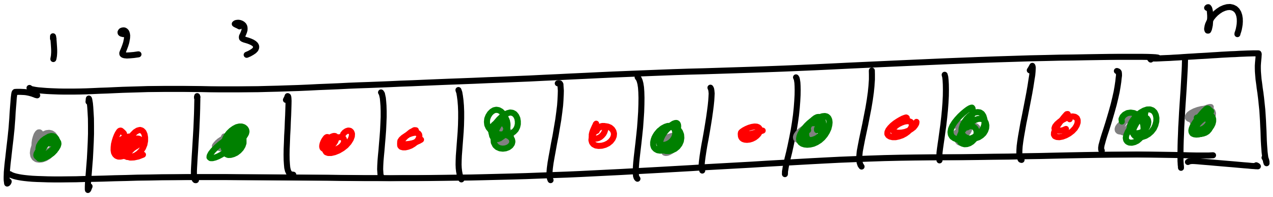
COL 702 Aug 2, 2018



We want to pick a green ball
The colour of ball is not known till
we actually pick/access/test

The proportion of green : red is roughly 1:1

So sequentially picking them yields
$\Omega(n)$ algorithm — $\frac{n}{2}+1$ to be precise

Among the $1 \ldots n$ locations choose
an integer $i \in [1..n]$ uniformly at random
using a random no. generator (RNG)

Repeat
generate $i \in_{\mathcal{U}} [1..n]$
Test if $A[i]$ is green
Until we have found a green ball

Input is worst case and no
one has the control of the RNG
RANDOMIZED Algorithm

Suppose the balls were randomly distributed in the array —

(i) Among all possible distributions/permutations, every permutation is equally likely

(ii) In Every location i $\Pr[A[i] = \text{green}] = \Pr[A[i] = \text{red}] = \frac{1}{2}$

The probability that the first $k \geq 1$ location are red $= \frac{1}{2^k}$ (using independence)

(probing in sequence where there are $\frac{n}{2}$ green balls)

$\leq \frac{1}{2^k}$ using conditional prob

(given the first $k-1$ balls are red ...)

What is the "Expected" no. of balls we probe before we find a green ball?

Let X be a r.v. that is the # of balls picked up before we find a green ball

$$1 \leq X \leq \frac{n}{2} \leq n$$

$$\Pr[X = i] \leq \frac{1}{2^i}$$

$$E[X] = \sum_{i=1}^{n} i \Pr[X=i] = O(1)$$

Probabilistic inequalities

What is the prob that a r.v.

$$a \leq X \leq b$$

For non-negative r.v.

Markov inequality

$$\Pr[X \geq k \cdot E[X]] \leq \frac{1}{k}$$

Average case analyses is the behavior (Time / Space) of random input under some known distribution

Randomized algorithm $\mathcal{A}$
is function of Input I and
random bits r

Running Time $T = f(I, r)$

<u>Quicksort</u> : Use a pivot to partition the input into two subsets and apply the same algorithm recursively to the two partitions

$x_{n/4}$ $\qquad\qquad\qquad$ $x_{3n/4}$

$$x_1 < x_2 < x_3 \quad \boxed{x_{\frac{n}{2}}} \quad < x_n$$

x_1'	x_2'	x_3'	$\cdot$		x_n'

Matrix multiplication

$A^{n\times n}$, $B^{n\times n}$ $A_{i,j}, B_{i,j} \in \{0,1\}$

$$\begin{bmatrix} \boxed{0 \quad 0 \quad 1} \\ 1 \quad 0 \quad 1 \\ 1 \quad 1 \quad 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \bmod 2 \\ \\ \end{bmatrix}$$

M. Multiplication $O(n^3)$

<u>Input</u> : A, B, C

$$AB \stackrel{?}{=} C$$

$$AB - C \stackrel{?}{=} 0$$

<u>Idea</u> Consider a 0/1 vector X

$$(A(BX)) \stackrel{?}{=} (CX)$$

If the two sides are equal we answer YES

otherwise ans NO