CSL 702 Lecture 3

Dijkstra's algorithm for +ve wt graphs

BF : $O(|V||E|)$ which can be $\Omega(n^3)$ running time

$O(n^3)$

Please use 'O' 'Ω' appropriately

'o' small 'Oh' ω : small Omega

strictly asymptotically smaller n is $o(n\log n)$

$\frac{n}{n\log n} \to 0$ $_{n\to\infty}$

asymp. strictly greater than $n\log n$ is $\omega(n)$

$\leq, <, >, \geq$

Basic operation Relax(u,v)

if $\delta(v) < \delta(u) + w(u,v)$
then $\delta(v) \leftarrow \delta(u) + w(u,v)$
Link u as predecessor of v

disjoint
S : set of vertices v, $\delta(v) = D(v)$
T : " " $\delta(v) \geq D(v)$

Initially $\delta(s) = D(s) = 0$ other $\delta(v) = \infty$

Try to include vertices $w \in T - S$ into S
Pick the smallest value of $\delta(w)$ from T

Once w moves to S, relax all edges (w, x) $\quad (w,x) \in E$

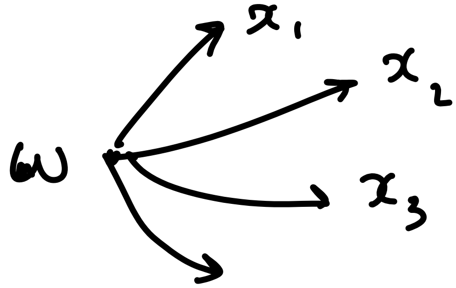
until $T = \phi$

Running Time : $(|V| - 1)$ iterations

1. Pick the smallest label : Heap on T

$O(\log n)$ to pick the smallest

2. Relax all edges (w,x)



neighbours of w

$\boxed{\text{degree of } w}$

$\delta(x_1)$ $\delta(x_2)$ $\cdot$ $\delta(x_k)$

must be updated

$\deg(w) \leq |V|$

Decrease key : $O(\log n)$

Cost : $O(\deg(w) \cdot \log n)$

Total cost of a single iteration $O(\log n + \deg(w) \log n)$

Over $|V|$ iterations $O(|V| \log n + |V|^2 \log n)$

$$\sum_{i=1}^{|V|} \left(\log n + \deg(w) \cdot \log n \right)$$

$$\leq |V| \cdot \log n + \log n \underbrace{\sum_{w \in V} \deg(w)}_{\leq |E|}$$

$$\leq |V| \log n + |E| \log n = (|V| + |E|) \log n$$

<u>Possibilities of improvement</u>: Perhaps Decrease key can improve

<u>Why is Dijkstra's algorithm always correct?</u>:

<u>Claim</u>: When we move $w \in T$ to S
$\delta(w) = D(w)$

<u>Observation</u>: The labels of vertices in successive iterations included in S are in non-decreasing order.

<u>Proof by contradiction</u>: Suppose $D(w)$ is not the shortest path distance of w
$\delta(w)$ was the min of all labels in T

Then is a shortest path from s to w

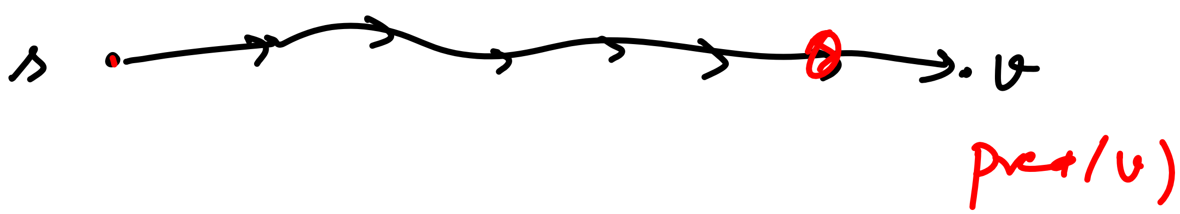
Clearly $x_{k-1} \notin S$ $x^* \to y$ is an edge

Case: no zero length path from x^* to w

$D(y) = \delta(y) > \delta(x_k)$

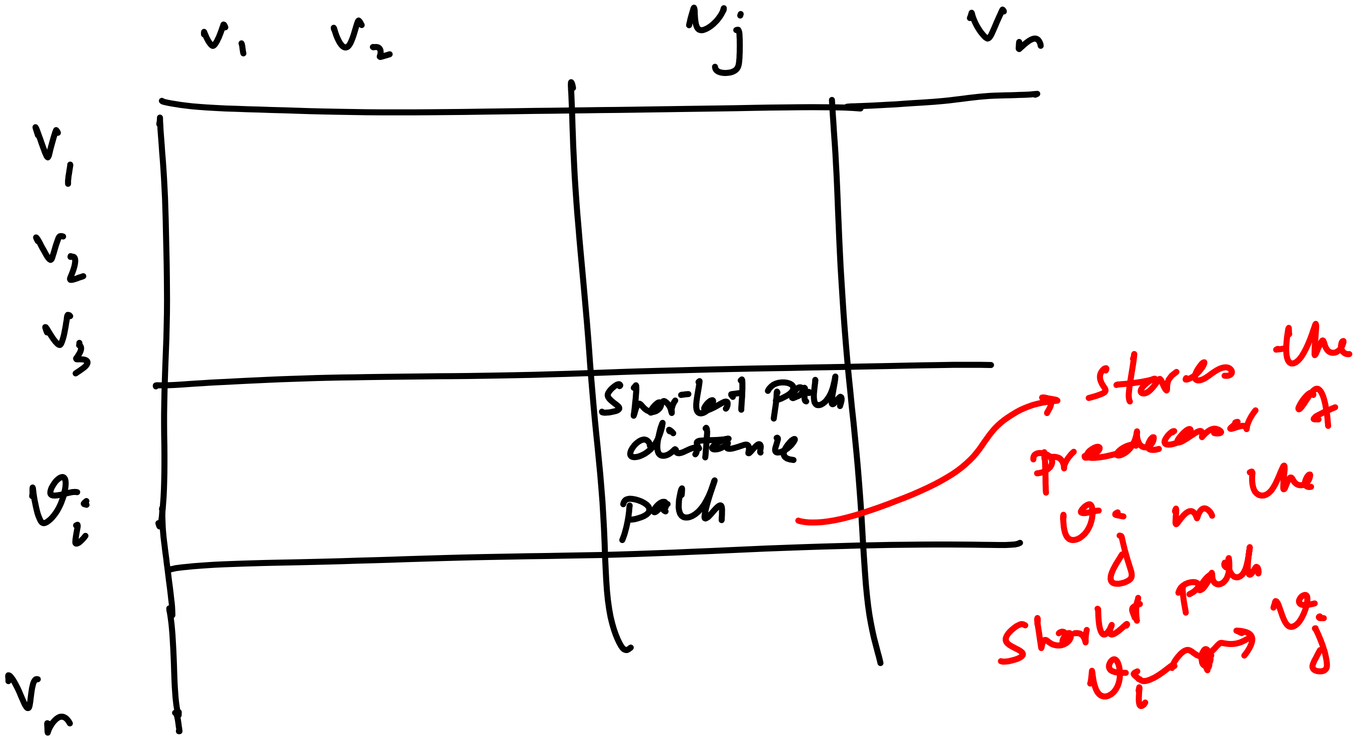
Clearly w doesn't have the smallest label: contradiction

What about shortest "paths" rather than shortest path distances $D(v)$

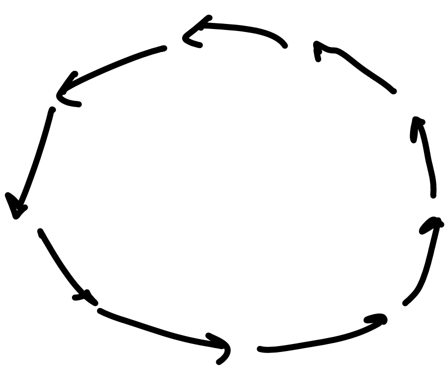


All pairs shortest paths

Invoke SSSP(v) for all all vertices $v \in V$



A single path can have length $|V|-1$



What is the structure of the shortest path (SSSP)