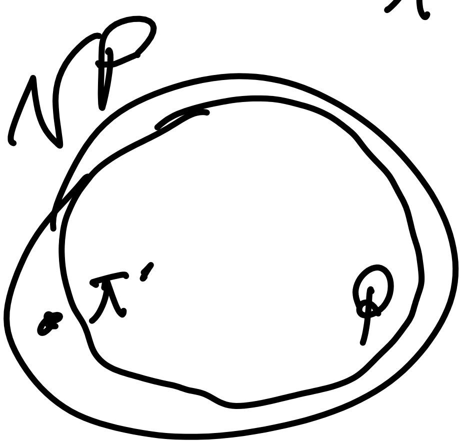
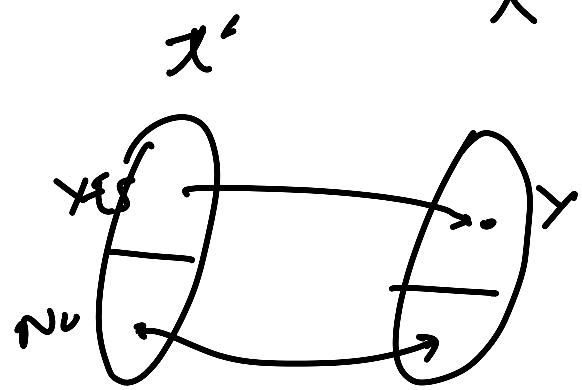
CSL 356 Lecture 41

A problem π is **NP Hard** if all problems in the class NP can be polynomially reduced to π.

i.e. If $\pi' \in NP$ then

$$\pi' \leq_{poly} \pi$$





If $\pi \in NP$ then π is **NP complete**

NP hard / complete "under" polytime reduction

Recall that poly-time reduction satisfy transitivity

$\pi' \leq_{poly} \pi''$ and $\pi'' \leq_{poly} \pi^*$

$\Rightarrow \pi' \leq_{poly} \pi^*$

If π is NPC and there is a polytime algorithm for π

1. $\Rightarrow$ poly-time algorithms for all problems in NP.

2. If we can show that there cannot exist a poly-time algorithm for π, then $P \neq NP$

$$\boxed{\text{How does one show the } \pi \text{ is NP complete? ??}}$$

Suppose π^1 is NPC and
π^2 is NPC

$\Rightarrow \quad \pi^1 \le_{poly} \pi^2$

and $\quad \pi^2 \le_{poly} \pi^1$

To show a new problem π^3 is NPC

① π^3 is in NP

② $\pi^1 \le_{poly} \pi^3$

(since $\pi \in NP$ $\quad \pi \le_{poly} \pi^1$

<u>Cook-Levin theorem</u>: The satisfiability problem of Boolean formula is NPC.

Give n boolean variables

say $x_1, x_2, \ldots x_n$

$x_i \in \{T, F\}$

then given any Boolean formula

say

V: or
Λ: and
‾ : negation

$(x_1 \vee x_3) \wedge (x_4 \vee x_5)$

$\wedge \quad x_1 \wedge (x_4 \vee \bar{x}_6) \ldots$

is there an assignment of x_i's such that the expression is True.

<u>Cook-Levin thm (stronger)</u>: The satisfiability problem of a boolean formula given as Conjunctive Normal Form (CNF) with exactly 3 literals per clause is NP complete

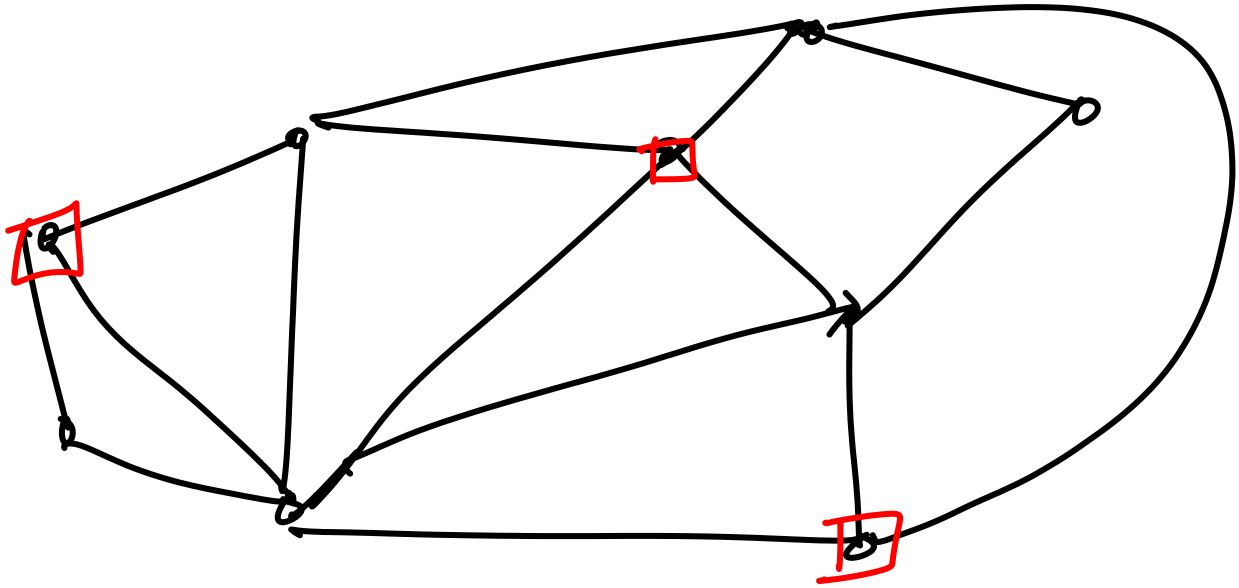
$$(x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_2 \vee \bar{x}_3 \vee x_4)$$

$$\wedge \; (\qquad) \; - \; - \; - \; (\qquad)$$

Say m clauses each having 3 literals (a literal is a boolean variable or its complement)

3 CNF formula

Vertex cover problem



Does every edge have at least one of its endpoints in the cover (marked by red)

Does there exist a Vertex cover of size K $(K \leq n)$?

Is V.C. in NP?

It suffices to show that

$$3CNF \leq_{poly} V.C.$$

Given any instance of the 3CNF problem, say a formula F we have to map it to some instance of the V.C. problem, say $\rho(F)$ such that

$$\rho(F) \begin{array}{l} \nearrow G \\ \searrow K \end{array}$$

such that

G has a vertex cover of size K iff F is satisfiable.

<u>and</u> ρ must be computable in polynomial-time.