

# Bilateral Filtering

## Gaussian Filter:

- Do not average across edges
  - No matter their orientation
- Bilateral filter, Mitigated Gaussian:

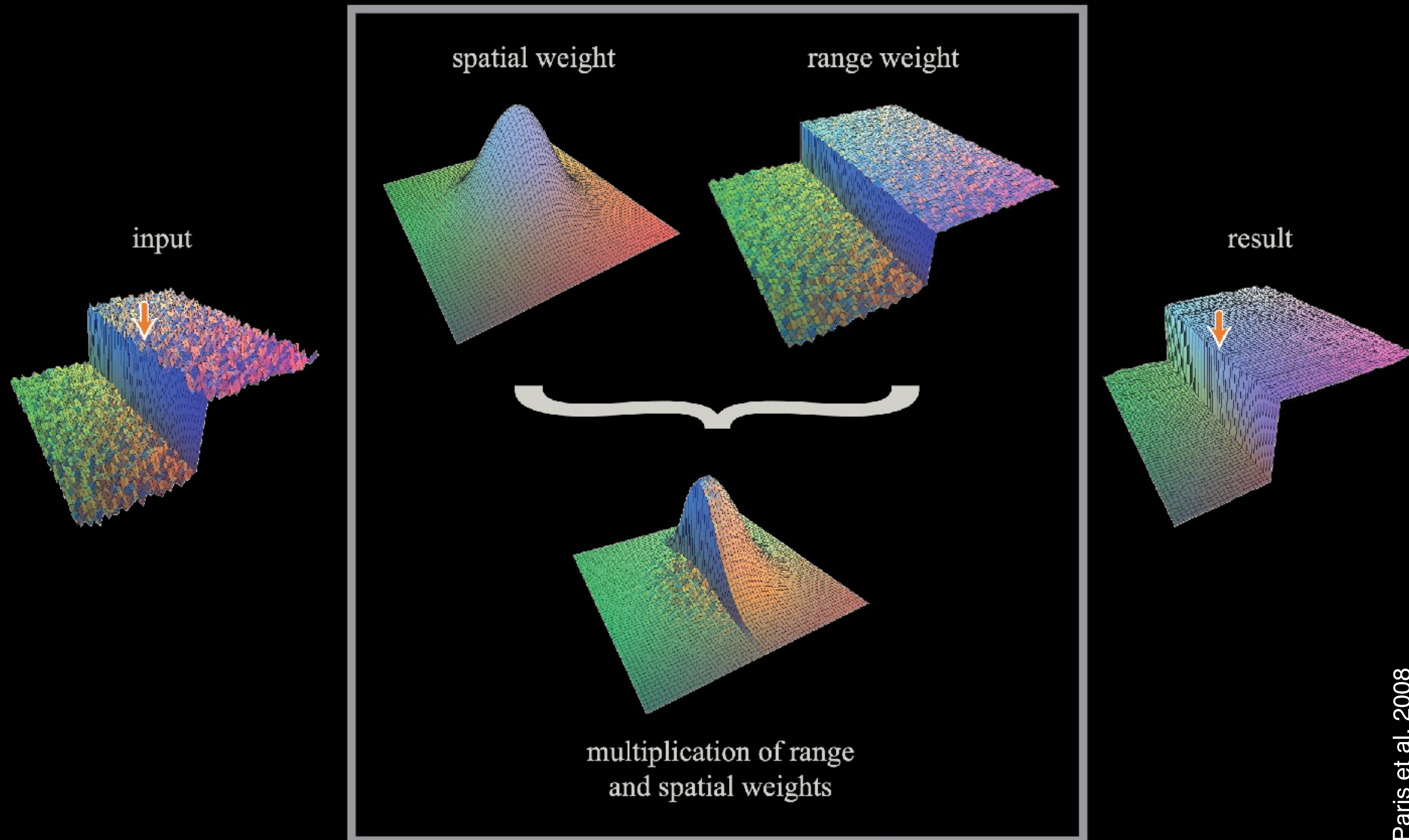
$$\hat{c}[p] = \frac{\sum_{|q-p| \leq R} g(p-q) c[q]}{\sum_{|q-p| \leq R} g(p-q)}, \text{ where } g(p, q) = e^{-\frac{|p-q|^2}{2\sigma^2}}$$

- If then neighbor  $q$  has very different intensity (low  $g_c$ ), scale its weight down

$$\hat{c}[p] = \frac{\sum_{|q-p| \leq R} g_s(p-q) g_c(c[p] - c[q]) c[q]}{\sum_{|q-p| \leq R} g_s(p-q) g_c(c[p] - c[q])}$$

👉 Is this a linear operation?

# bilateral filter weights of the central pixel





512x512 Grainy image



Gaussian Filtered (7x7,  $\sigma=2$ )



512x512 Grainy image



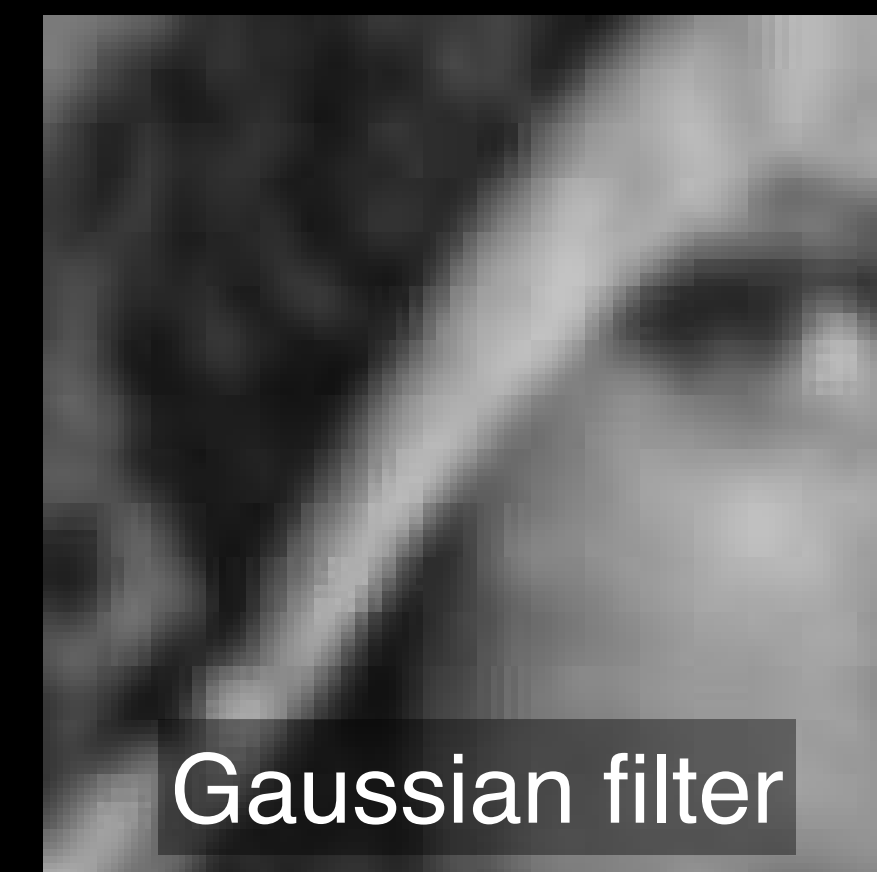
Gaussian Filtered (7x7,  $\sigma=2$ )



Bilateral Filtered (7x7,  $\sigma_s=4$ ,  $\sigma_r=50$ , w 8-bit color)

# Beyond Local Neighborhood

- Weighted averaging removes certain sporadic noise
  - More neighbors are likely to be uncorrupted, and likely to have similar intensities
    - But noise may be focussed in neighborhoods, images may have high contrast
- Order statistics have a slightly different behavior
  - But are still local
- Also use similar data from 'far away'



# Non-Local Means Filtering

- Average over nearby pixels with similar neighbourhoods

$$\hat{c}[p] = \frac{\sum_{\forall q} w(p, q) c[q]}{\sum_{\forall q} w(p, q)}$$

Compare to previous  $\sum_{|q-p| \leq R} w(p - q) g(q)$

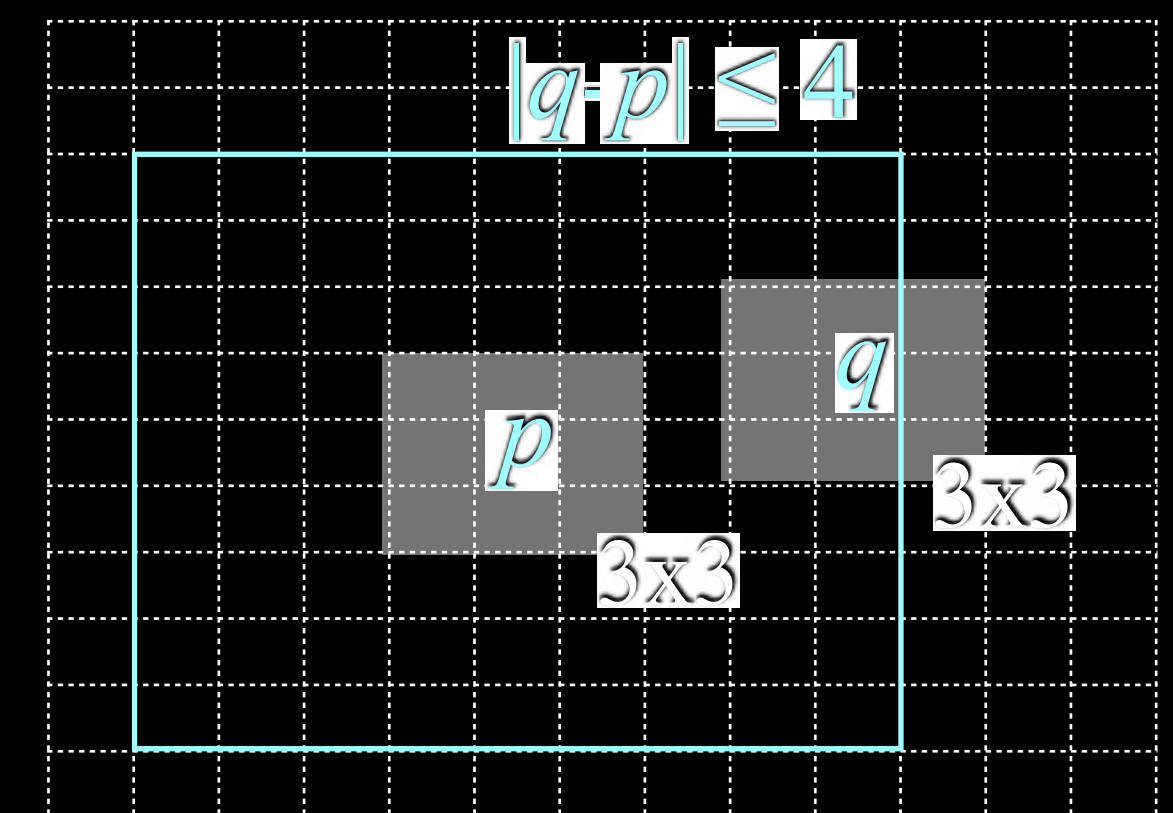
$$w(p, q) = e^{-\frac{\|c(\mathcal{N}_p) - c(\mathcal{N}_q)\|^2}{2\sigma^2}}$$

Compare to  $e^{-|p-q|^2/2\sigma^2}$

$\mathcal{N}_p$  denotes a neighborhood of pixel  $p$  (say,  $m \times n$  in 2D),

$\|g\|$  indicates sum of squared values of  $g$

Somewhat local..

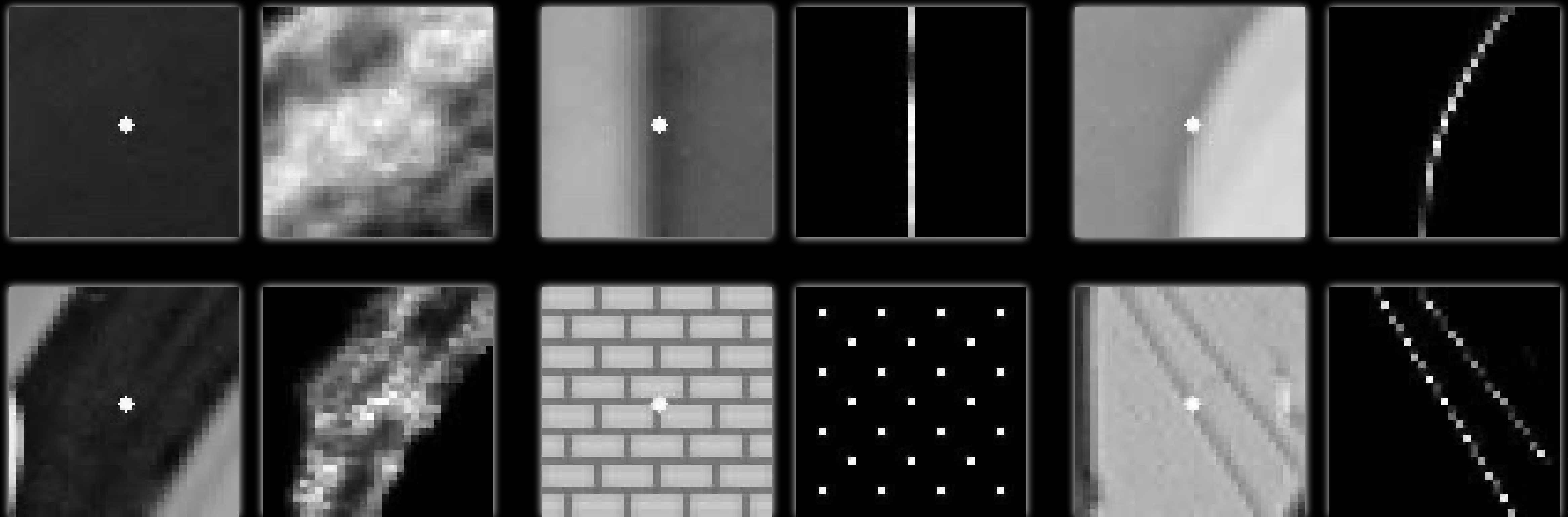
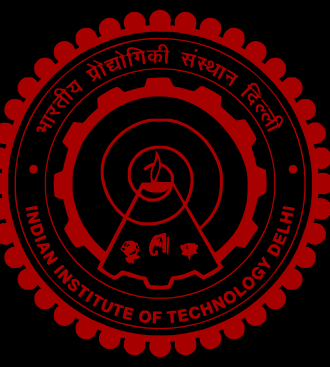


Improve:

A more geometric measure of similarity

Adapt to noise location, if available

# Examples of Distance



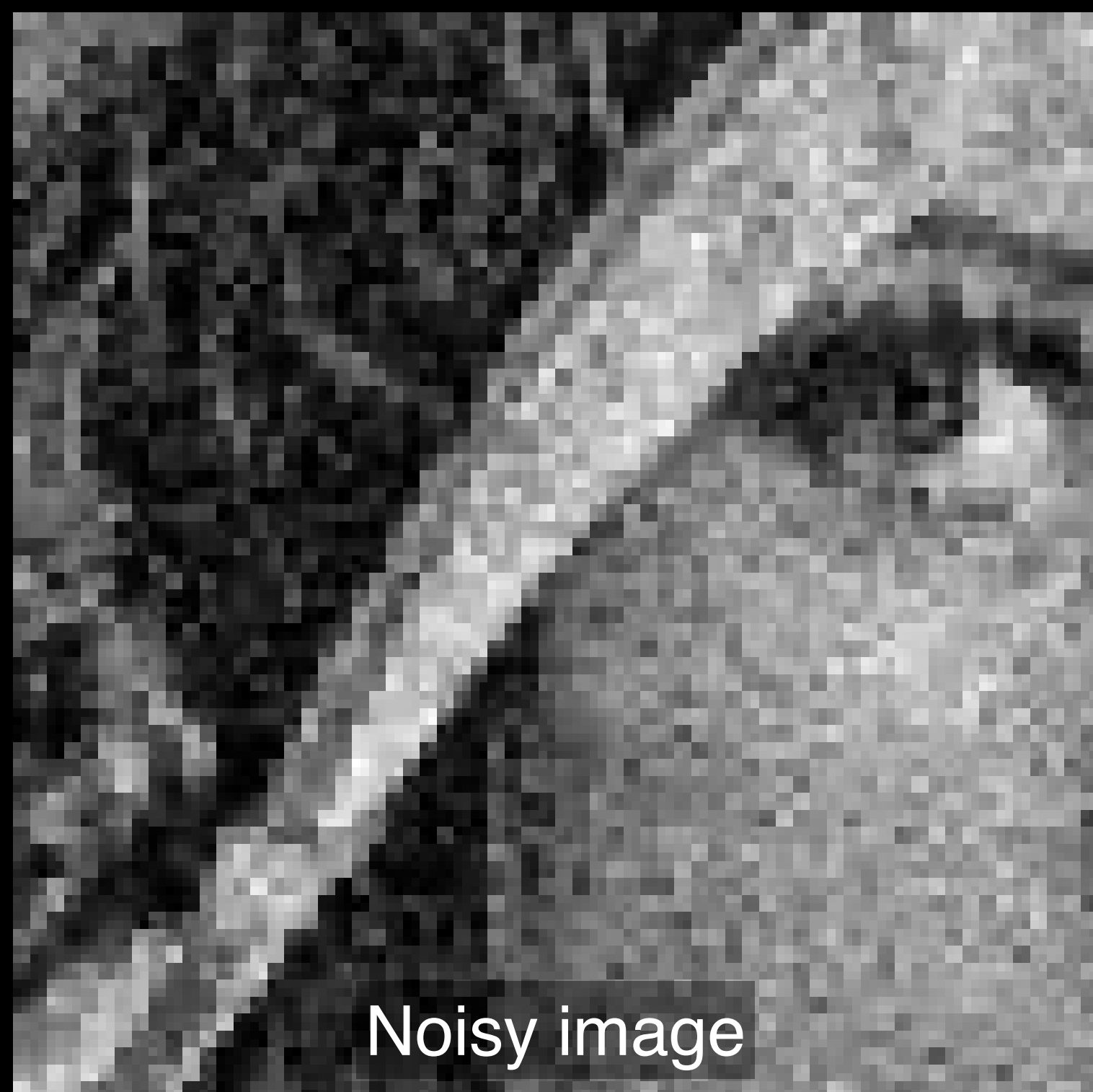
[Buades et al. 2005]

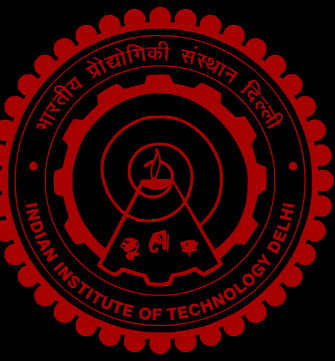


512x512 Grainy image



Non-local means, 7x7 patch, 21x21 neighborhood





# Deconvolution

Undo the effects of a filter

$$\hat{c} = c * f \longleftrightarrow \hat{C} = C \times F$$

Given,  $\hat{c}$  and  $f$ , compute  $c$

Compute  $CF$  and  $\bar{F}$ , then compute  $c = \mathcal{F}^{-1} \frac{CF}{F}$

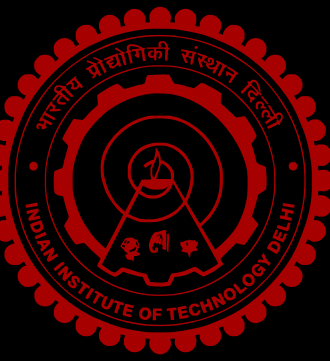
But due to noise, we actually get  $G = CF + N$  instead of  $CF$

- Find convolution that reverses the effect of both noise  $n$  and filter  $f$
- Approach: solve for  $w$ , such that  $c \approx w * g = \bar{c}$ , i.e.,  $\bar{C} = WG = WCF + WN$
- i.e., to minimize the expected MSE,  $\mathbb{E} \left( \sum | \bar{c}[x, y] - c[x, y] |^2 \right)$ , find  $W$  that minimizes  $\mathbb{E} \left( \sum | W[\mu, \nu]G[\mu, \nu] - C[\mu, \nu] |^2 \right)$

$\Rightarrow$  Solution:

$$W(\mu, \nu) = \frac{|F(\mu, \nu)|^2 / F(\mu, \nu)}{|F(\mu, \nu)|^2 + S_\eta(\mu, \nu) / S_c(\mu, \nu)}$$

Power Spectral Density: distribution of amplitude<sup>2</sup> across  $\mu, \nu$



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(Wiener filter)

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1/Signal to Noise Ratio

$(\text{SNR}_{\text{db}} = 10 \log_{10}(\text{SNR}))$



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Undo the effects of a filter

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(Wiener filter)

- For high SNR,  $S_\eta(\mu, \nu) \ll S_f(\mu, \nu)$ ,  $W(\mu, \nu) \approx 1/F(\mu, \nu)$
- For low SNR,  $S_\eta(\mu, \nu) \gg S_f(\mu, \nu)$ ,  $W(\mu, \nu) \approx 0$
- Usually,  $S_f, S_\eta$  are not known, try several constant estimates  $S_\eta(u, v)/S_f(u, v) = K$



Original

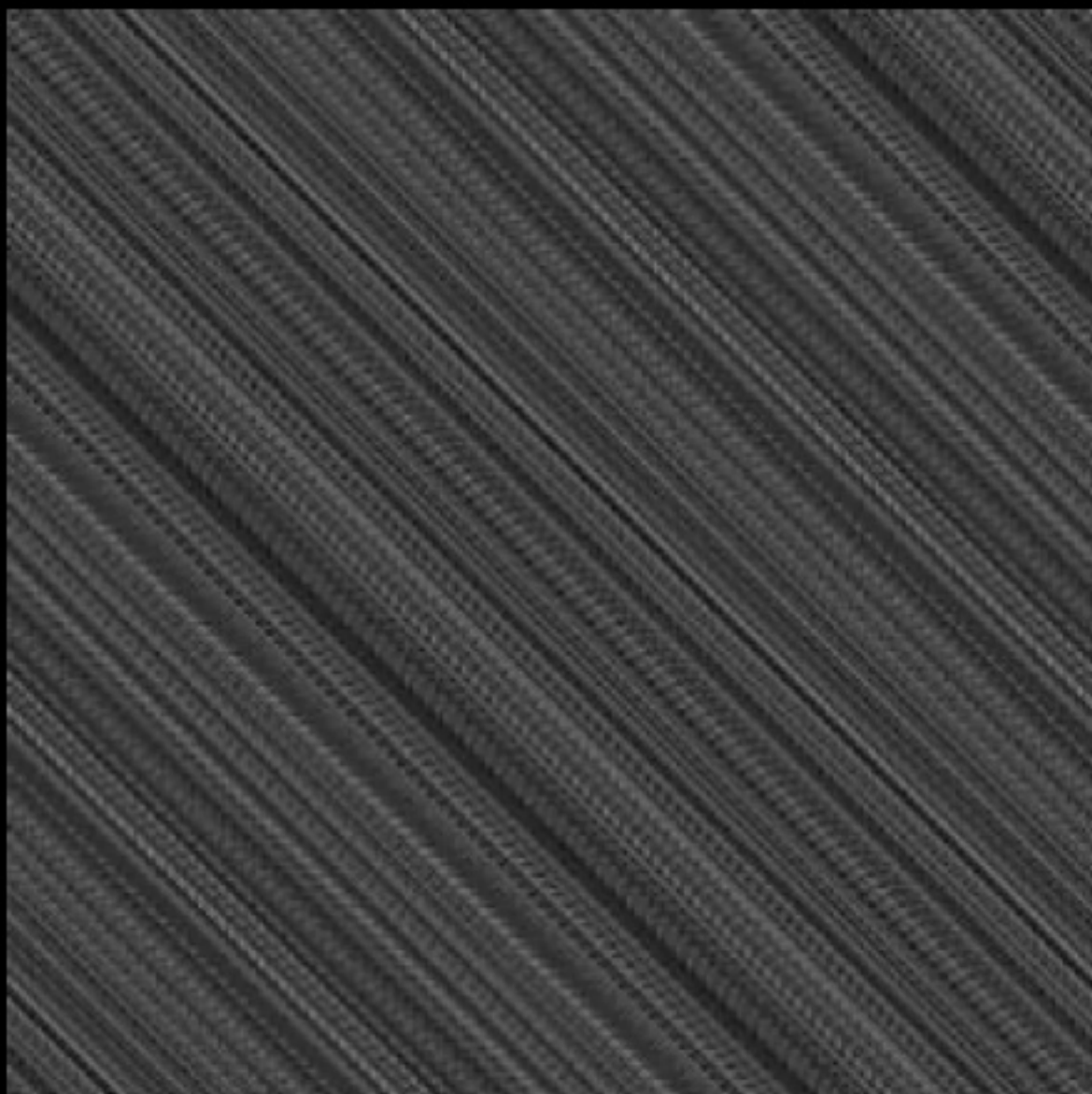


Blurred

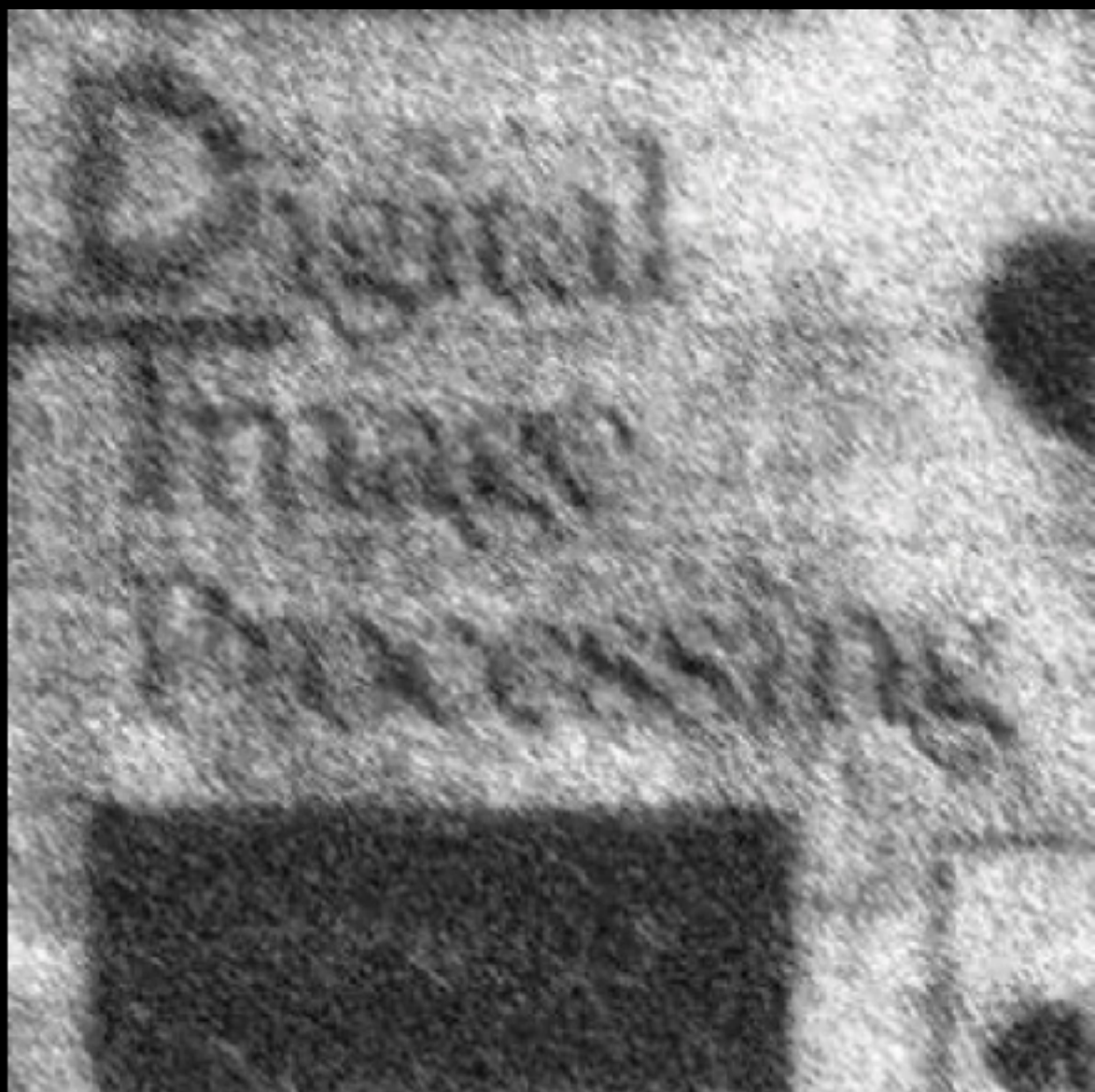
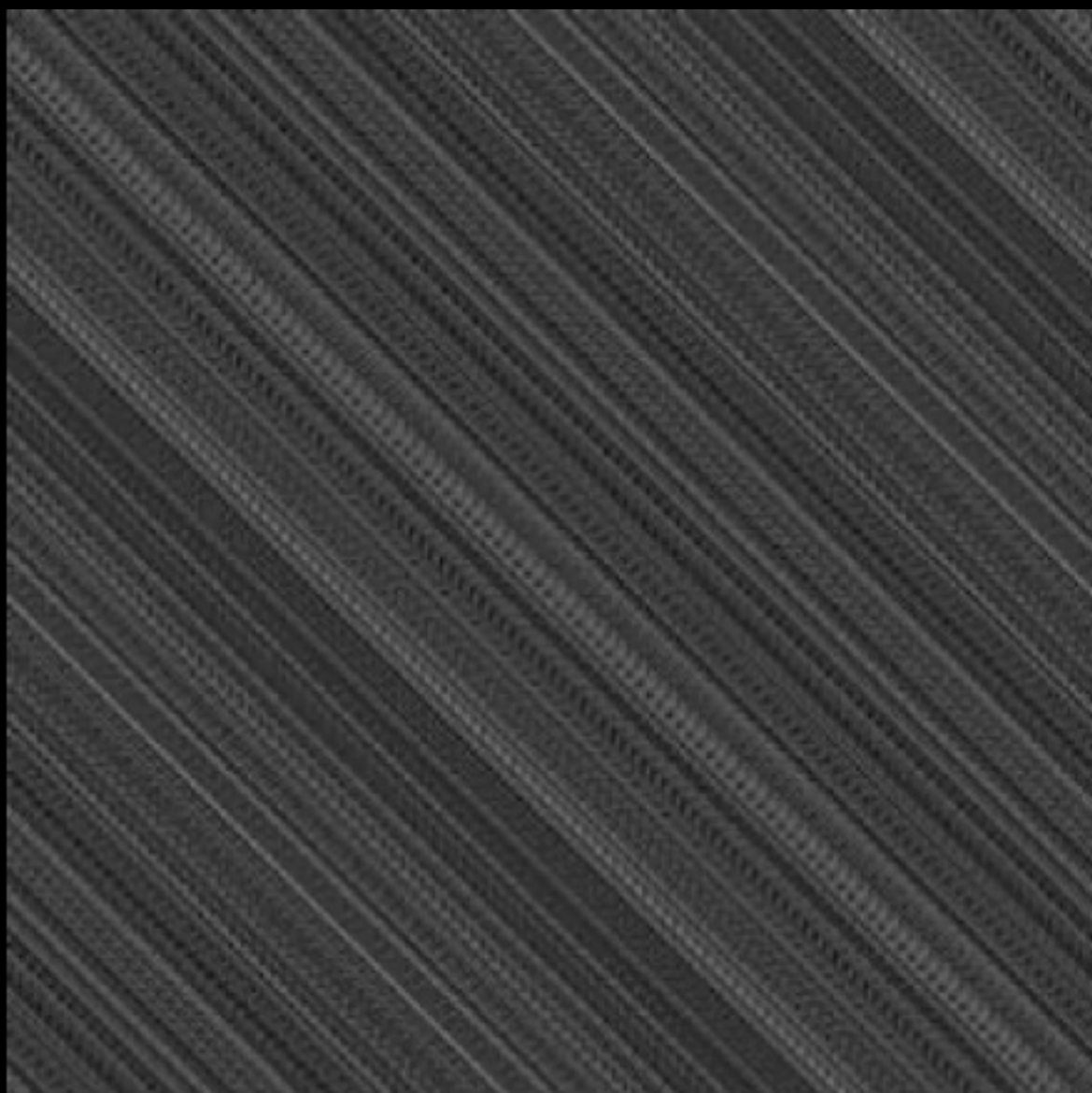
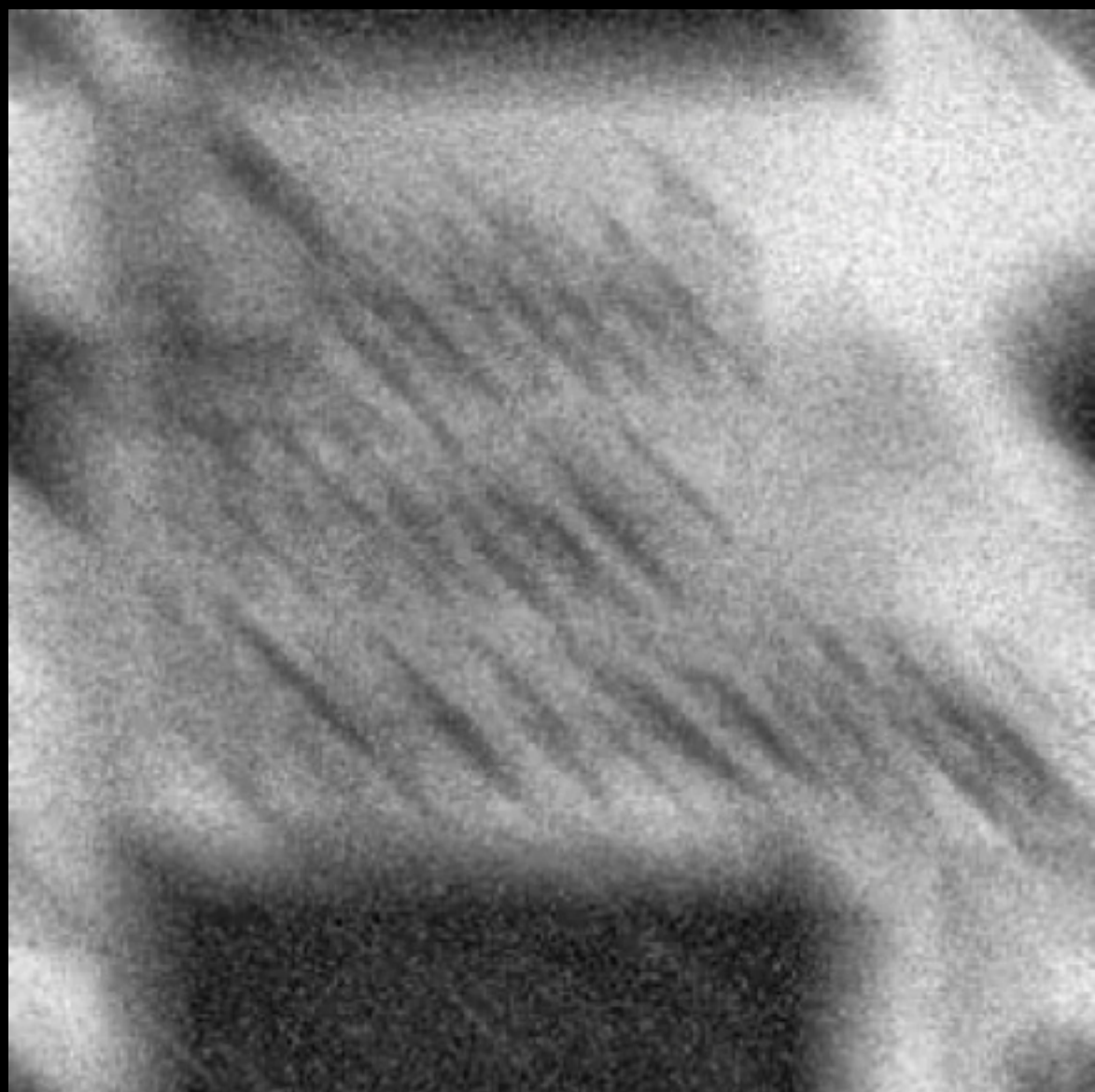


Wiener filter

Low noise



High noise



Input image

Inverse filter

Wiener filter