

COL7655

Foundations of Visual Computing

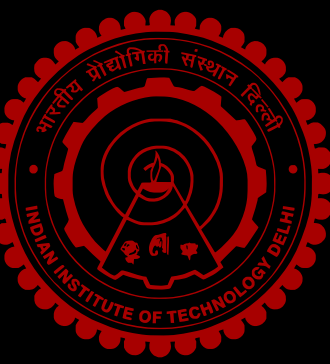
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Camera Recovery

Thanks to Chetan Arora for help with slides



Recovering Transformation Matrix

PnP or Pose estimation

- Find M , such that:
$$\begin{bmatrix} | & | & \dots & | \\ O_0 & O_1 & \dots & O_n \\ | & | & \dots & | \end{bmatrix} = M \begin{bmatrix} | & | & \dots & | \\ P_0 & P_1 & \dots & P_n \\ | & | & \dots & | \end{bmatrix} \Rightarrow \bar{O} = M\bar{P}$$

But $\bar{P}$ is not invertible

- Pseudo-inverse using SVD: $\bar{P} = \overset{\text{Diagonal } \uparrow}{U} \overset{\text{Orthonormal } \uparrow}{\Sigma} \overset{\text{Orthonormal } \uparrow}{V^T}$
- Sometimes $\bar{P}\bar{P}^T$ may be invertible

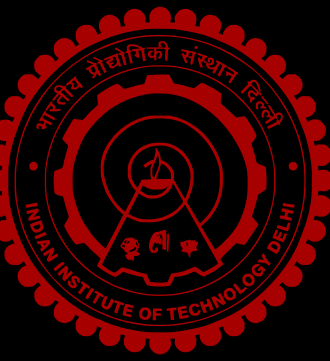
$$\bar{O}\bar{P}^T = M\bar{P}\bar{P}^T \Rightarrow M = \bar{O}\bar{P}^T(\bar{P}\bar{P}^T)^{-1}$$

$$\Rightarrow \bar{O} = MU\Sigma V^T$$

$$\Rightarrow M \approx \bar{O}V\Sigma^+U^T$$

Σ^+ = replace each element e of Σ^T with e^{-1}
(If $e \approx 0$, do not replace)

OK, for affine transformation.
How to handle projection?



Computing Intrinsic and Extrinsic Matrices

Includes projective transformation

p : pixel coord

$$\begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{bmatrix} = \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = \begin{bmatrix} M_0^T \\ M_1^T \\ M_2^T \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

Intrinsic x Extrinsic

World coord
 $P \leftarrow$

$$\begin{bmatrix} P^T & 0^T & -uP^T \\ 0^T & P^T & -vP^T \end{bmatrix} \begin{bmatrix} M_0 \\ M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

True for every (P_i, p_i) pair:

$$u = \frac{M_0^T P}{M_2^T P} \Rightarrow M_0^T P - u M_2^T P = 0 \Rightarrow P^T M_0 + 0 M_1 - u P^T M_2 = 0$$

$$v = \frac{M_1^T P}{M_2^T P} \Rightarrow M_1^T P - v M_2^T P = 0 \Rightarrow 0 M_0 + P^T M_1 - v P^T M_2 = 0$$

$$\boxed{QM_c = \mathbf{0}}$$

At least 6 (P_i, p_i) pairs required to compute M

~~Assume $m_{23} = 1$?~~

$$Q \rightarrow \begin{bmatrix} P_0^T & 0^T & -u_0 P_0^T \\ 0^T & P_0^T & -v_0 P_0^T \\ P_1^T & 0^T & -u_1 P_1^T \\ 0^T & P_1^T & -v_1 P_1^T \\ \vdots & & \end{bmatrix} \begin{bmatrix} M_0 \\ M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

12 Unknowns?

But if M is a solution, so is λM

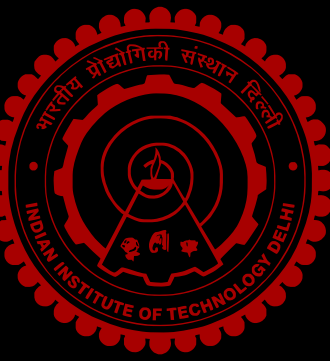


Direct Linear Transform (DLT)

Constraint on matrix values

- Use $n \gg 6$ points, and find the least error solution (for M_c)
 - We rarely have perfect measurements for points p_i and P_i anyway
- Minimize $\|QM_c\|^2$, subject to the constraint $\|M_c\|^2 = 1$ (gives non-trivial solution)
 - The solution is the eigenvector of $Q^T Q$ with the smallest eigenvalue (Homogeneous Least Squares problem)
- SVD provides a solution: $M_c = U\Sigma V^T$
 - *i.e.*, the last column of V (the singular vector with the smallest singular value)
- Finally, assemble M from M_c

Note: Collinear and Coplanar P_i can reduce the rank or make the system unstable



Separating Intrinsic and Extrinsic Matrices

Factorize M

$$M = \begin{matrix} \text{Intrinsic} \\ \text{Matrix} \\ \text{(K)} \end{matrix} \begin{matrix} \text{Perspective} \\ \text{Matrix} \\ \text{(P)} \end{matrix} \begin{matrix} \text{Extrinsic} \\ \text{Matrix} \\ \text{(R|T)} \end{matrix}$$
$$M = \begin{bmatrix} f_x & s & O_x \\ 0 & f_y & O_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{00} & r_{01} & r_{02} & t_0 \\ r_{10} & r_{11} & r_{12} & t_1 \\ r_{20} & r_{21} & r_{22} & t_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(Notation switch: $u, v \rightarrow x, y$)

$$M = K (R|T) = [KR \mid KT]$$

$$= \begin{bmatrix} f_x & s & O_x \\ 0 & f_y & O_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{00} & r_{01} & r_{02} & t_0 \\ r_{10} & r_{11} & r_{12} & t_1 \\ r_{20} & r_{21} & r_{22} & t_2 \end{bmatrix}$$

R is Orthonormal: $R^T R = \mathbf{I}$

- RQ-factorization of KR gives upper-triangular K and orthonormal R
- With K now known, and KT given, find T as $K^{-1} [KT]$
 - Actually, just use back-substitution, given K is upper triangular and T is column matrix

Can we also support non-linear intrinsics?

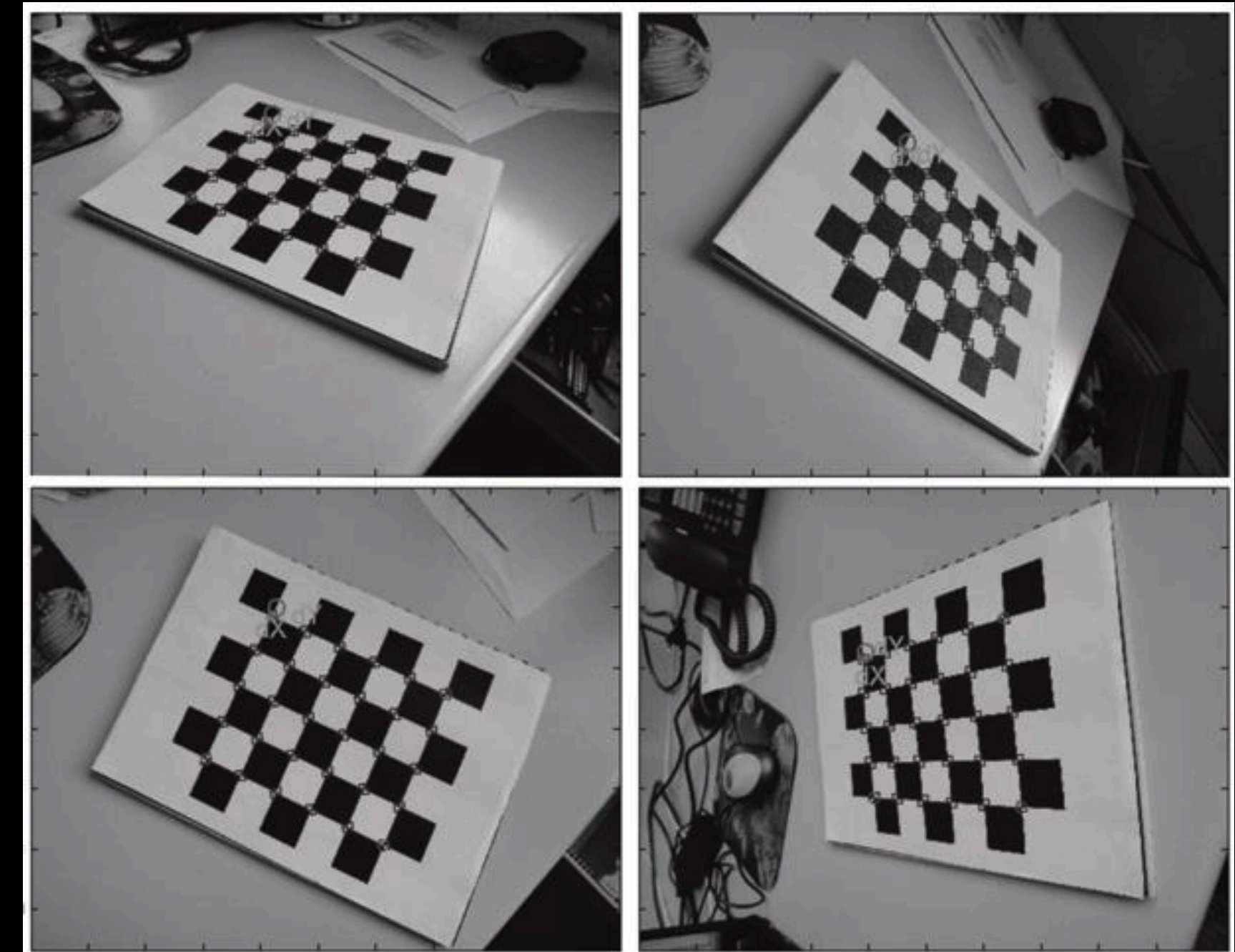
Zhang's Camera Calibration

1. Precisely measure the 3D location of points on a single flat image
2. Image the grid from ~30 camera distances, directions, and orientations
3. Detect corners in images with sub-pixel accuracy
4. Compute “ M ” matrix for each camera position
5. Starting with an initial guess for K matrix, refine it using Levenberg-Marquardt bundle adjustment

$$\sum_{i=0}^{n-1} \sum_{c=0}^{m-1} \|p_{ic} - f(K, R_i, T_i, P_c)\|^2$$

Allows a general projection function including non-linear lens distortion

For each image i and corner point c



Zhang's Method

Compute Homography

- If all P_i lie on a plane, we can arrange that the plane is $Z_w = 0$

$$\begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{bmatrix} = \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ \boxed{0} \\ 1 \end{bmatrix}$$

Zeroed

Homography matrix
Projective mapping
from plane to plane

$$\begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{bmatrix} = \begin{bmatrix} m_{00} & m_{01} & m_{03} \\ m_{10} & m_{11} & m_{13} \\ m_{20} & m_{21} & m_{23} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix} = \begin{bmatrix} H_0^T \\ H_1^T \\ H_2^T \end{bmatrix} P$$

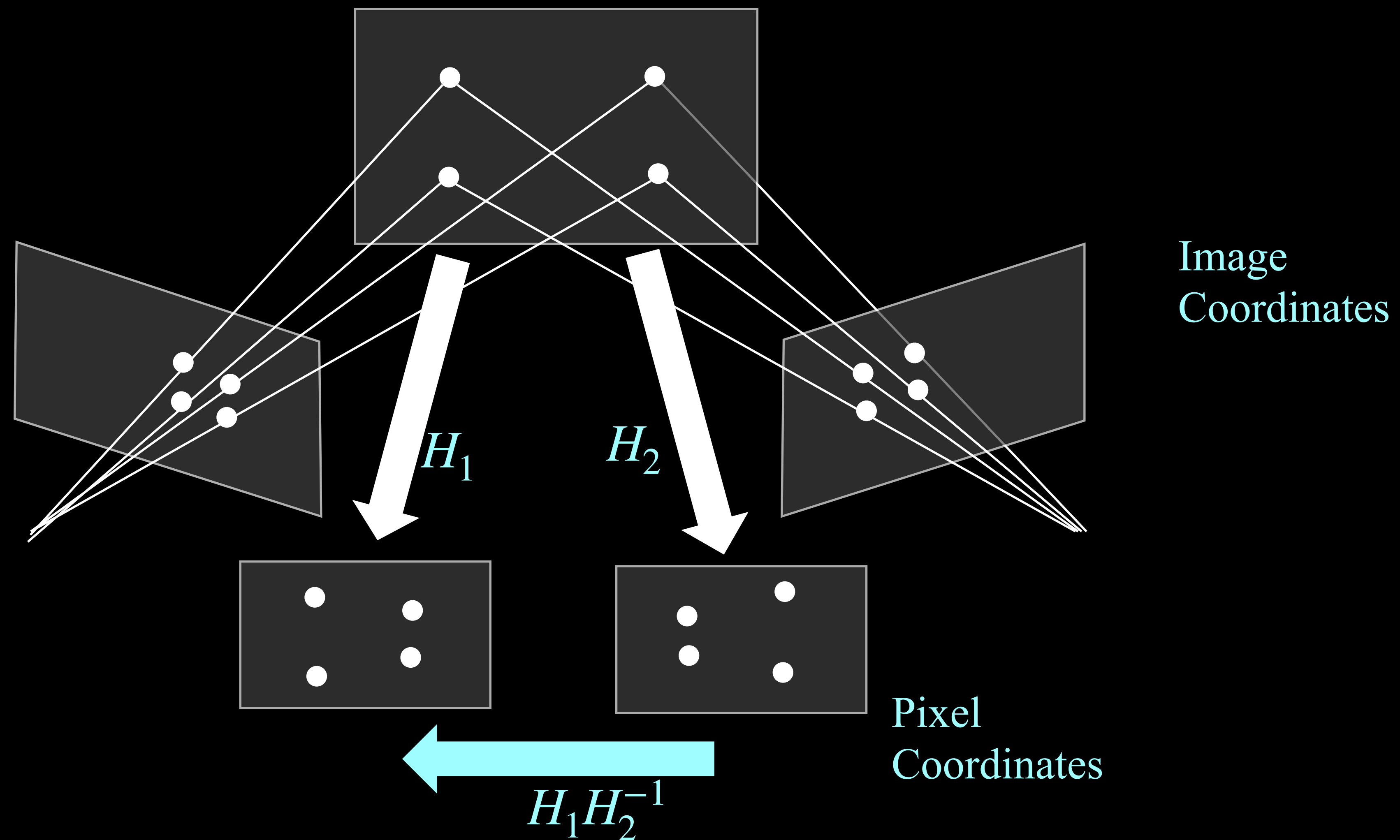
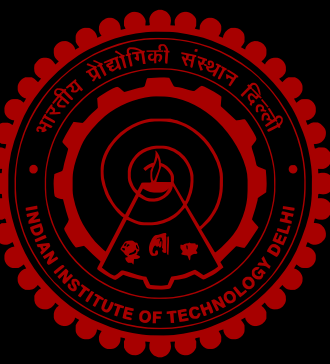
$$\begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{bmatrix} = \begin{bmatrix} f_x & s & O_x \\ 0 & f_y & O_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{00} & r_{01} & r_{02} & t_0 \\ r_{10} & r_{11} & r_{12} & t_1 \\ r_{20} & r_{21} & r_{22} & t_2 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ 0 \\ 1 \end{bmatrix}$$

↓

$$\begin{bmatrix} f_x & s & O_x \\ 0 & f_y & O_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{00} & r_{01} & t_0 \\ r_{10} & r_{11} & t_1 \\ r_{20} & r_{21} & t_2 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix}$$

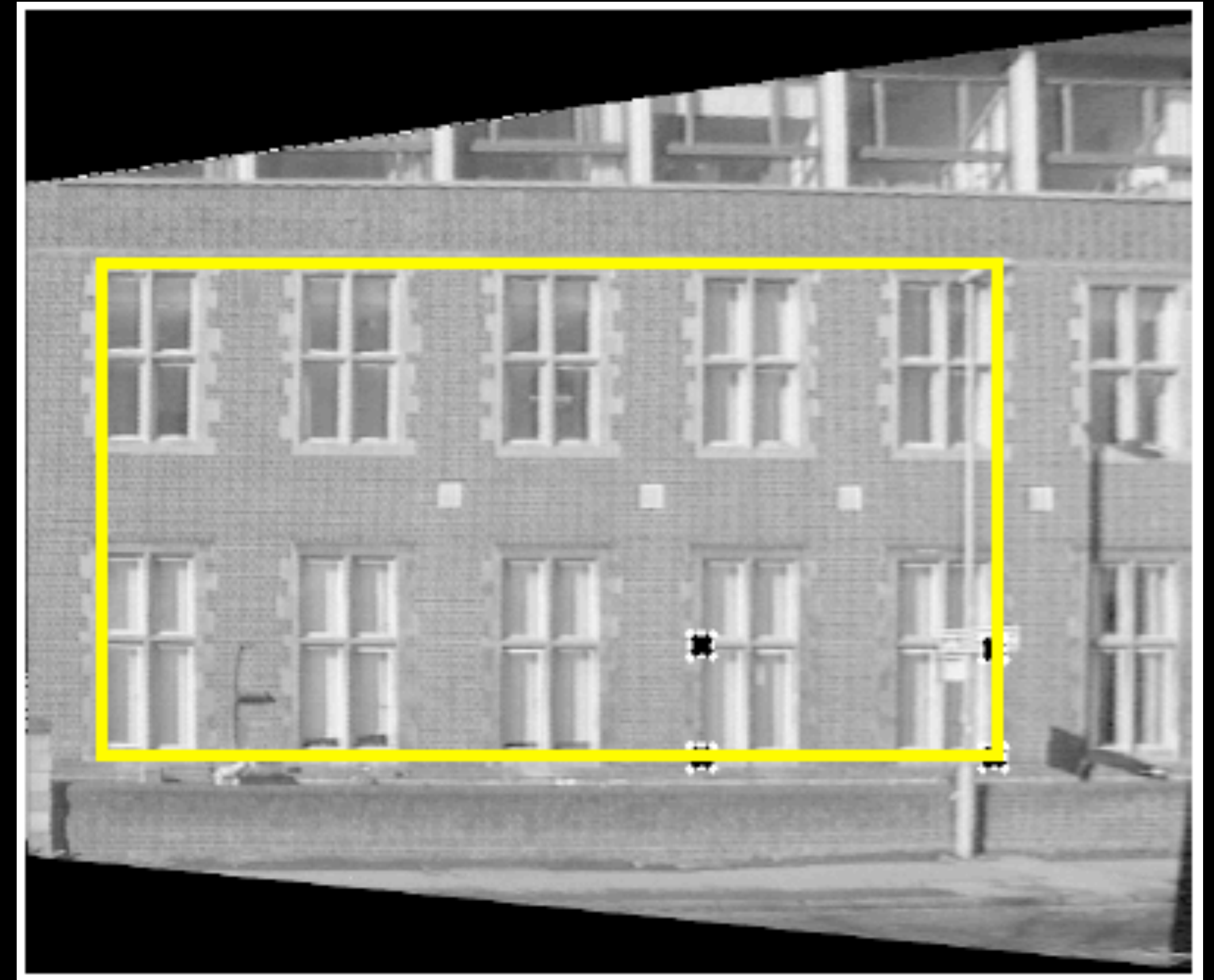
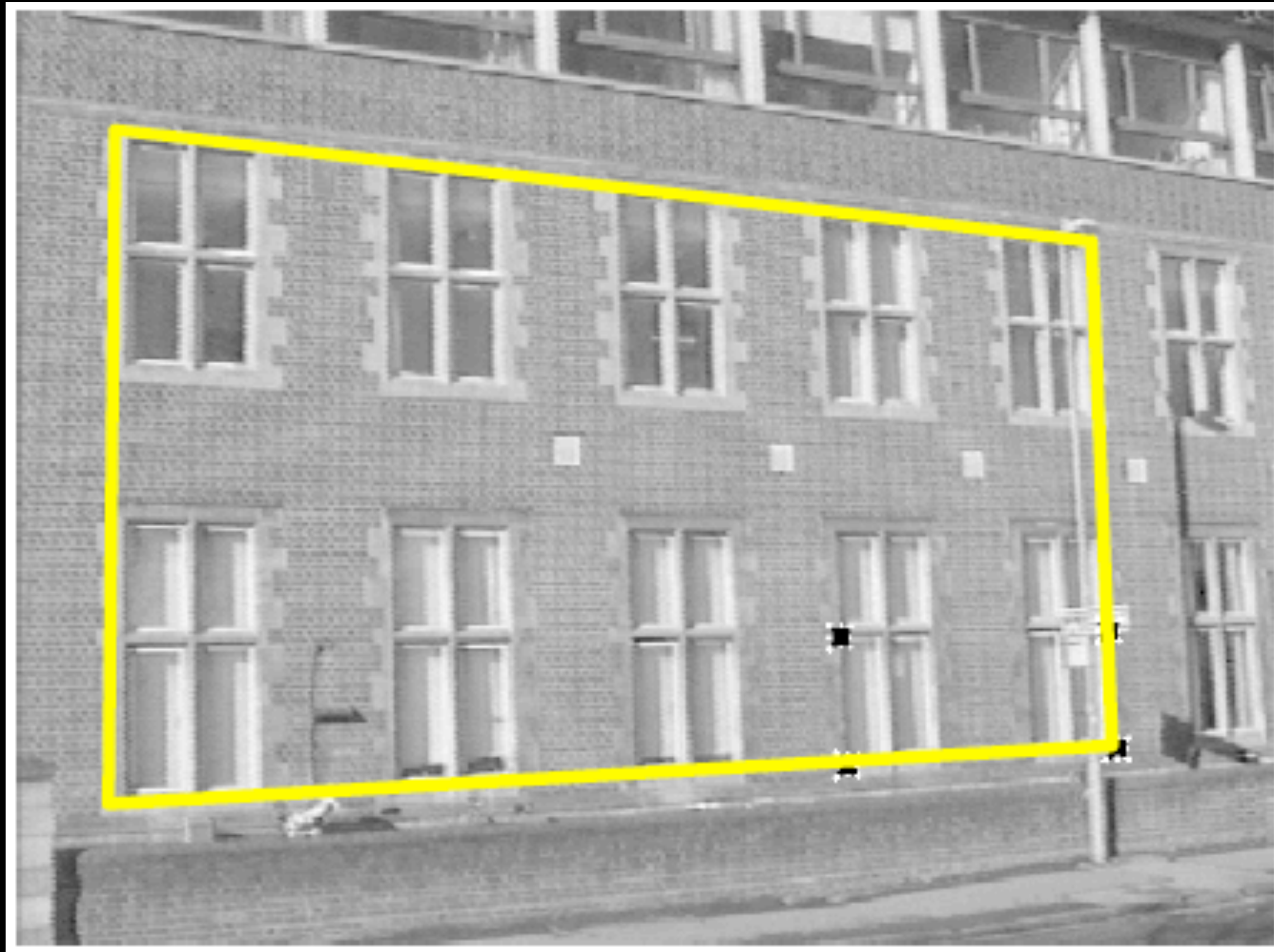
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Homography



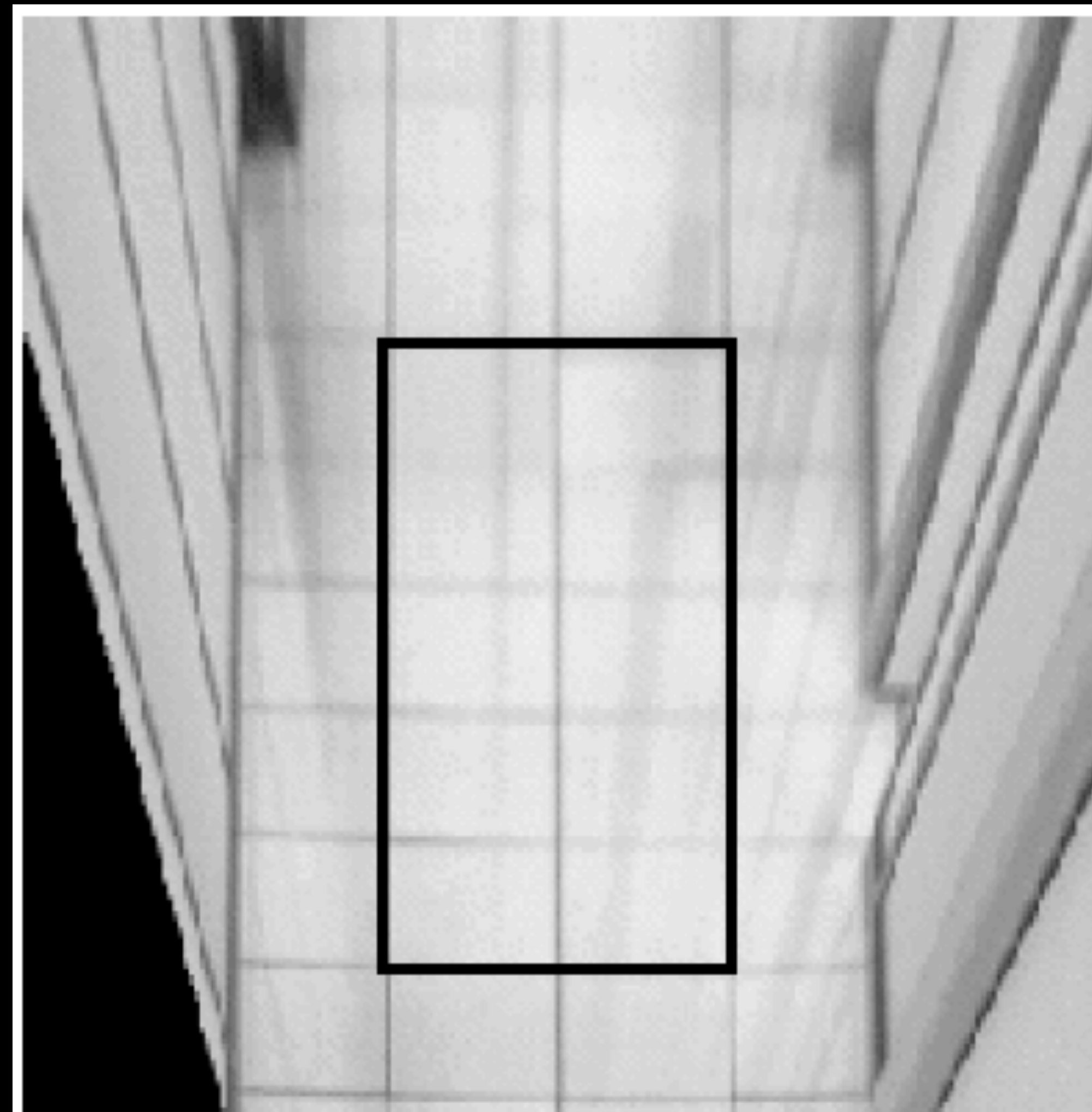
Map One Projection to Another

Undo perspective

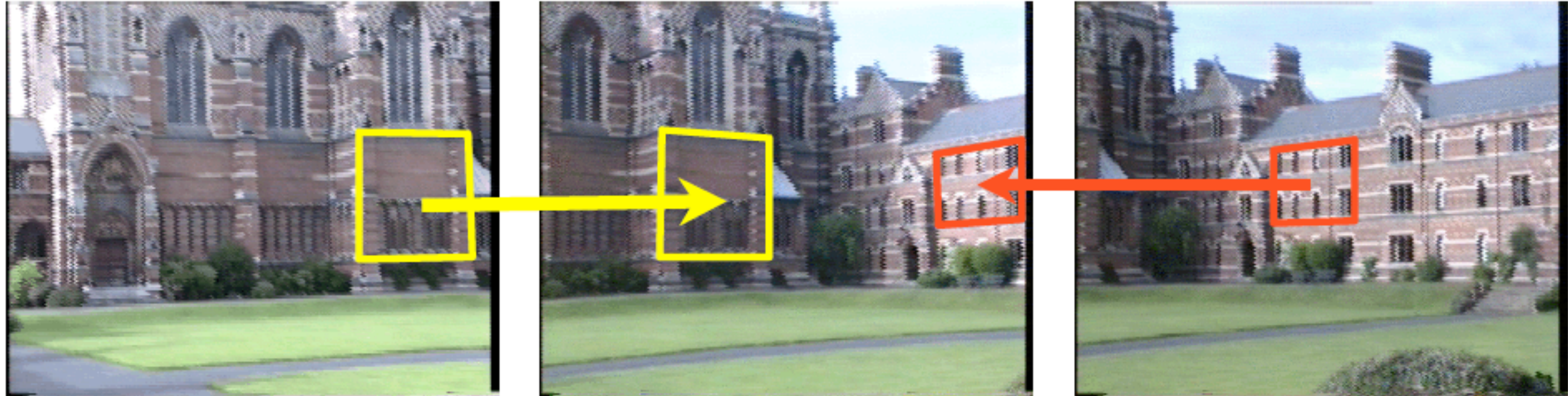


Map One Projection to Another

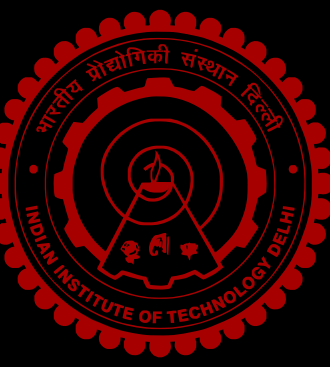
Undo perspective



Mosaicing



from Hartley & Zisserma



Zhang's Method

Compute Homography

- If all P_i lie on a plane, we can arrange that all points are on the plane $Z_w = 0$

$$\begin{aligned}
 \begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{bmatrix} &= \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ \boxed{0} \\ 1 \end{bmatrix} \\
 &\quad \text{Zeroed} \quad \text{Homography matrix} \\
 &\quad \text{Projective mapping} \\
 &\quad \text{from plane to plane} \\
 \begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{bmatrix} &= \begin{bmatrix} m_{00} & m_{01} & m_{03} \\ m_{10} & m_{11} & m_{13} \\ m_{20} & m_{21} & m_{23} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix} = \begin{bmatrix} H_0^T \\ H_1^T \\ H_2^T \end{bmatrix} P \\
 u &= \frac{H_0^T P}{H_2^T P} \Rightarrow P^T H_0 + 0H_1 - uP^T H_2 = 0 \\
 v &= \frac{H_1^T P}{H_2^T P} \Rightarrow 0H_0 + P^T H_1 - vP^T H_2 = 0 \\
 \Rightarrow \text{Solve} &\quad \begin{bmatrix} P^T & 0^T & -uP^T \\ 0^T & P^T & -vP^T \end{bmatrix} \begin{bmatrix} H_0 \\ H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
 &\quad \text{8 DOF} \quad \text{Use the DLT method,} \\
 &\quad \text{actually computing } H \\
 &\quad \text{instead of full } M
 \end{aligned}$$