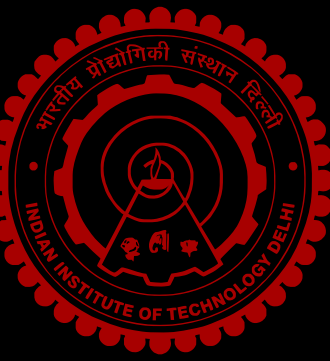
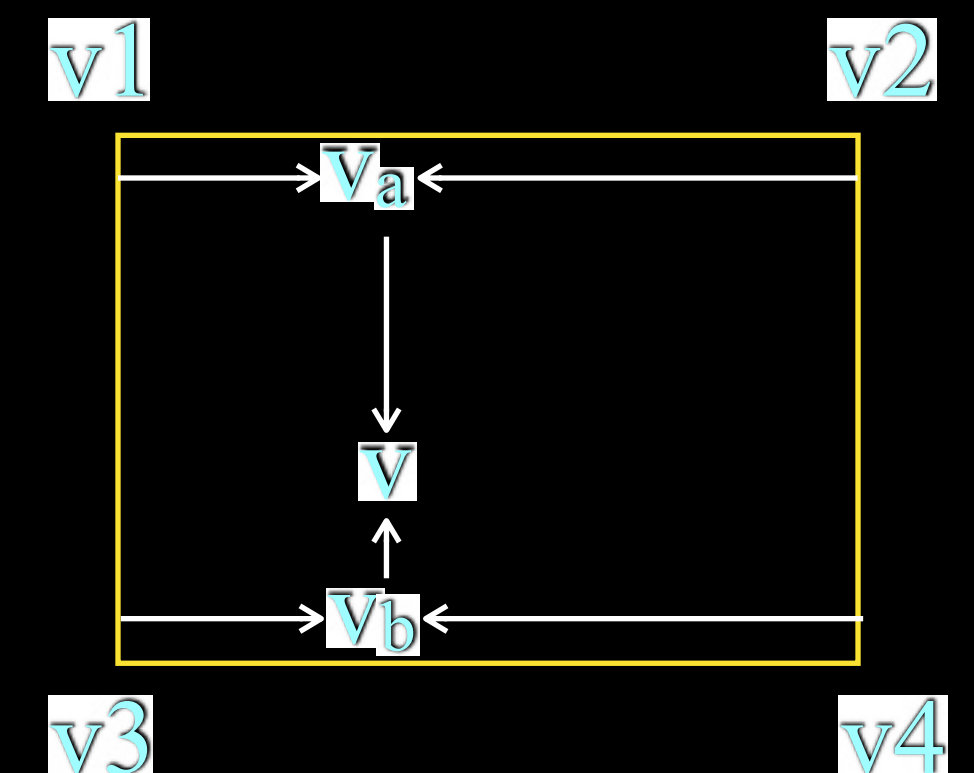




Types of Interpolation

- **Nearest neighbour:** Copy from nearest sample $f[i, j]$ where $i = \text{round}(x)$, $j = \text{round}(y)$
- Linear interpolation in one dimension (using two nearest samples)
 - Take values at 2 nearest samples $i_0 = \lfloor x \rfloor$, $i_1 = i_0 + 1$
 - Fit linear polynomial $f(x) = ax + b$ to sample values $f(i_0)$ and $f(i_1)$
- **Bilinear interpolation:** (in 2D, using four nearest samples)
 - interpolate separately in x and y
 - i.e., fit polynomial $f(x, y) = axy + bx + cy + d$ to 4 nearest samples
- **Bicubic interpolation:** Use cubic polynomial instead (in x and y)



((Combines more than
4 nearest samples))



Nearest neighbour



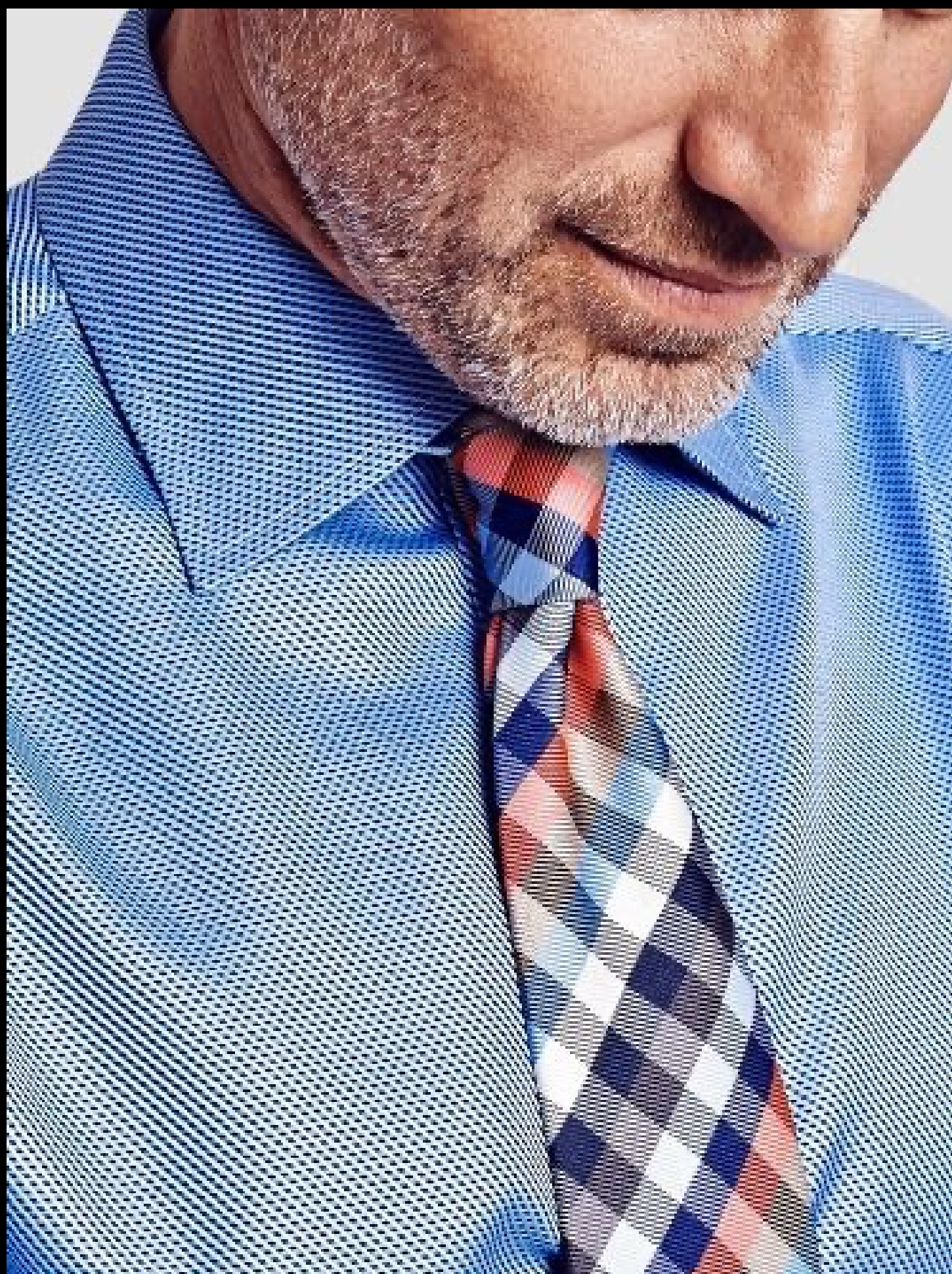
Bilinear



Bicubic

- Is this interpolation only good to fill in unknown samples?
 - How good are the interpolants?

Scaling Down



1/4x

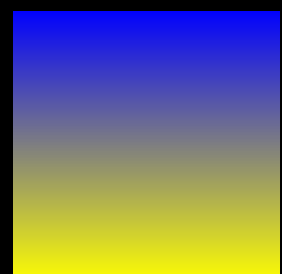
Drop every other pixel
in each dimension



(Scaled back)

Image Content

Mathematical Function of the Image

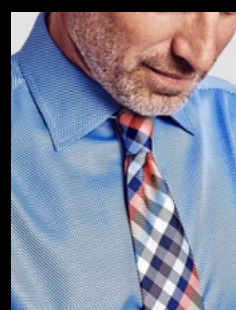
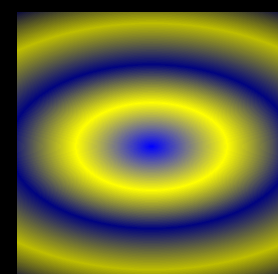
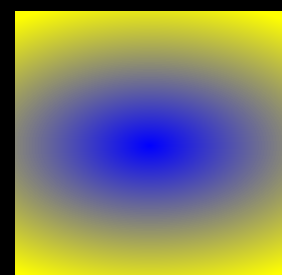


$(0, 1, 1)$

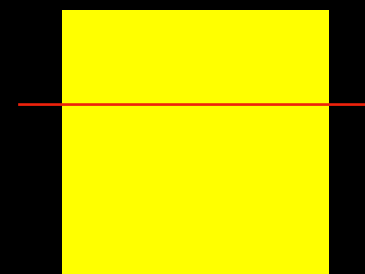
$(0, 0, 1)$

$(1, 0, 0)$

$(1, 1, 0)$



?



$C = (1, 0, 0)$

for (x, y) in $[0, 1] \times [0, 1]$, $C(x, y) = (1, 0, 0)$

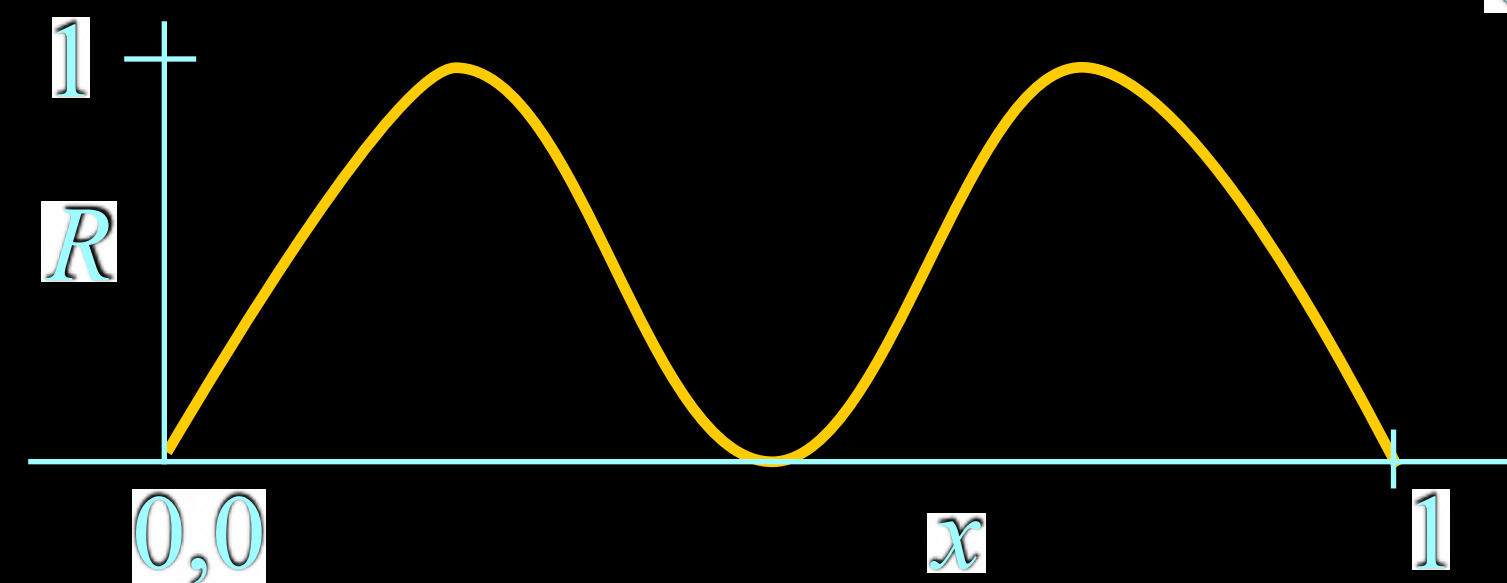
for (x, y) in $[0, 1] \times [0, 1]$,

$R(x, y) = 1, G(x, y) = 1, B(x, y) = 0$

1D slice: for a given y_0

for x in $[0, 1]$,

$R(x) = 1, G(x) = 1, B(x) = 0$



Representing Image Content

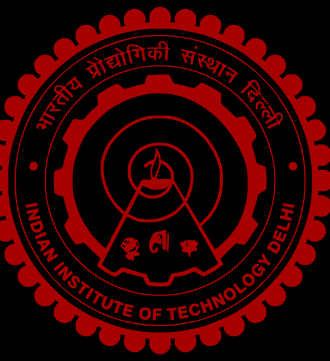
Does a polynomial represent the signal?

Let's understand the visual content of the image

And the impact of discretization and reconstruction (to continuous version)

Content roughly relates to how different a sample is compared to its neighbors
(Higher resolution usually means, they are more similar)

i.e., how quickly the color changes from pixel to pixel



Fourier Series

- A periodic function can be represented by a (possibly infinite) sum of sine and cosine functions

$$f(x) = \sum_{n=-\infty}^{+\infty} F_n e^{i2\pi \frac{nx}{T}}$$

(T is the period)

Each term has a frequency that is an integer multiple of x/T

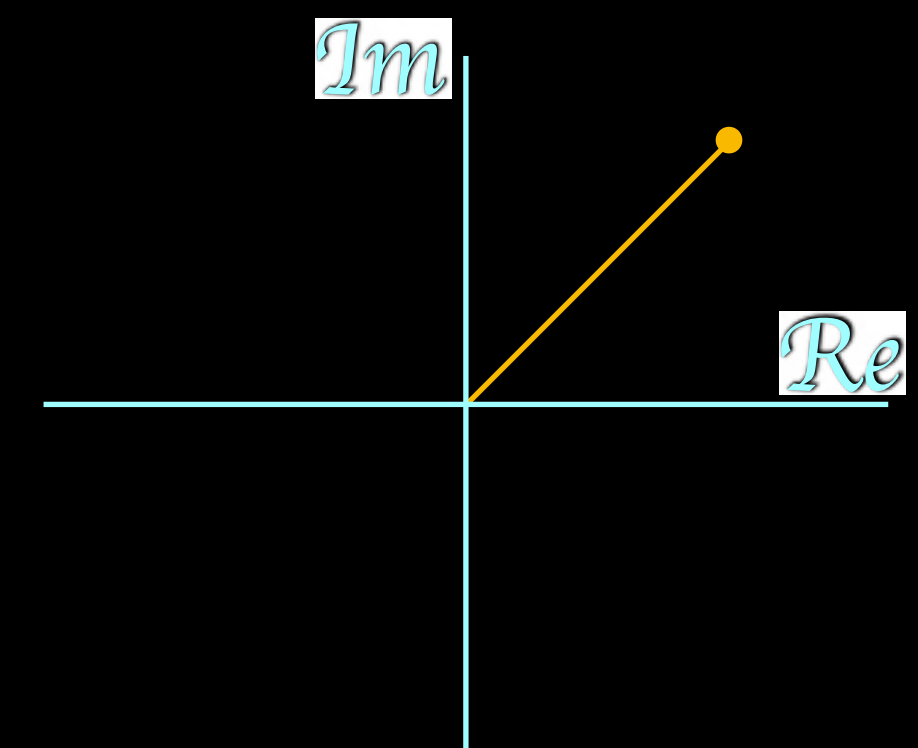
$$F_n = \int_0^T f(x) e^{-i2\pi \frac{nx}{T}} dx$$

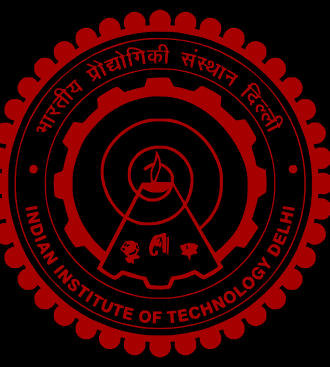
In complex domain:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\cos \theta = \frac{1}{2} e^{i\theta} + \frac{1}{2} e^{-i\theta}$$

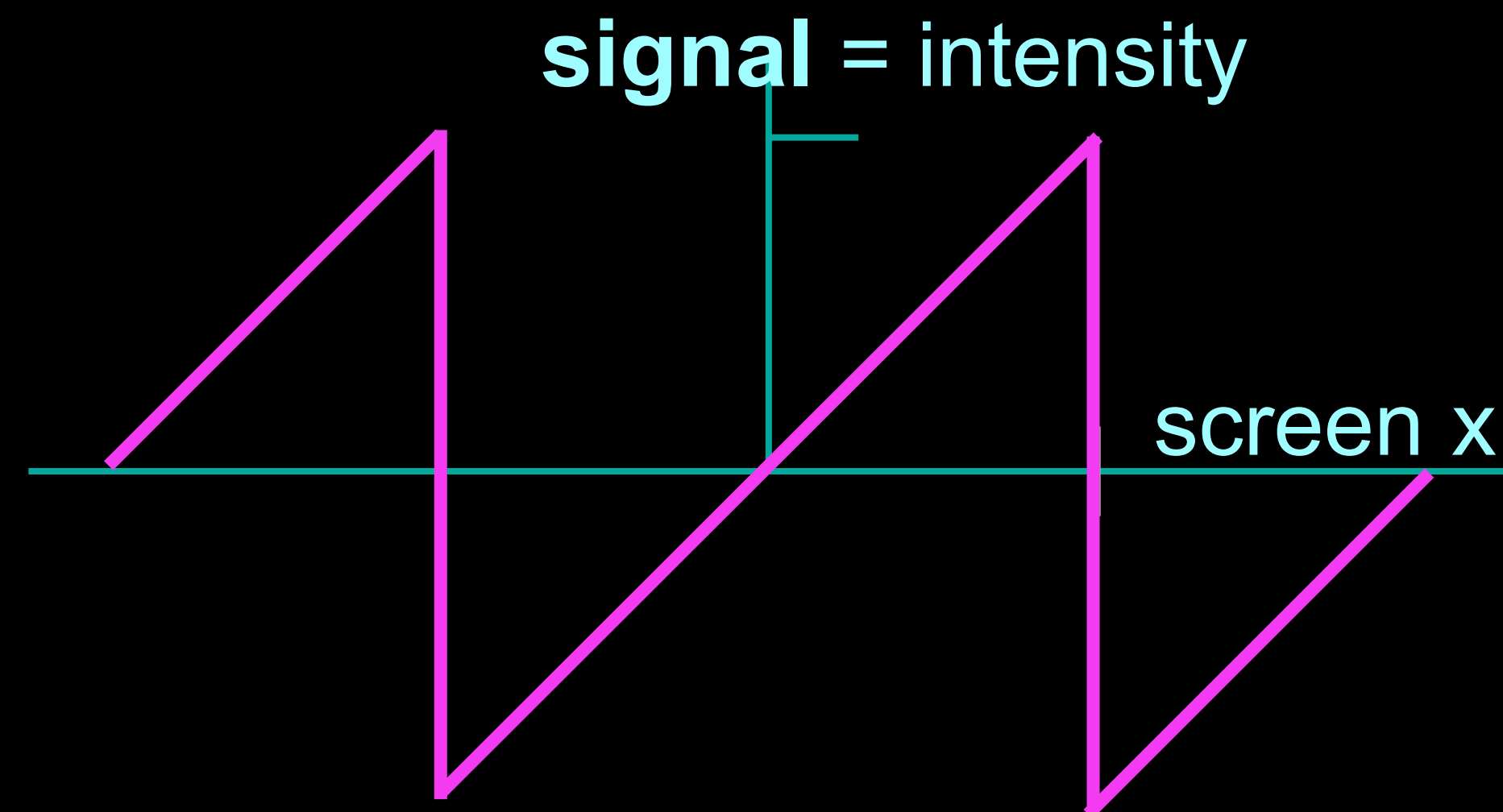
$$\sin \theta = -\frac{1}{2} i e^{i\theta} + \frac{1}{2} i e^{-i\theta}$$





Harmonics Summation

Fourier Series

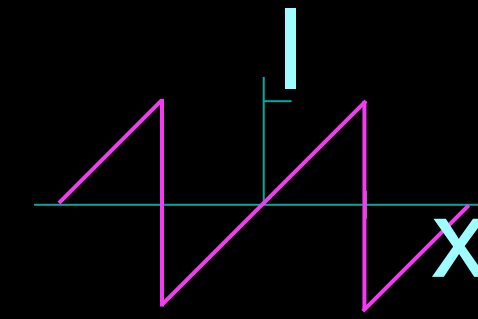
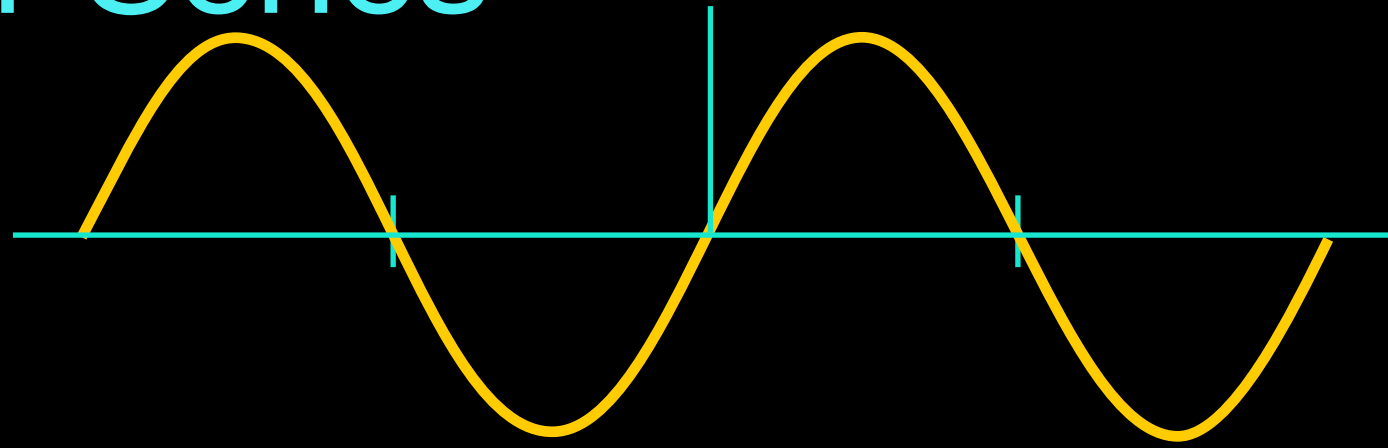




Harmonics Summation

Fourier Series

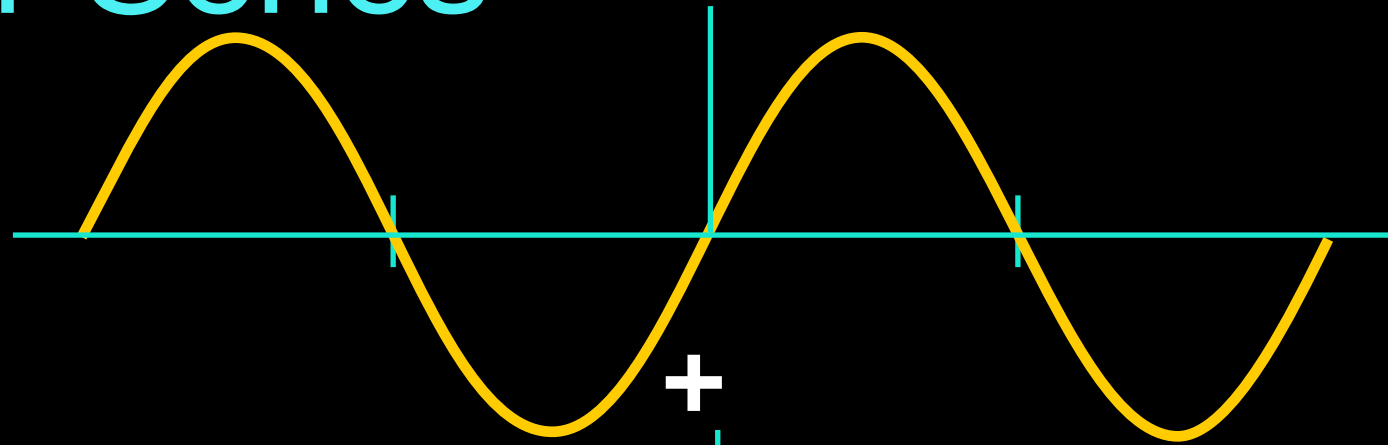
1



Harmonics Summation

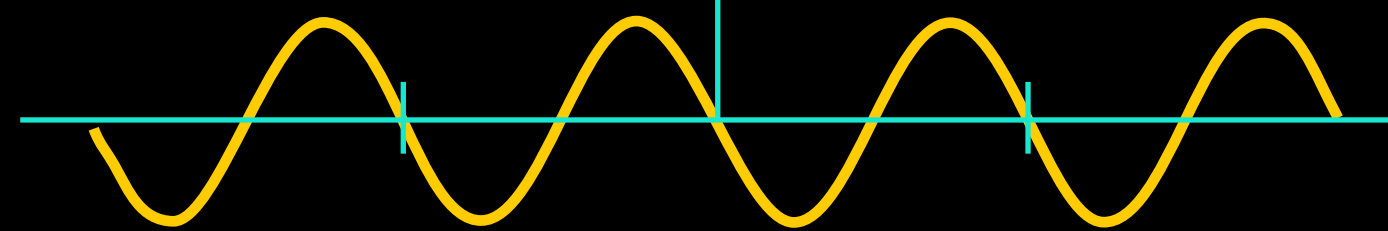
Fourier Series

1

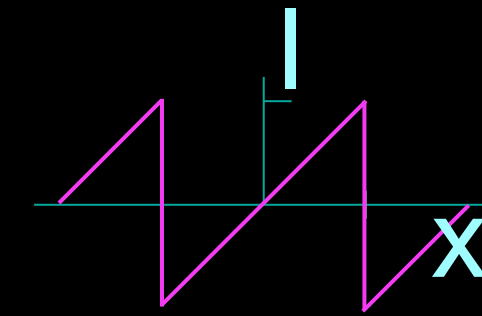
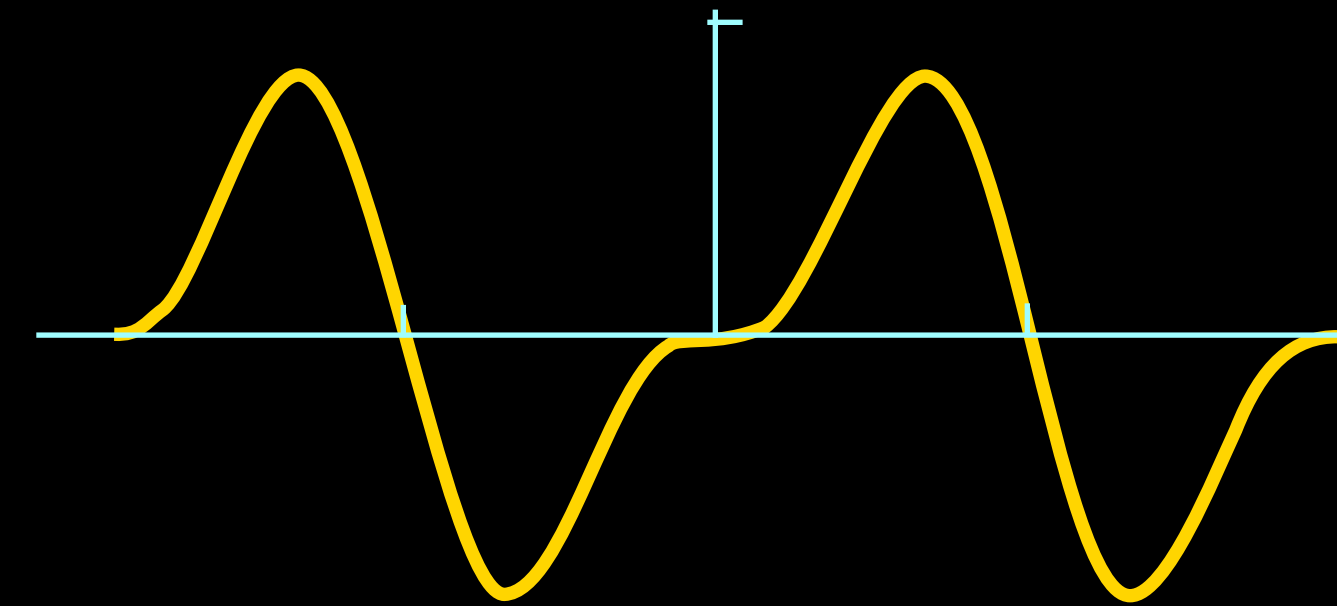


+

2

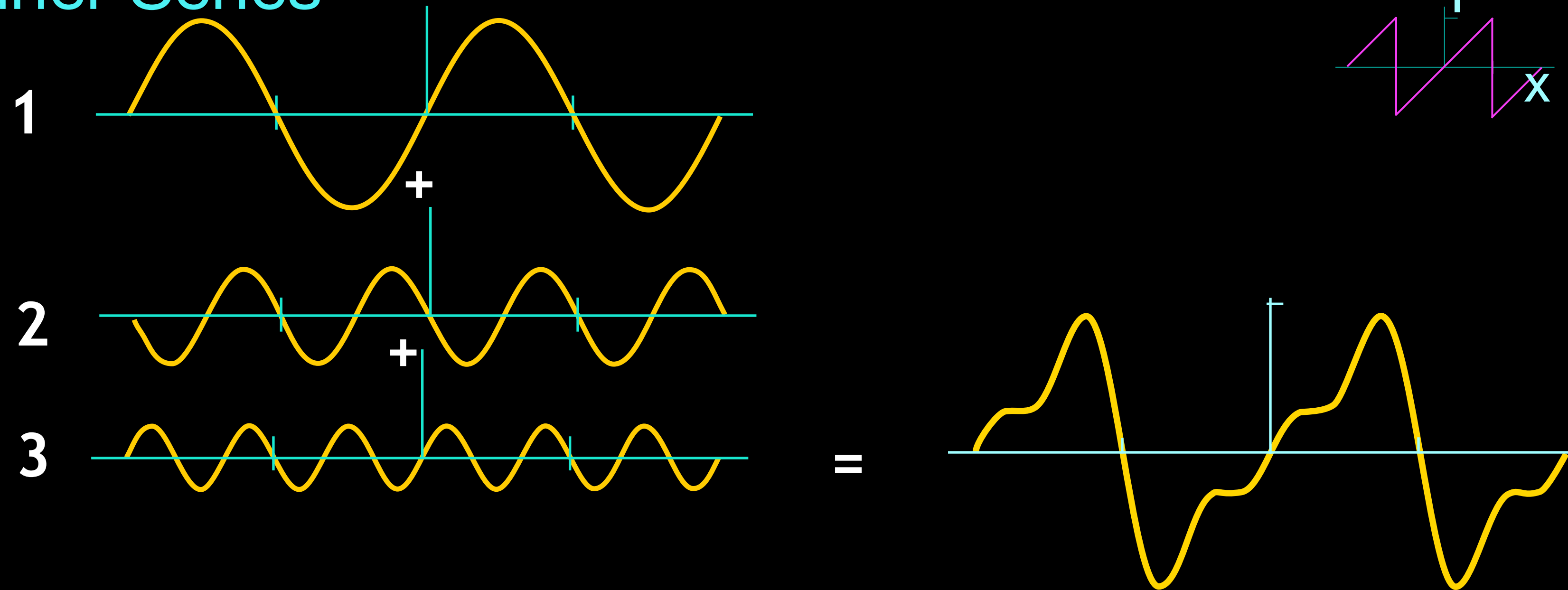


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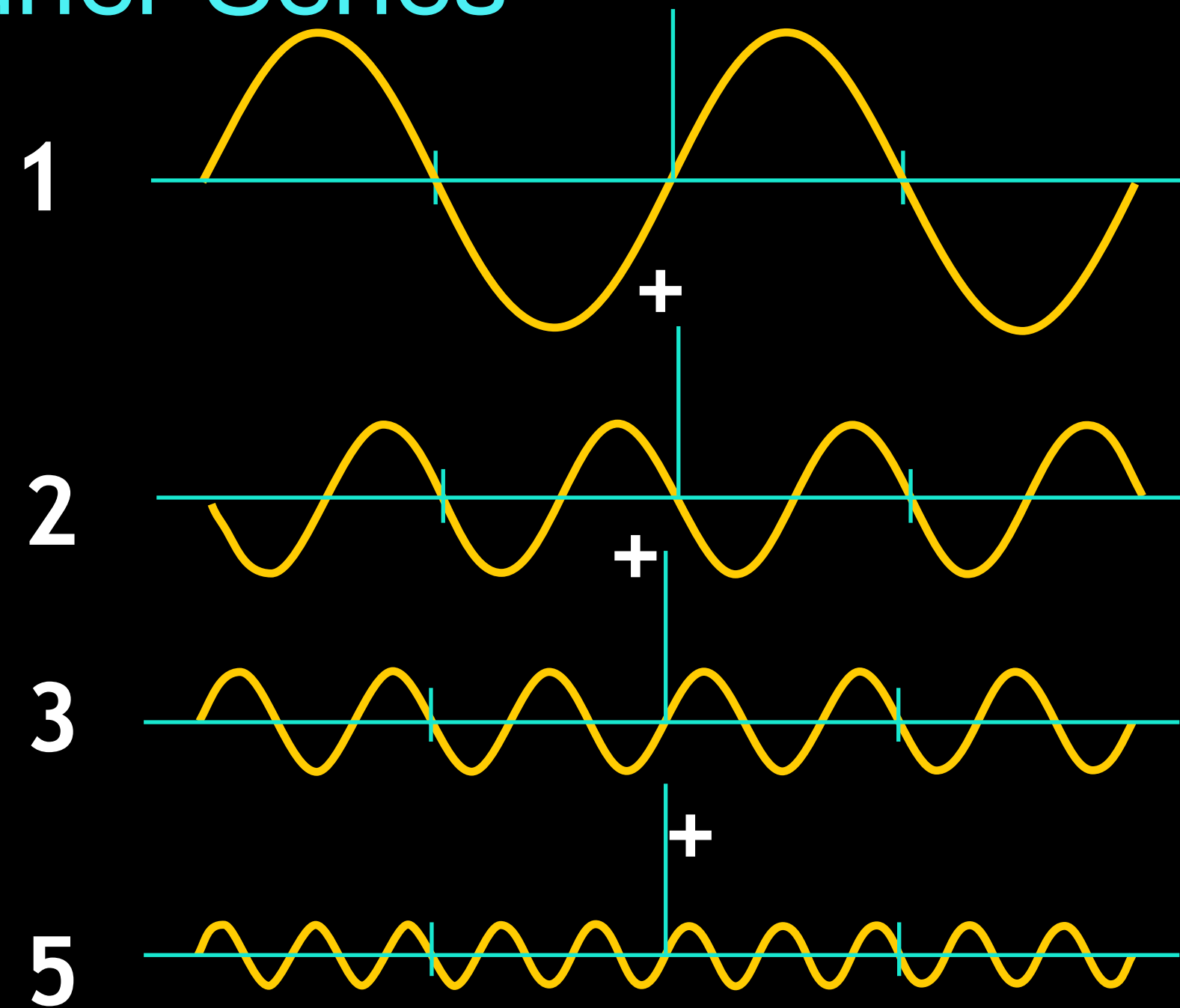
Harmonics Summation

Fourier Series

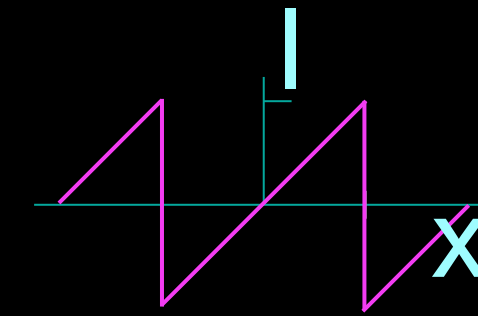
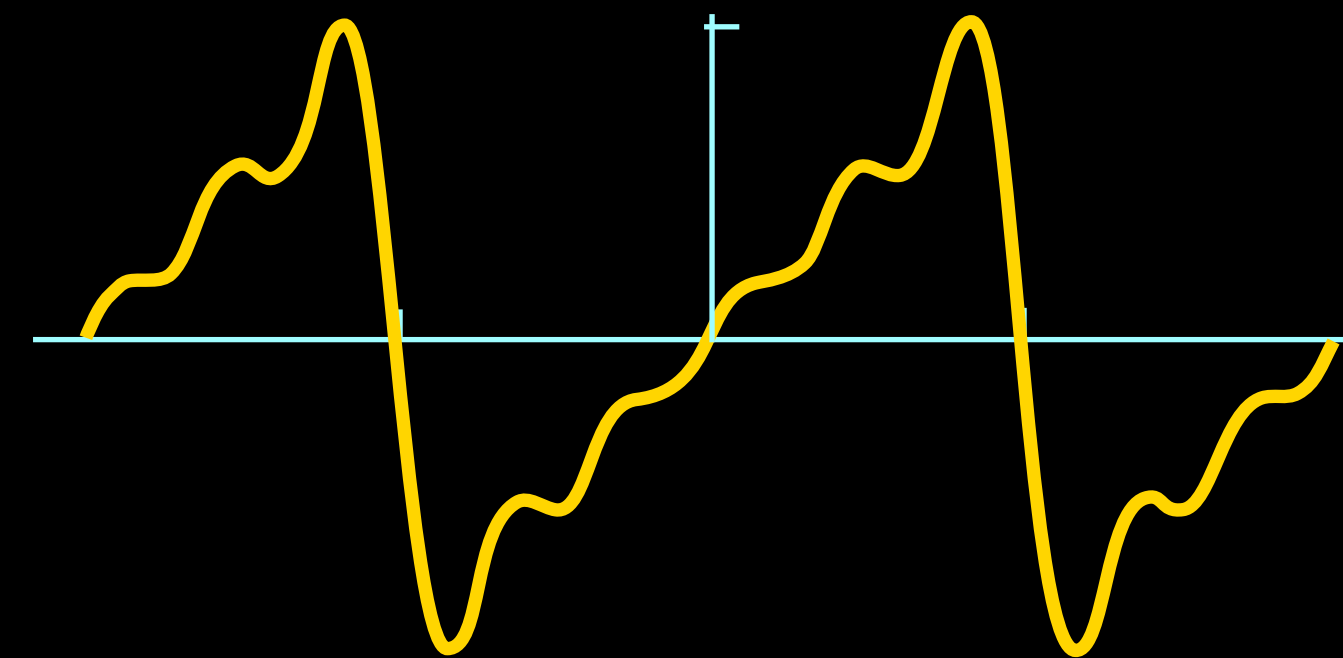


Harmonics Summation

Fourier Series

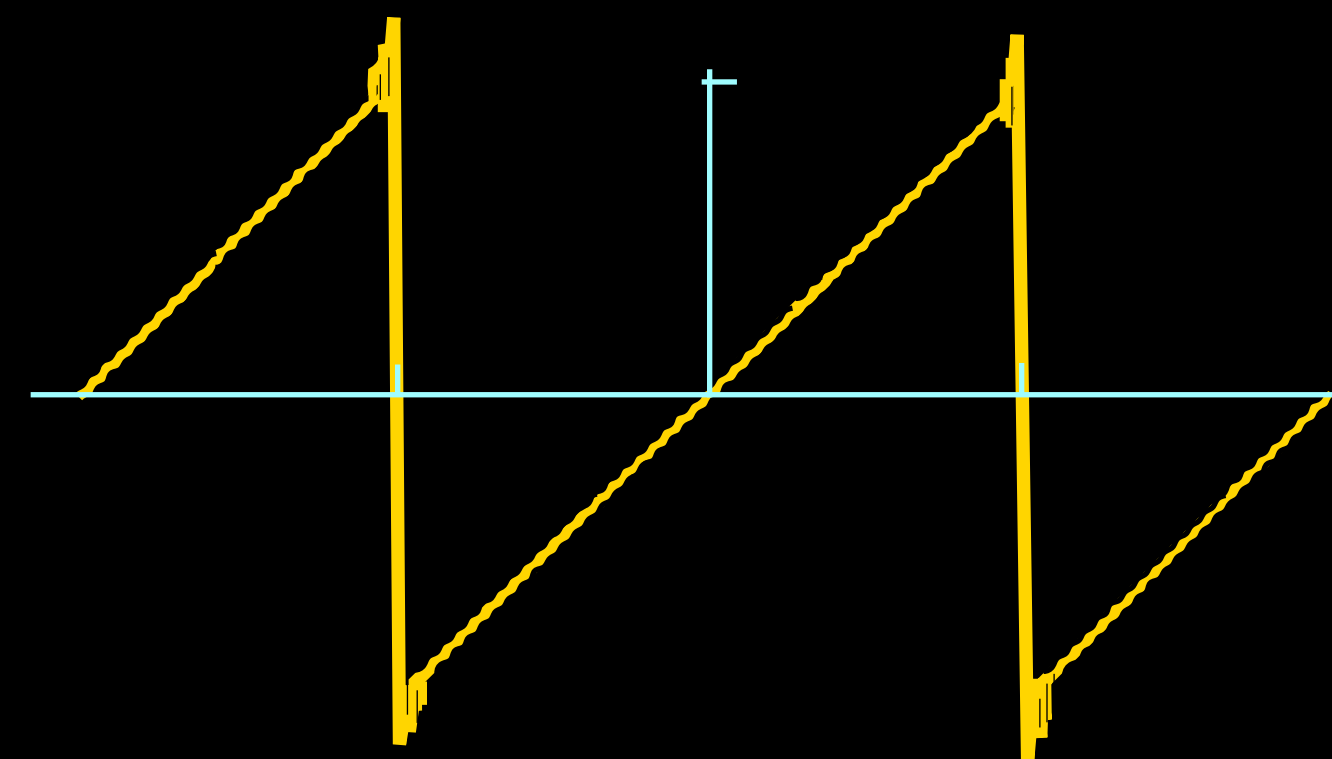
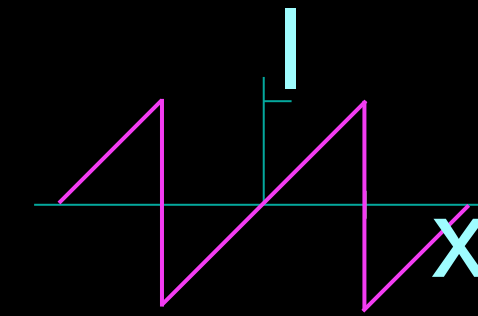
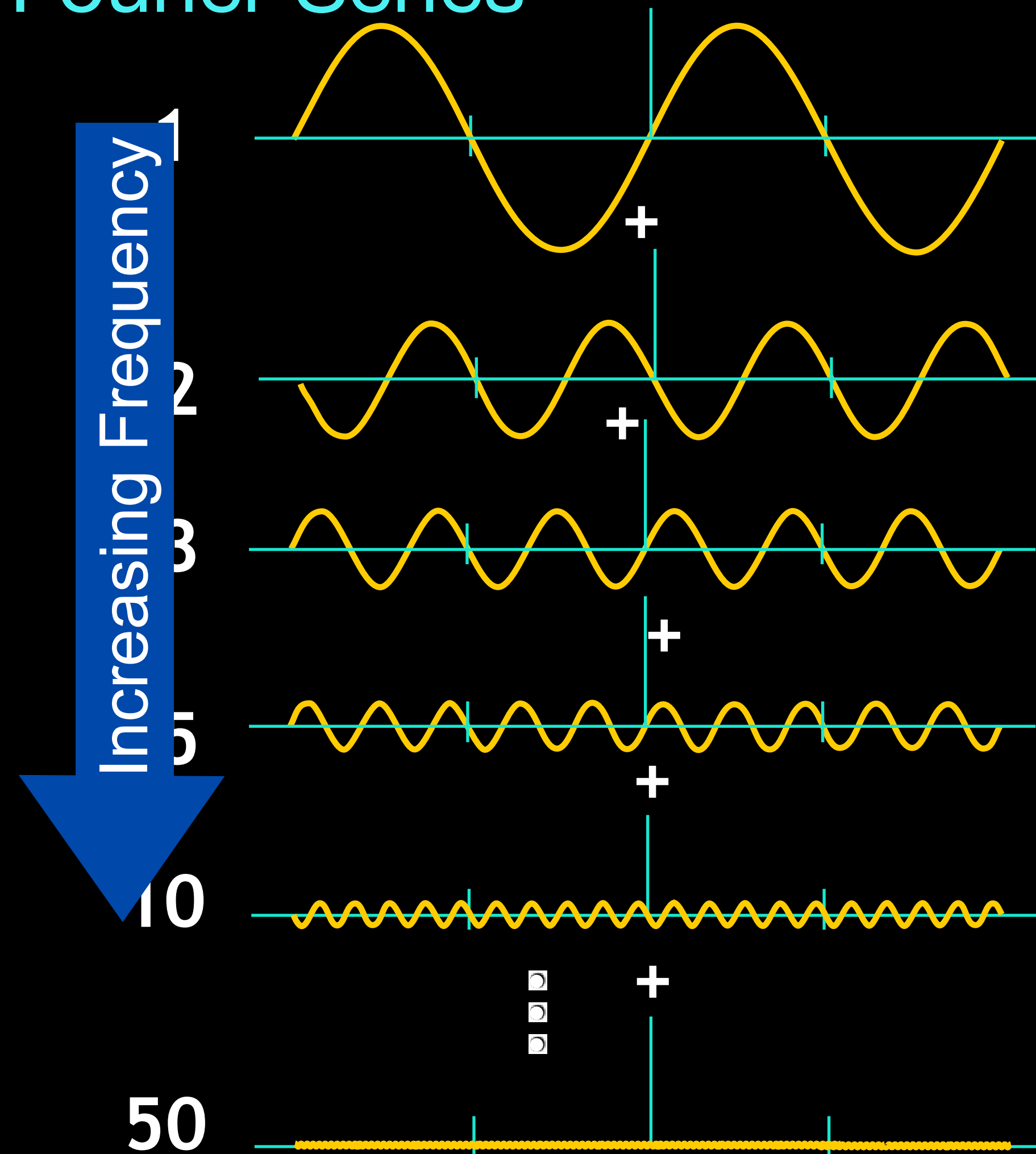


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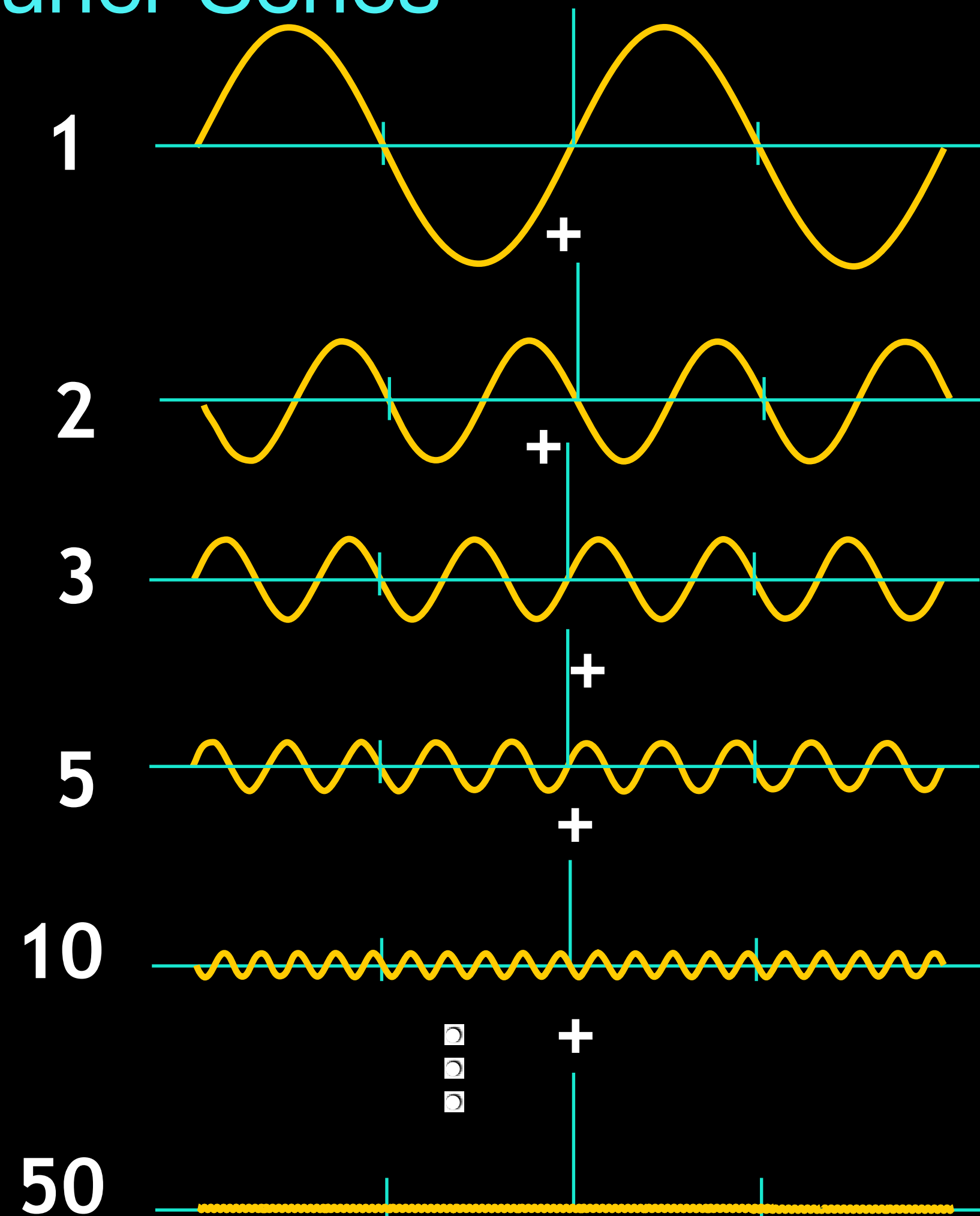
Harmonics Summation

Fourier Series

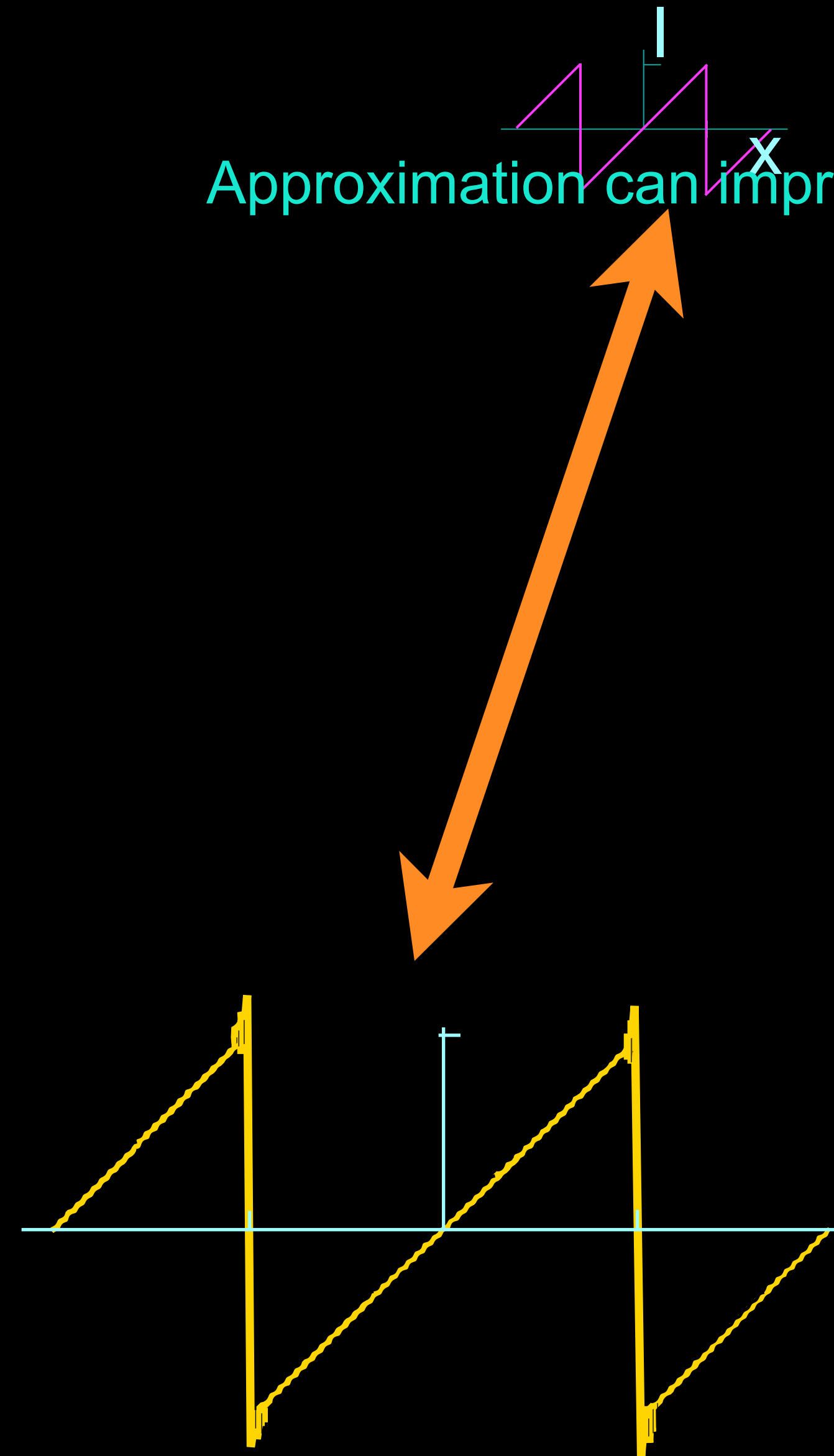


Harmonics Summation

Fourier Series

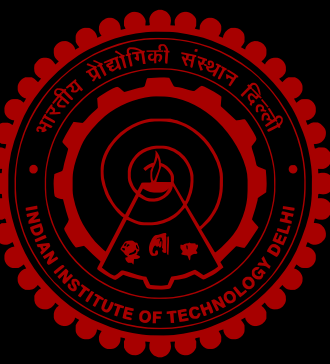


=



Approximation can improve with more terms

Gibb's ringing due to a small count



Fourier Transform

A periodic extension even for non-periodic functions

$$F(\mu) = \int_{-\infty}^{+\infty} f(x) e^{-i2\pi\mu x} dx$$

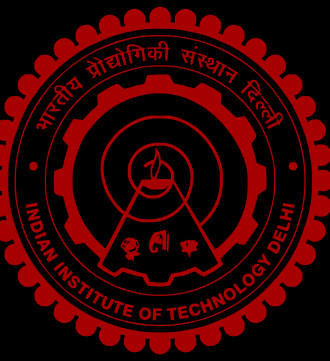


$$f(x) = \int_{-\infty}^{+\infty} F(\mu) e^{i2\pi\mu x} d\mu$$

Invertible:

If $F(\mu)$ is a Fourier transform of $f(x)$

$F(x)$ is a Fourier transform of $f(-\mu)$



Fourier Transform

A periodic extension even for non-periodic functions

$$F(\mu) = \int_{-\infty}^{+\infty} f(x) e^{-i2\pi\mu x} dx$$



Invertible:

If $F(\mu)$ is a Fourier transform of $f(x)$

$F(x)$ is a Fourier transform of $f(-\mu)$

$$f(x) = \int_{-\infty}^{+\infty} F(\mu) e^{i2\pi\mu x} d\mu$$

$$\cos(2\pi\mu x) + i \sin(2\pi\mu x)$$

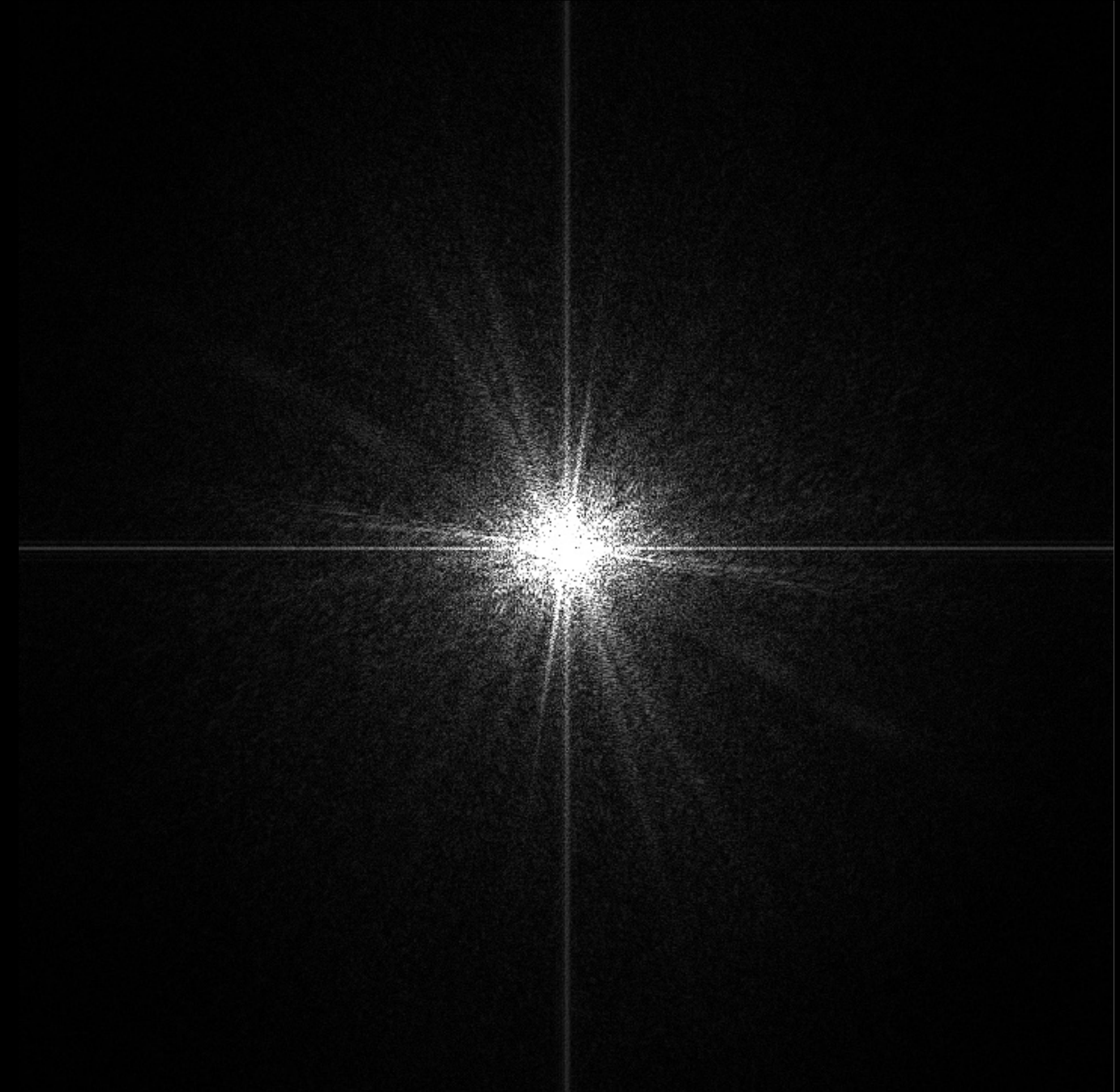
μ represents frequency

μ is the inverse of x : x^{-1}

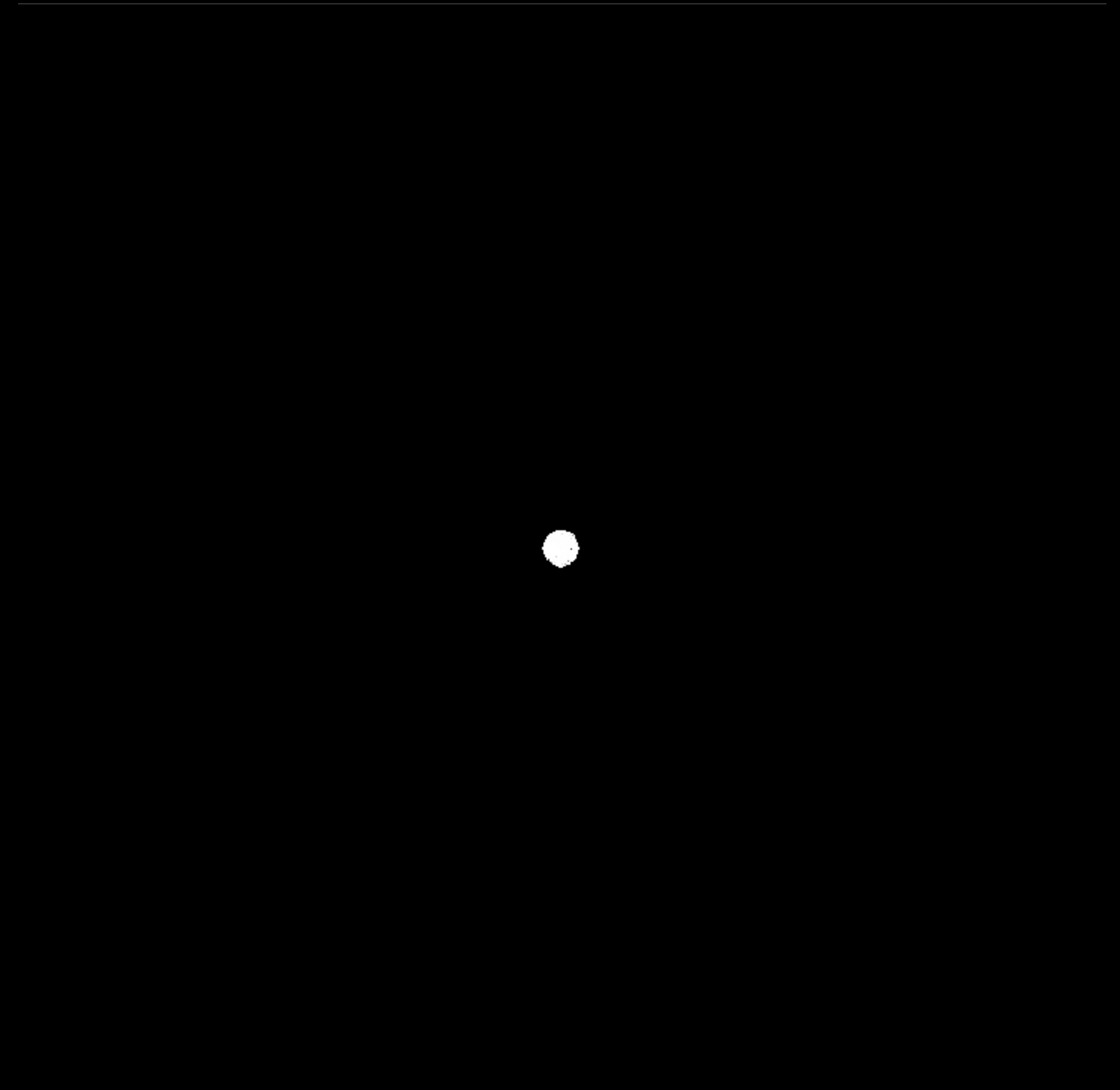
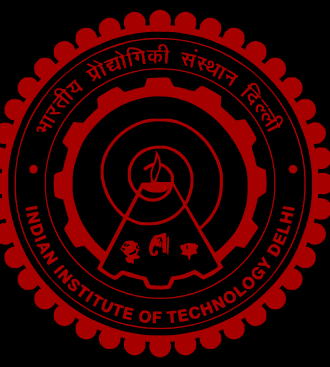
if x denotes position, μ denotes per unit position, aka frequency

“Hz” $\Rightarrow$ Per unit pixel (or other spatial unit)

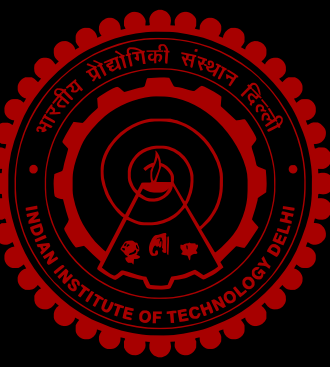
Fourier transform of an image



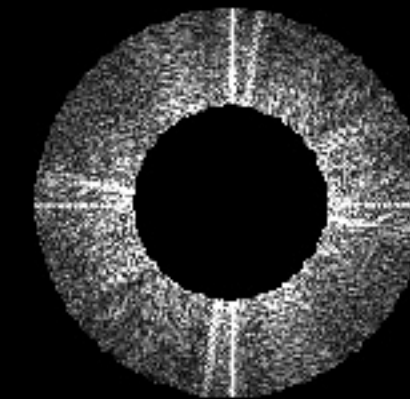
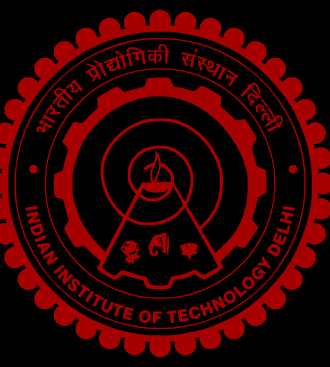
Low frequencies



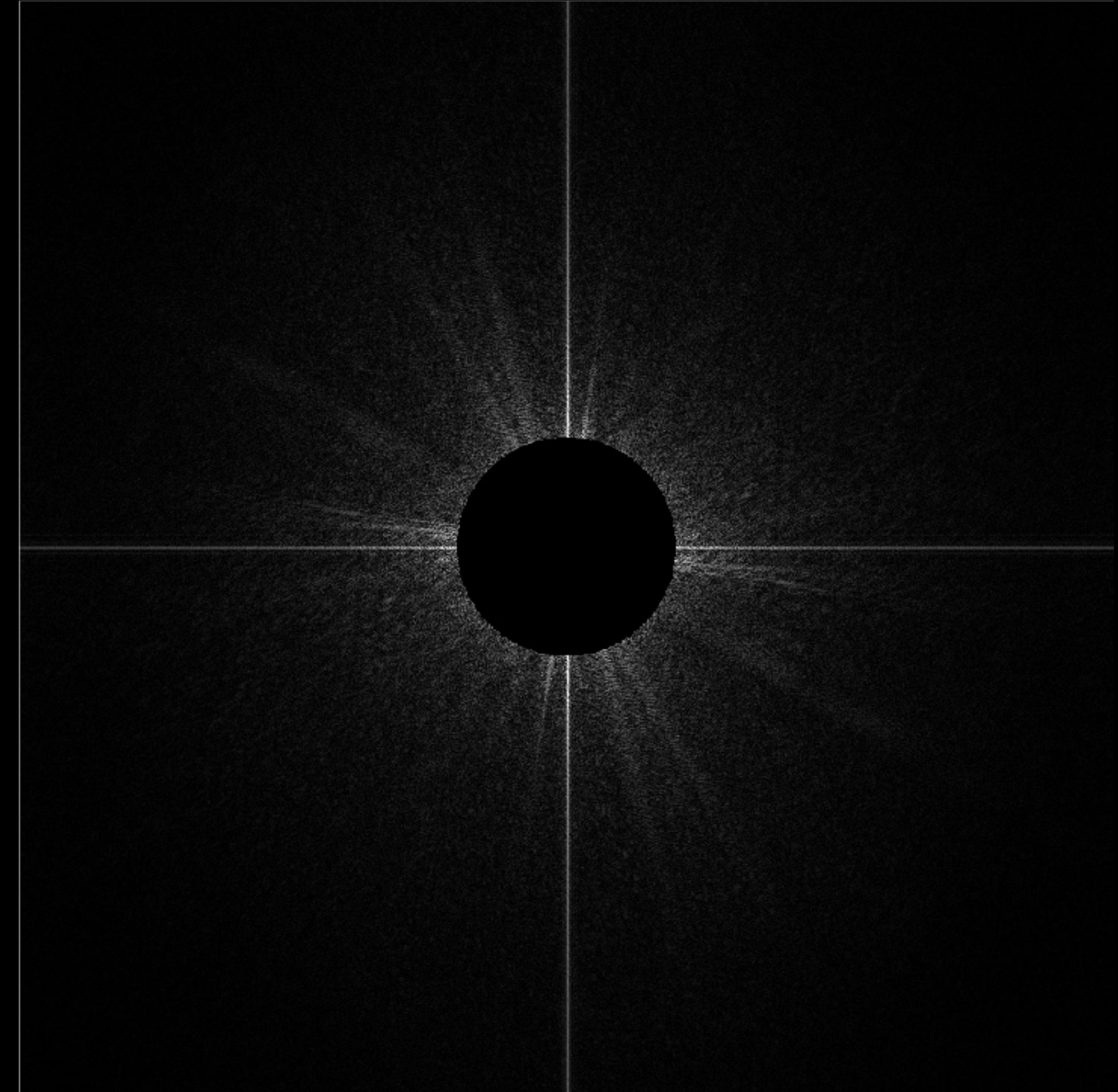
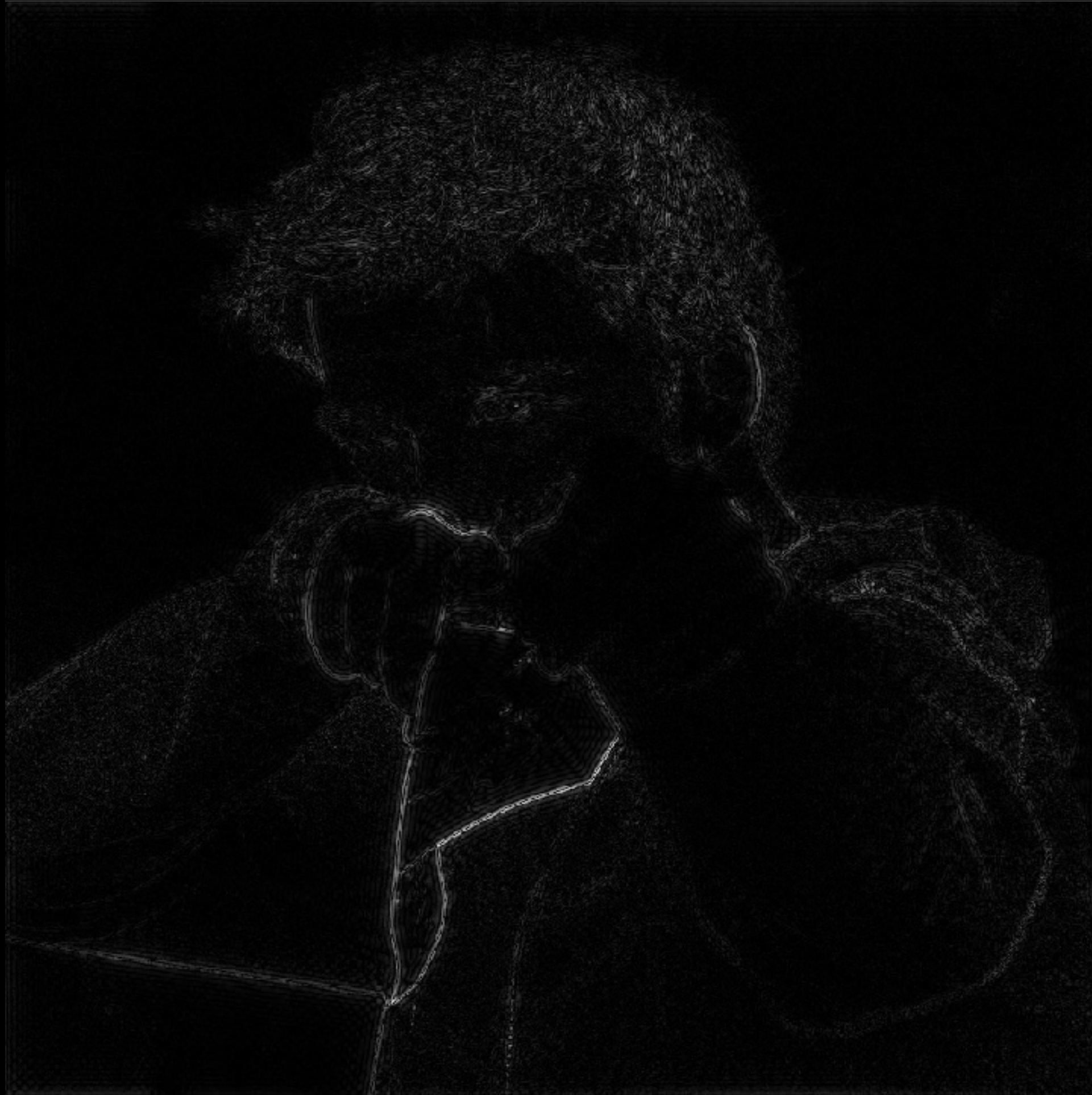
Medium frequencies



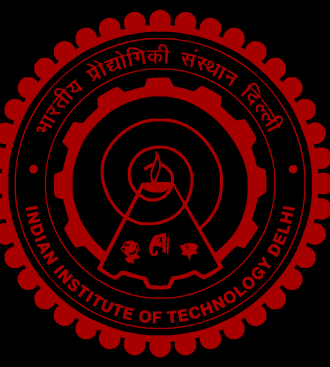
Medium frequencies



High frequencies



Remove High Frequency



Original



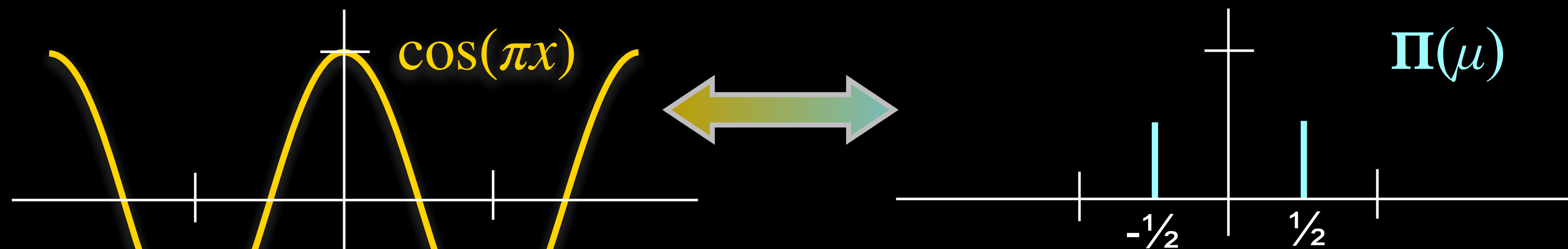
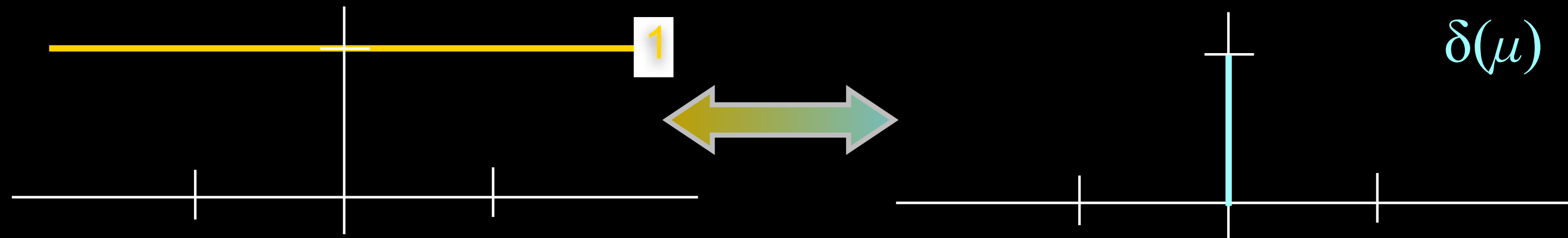
Lowest frequencies



Processing Images

- Hardly ever the need to represent the visual signal directly in Fourier domain
 - Except for application like compression
- Computation on frequencies can help with many transformations
 - Image smoothing
 - Feature extraction
 - Noise reduction
 - Image restoration
 - Pattern recognition

Fourier Transform Pairs

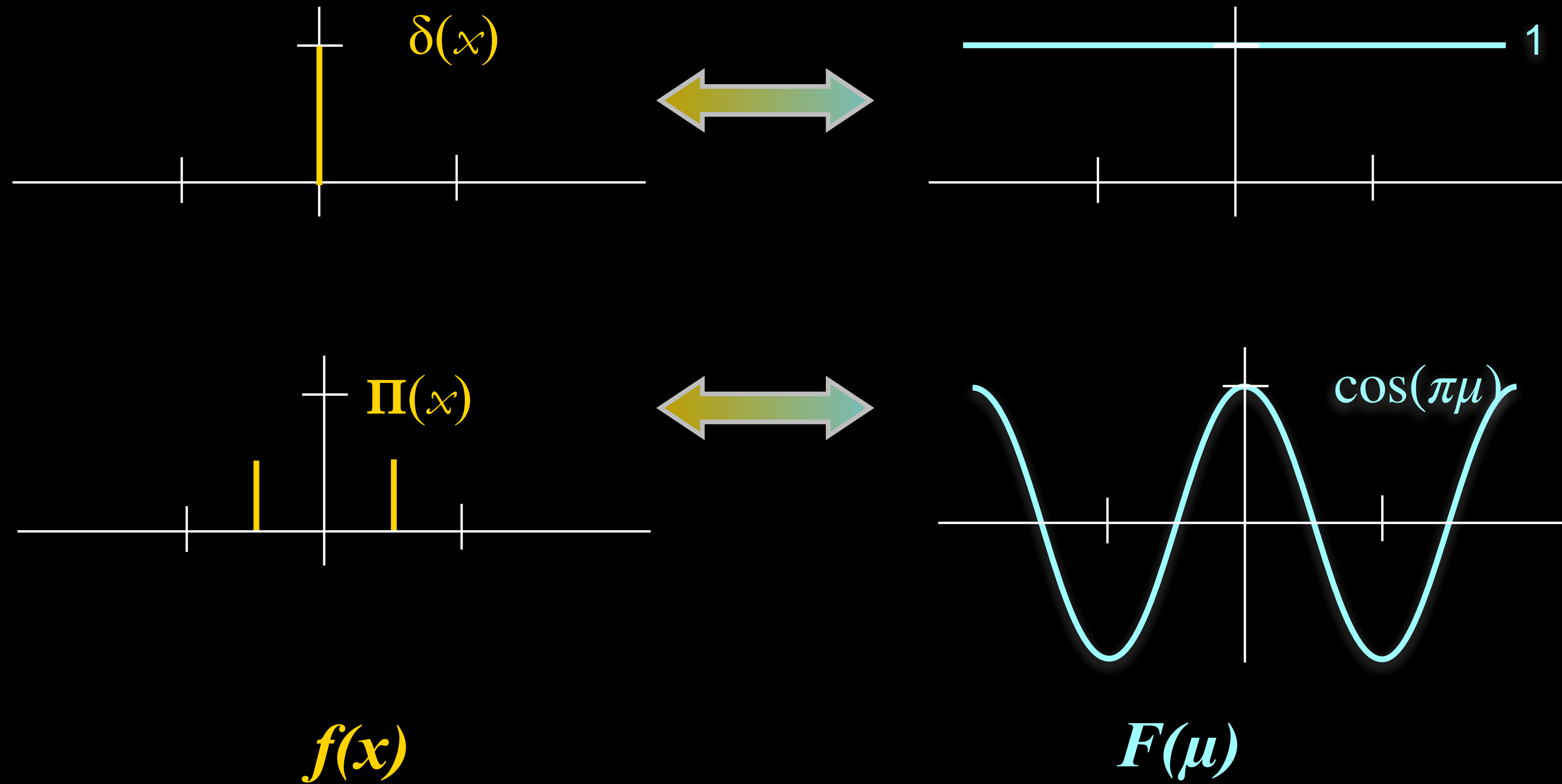
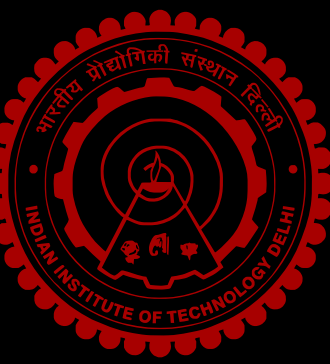


$$\cos \pi x = \frac{1}{2} e^{i\pi x} + \frac{1}{2} e^{-i\pi x}$$

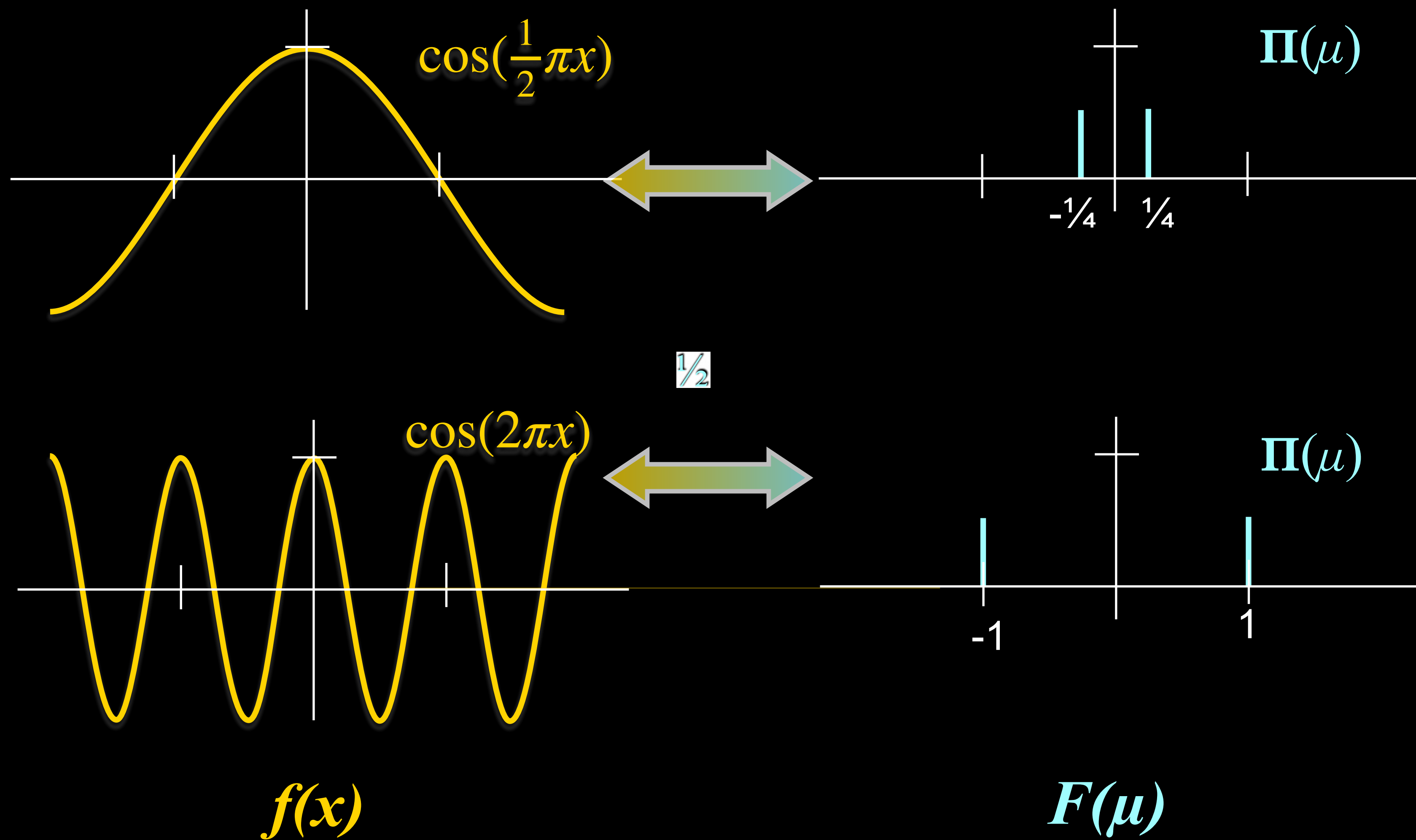
$f(x)$

$F(\mu)$

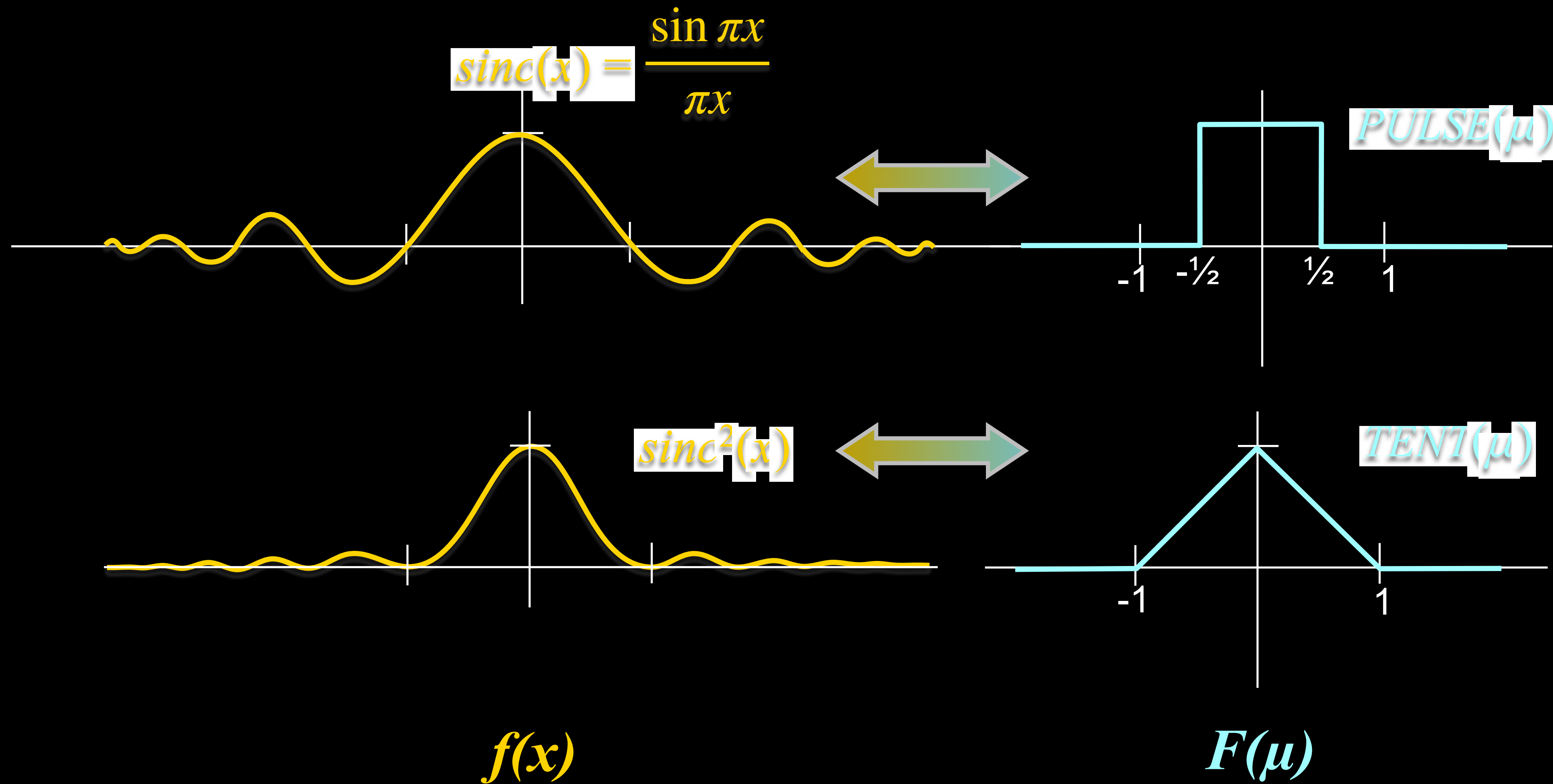
Reciprocity



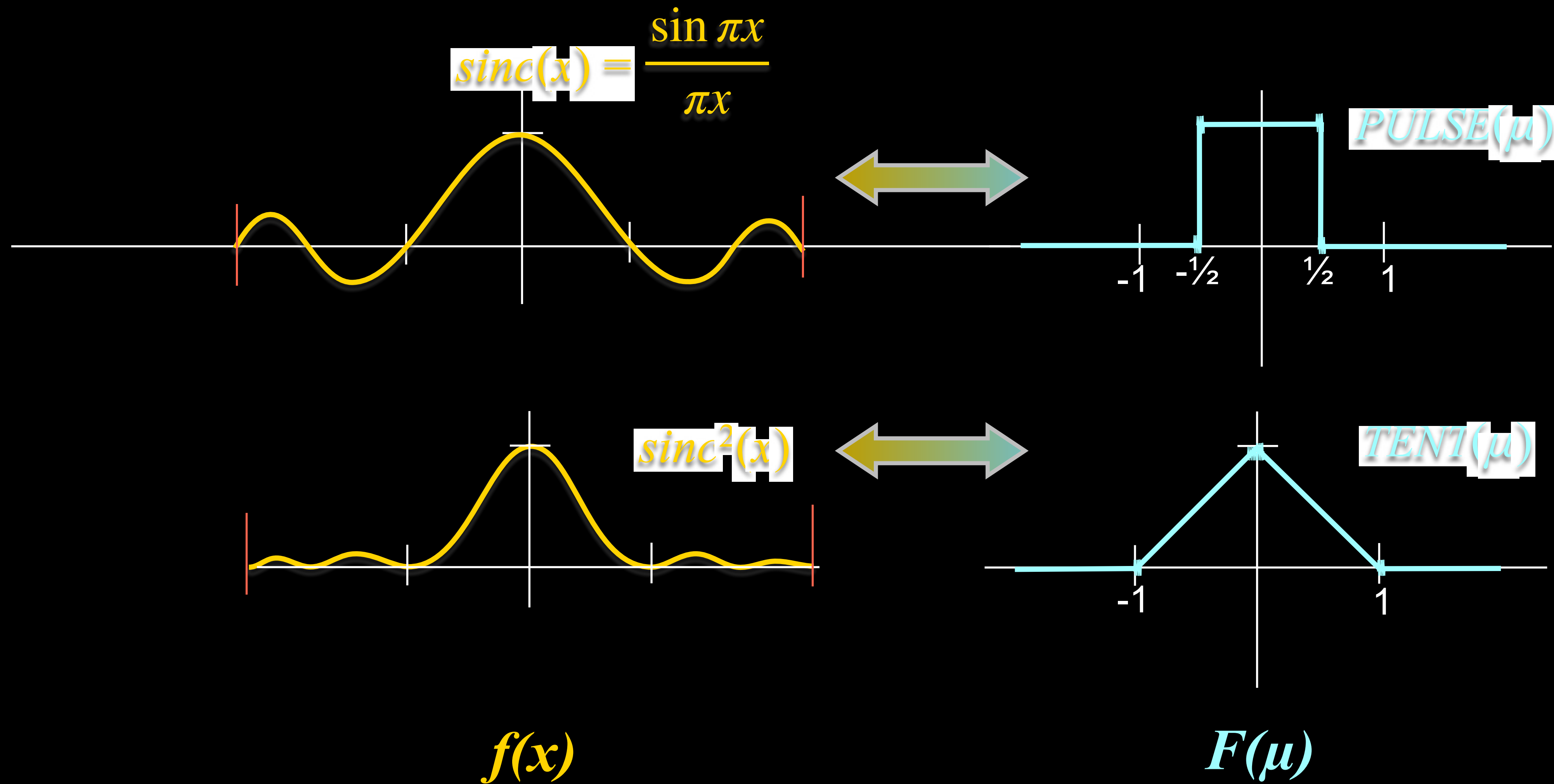
Inverse Scaling



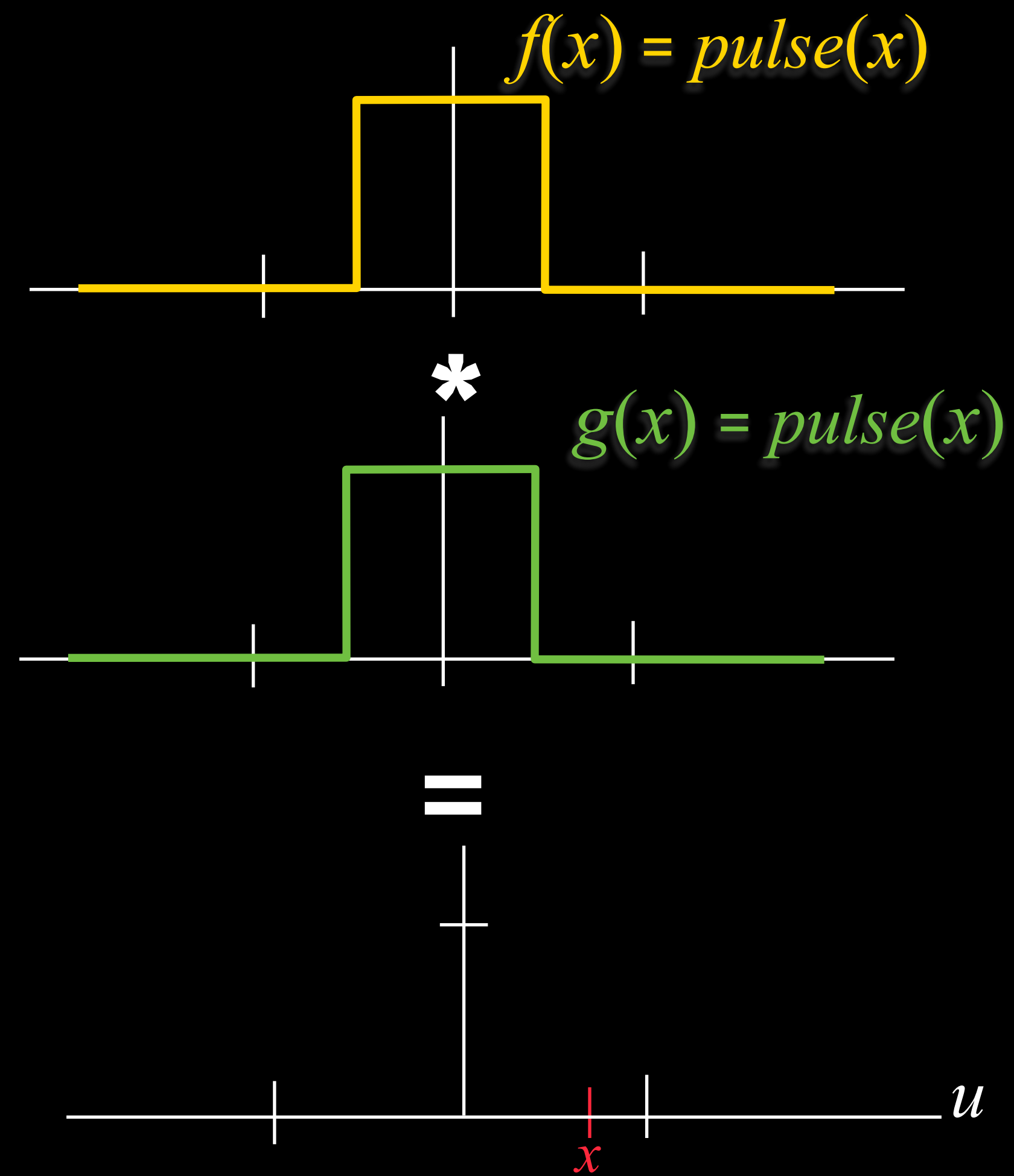
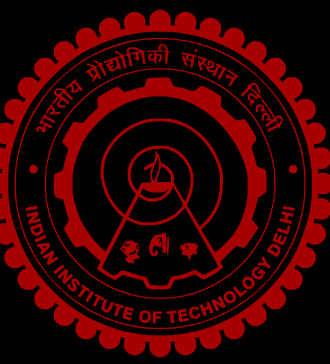
Transform Pairs



Band-limited Transform Pairs

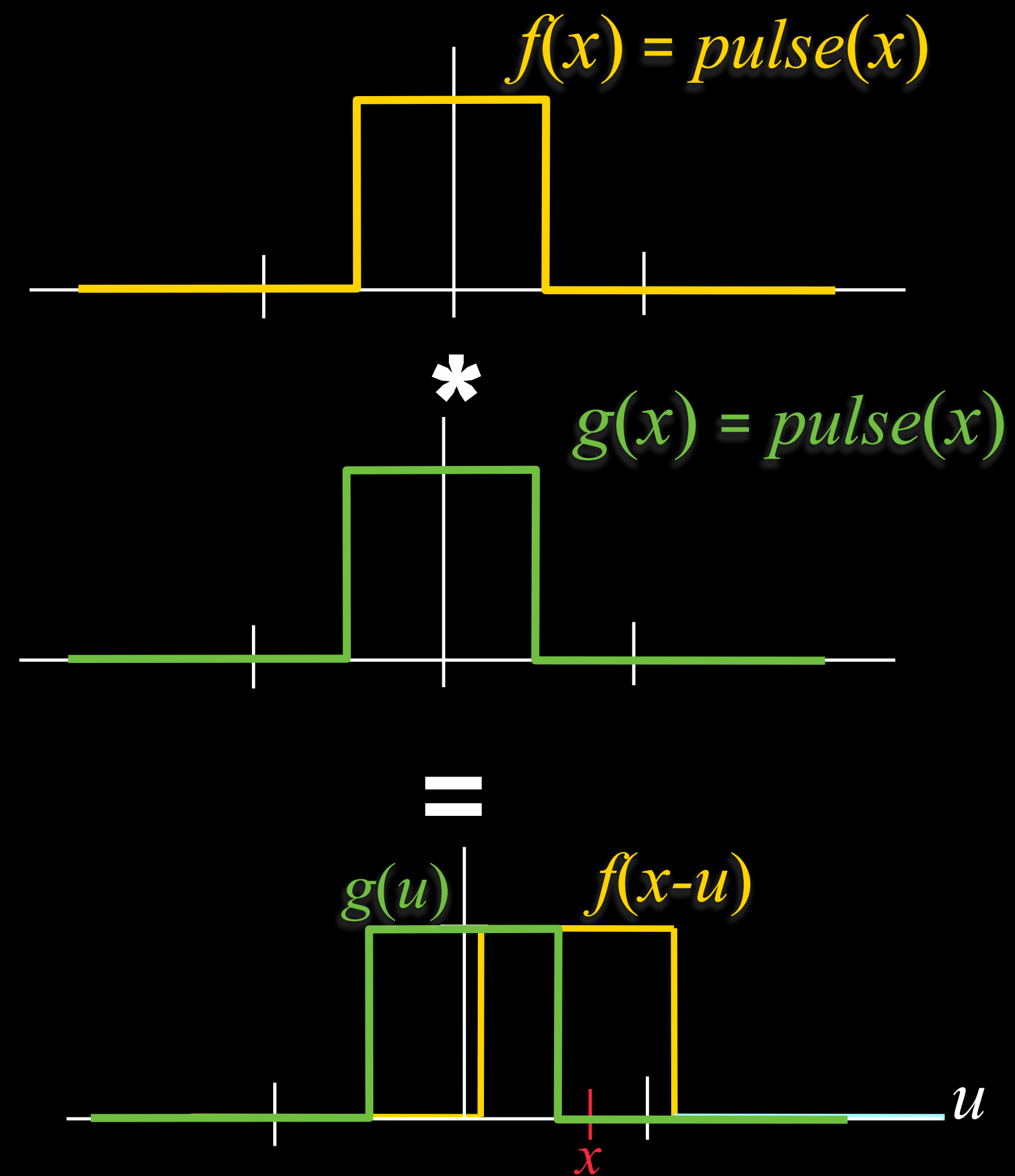


Convolution



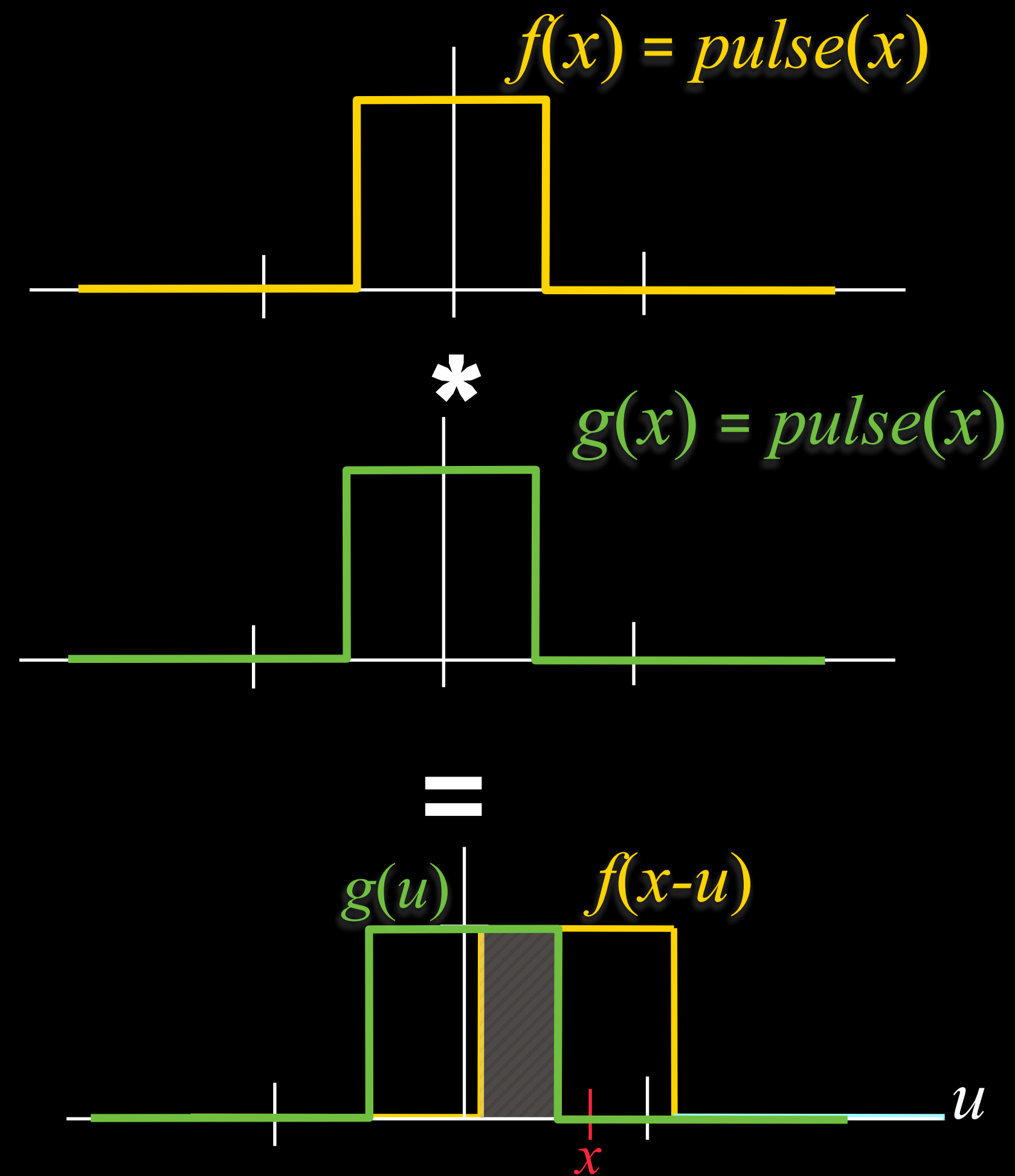
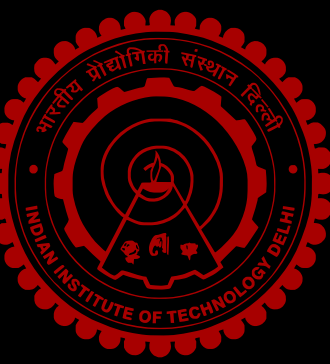
$$f(x) * g(x) = \int_{-\infty}^{+\infty} f(x - u)g(u)du$$

Convolution



$$f(x) * g(x) = \int_{-\infty}^{+\infty} f(x-u)g(u)du$$

Convolution



$$f(x) * g(x) = \int_{-\infty}^{+\infty} f(x-u)g(u)du$$