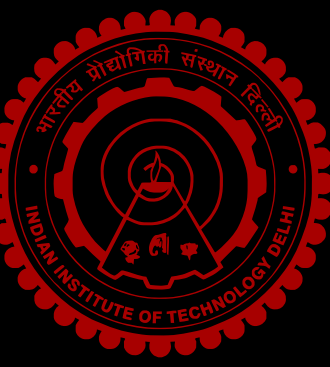
$$(x * f_1) * f_2 = x * (f_1 * f_2)$$

Convolution is associative
(also commutative, distributive)

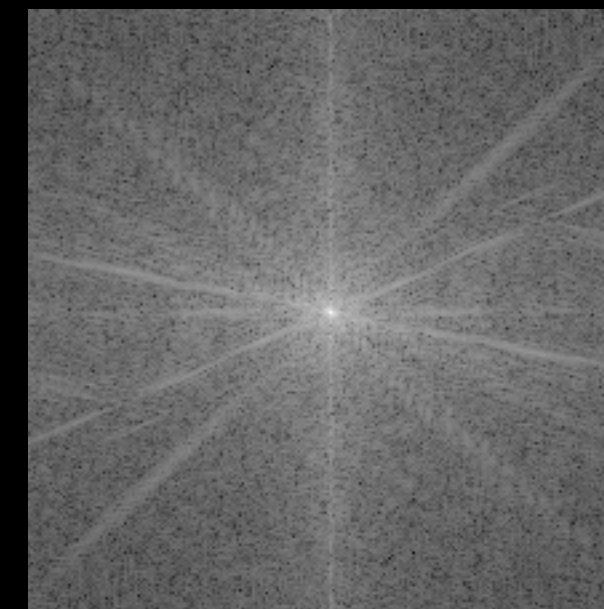
Filter Comparison



Frequency Domain



Lena



Cameraman