COL 702 Lecture 25 Nov 1

Constructing Graph Spanners

The girth of a graph (shortest cycle) is critical for bounding the "stretch" of a spanner.

In particular $\dfrac{\delta_{G_s}(x,y)}{\delta_G(x,y)} \geq g-1$

where g is the girth

A complete bipartite graph has girth $g=4$
$\Rightarrow$ any spanner has stretch ≥ 3

Observe that - the denser the graph
- the smaller is the girth

There are provable trade offs between # edges and girth of a graph

If a n-vertex graph $\geq m$ edges then girth $\leq f(m, n)$

for some function of m

There are graphs with $m = k n^{1+\frac{1}{k}}$ edges that have girth $\le 2k$ for any integer k

Eg. $k = 2$ $\quad n^{1+\frac{1}{2}} : n^{3/2}$ has a cycle ≤ 4 stretch 3

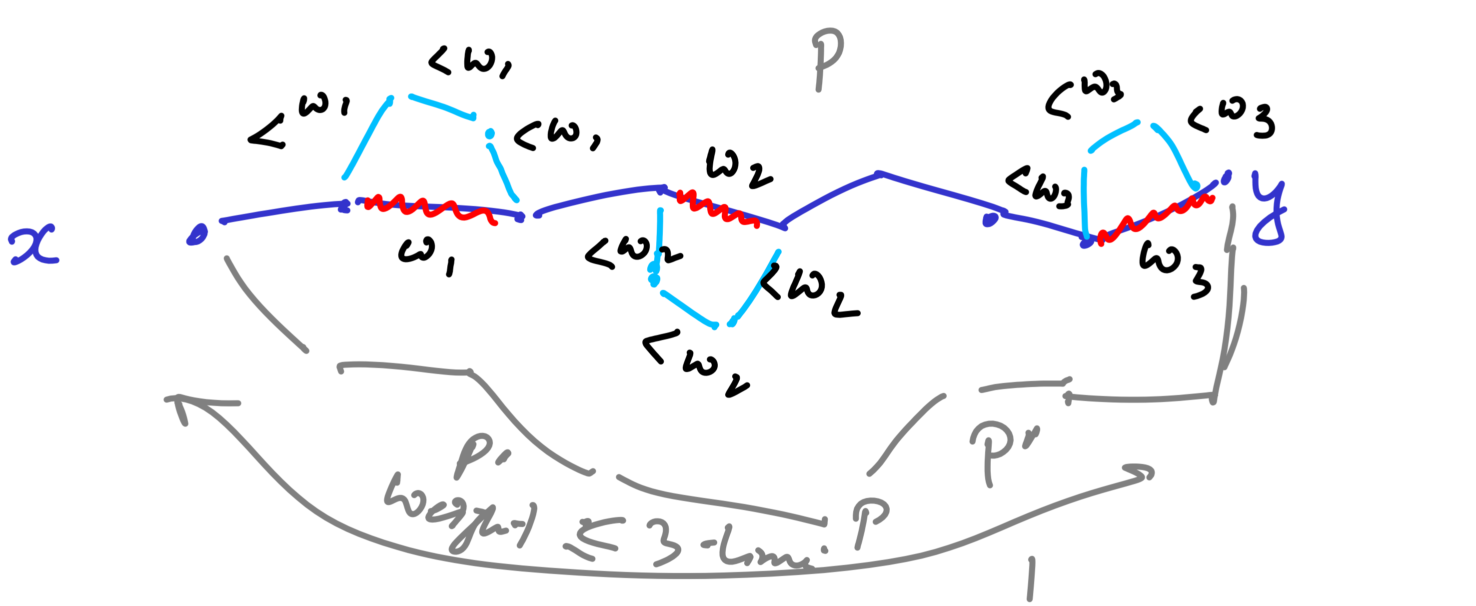
$k = 3$ $\quad n^{4/3}$ edges girth : 6 stretch 5

$\vdots$

$k = \log n$ $\quad m = \log n \cdot n \cdot n^{1/\log n} : n \log n$

girth - $2 \log n$ stretch ~ $\log n$

stretch : girth − 1



Phase 1 : ① Choose $\sqrt{n}$ vertices randomly, say $R \subset V$

$G : (V, E)$

$G_s : (V, E_s)$

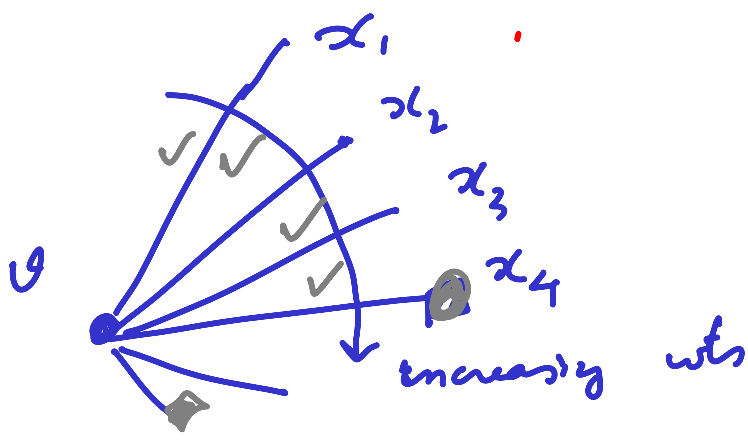
"choose every vertex independ. with prob $\frac{1}{\sqrt{n}}$"

For $v \in V - R$, we find the nearest sampled neighbour n_v in R and form clusters based on n_v

n_v is undefined if none of the neighbours $N(v)$ are sampled

So there are $\sqrt{n}$ clusters – one for each of the sampled vertices

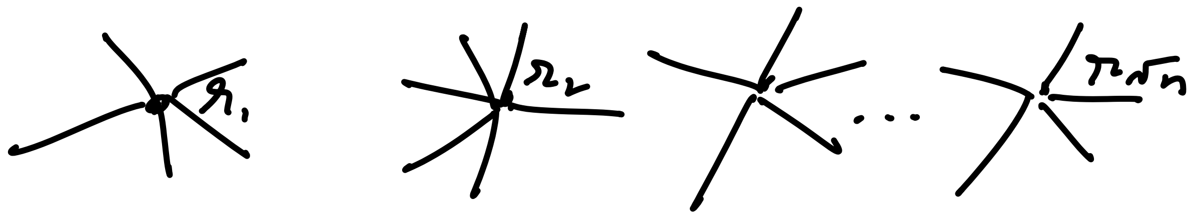
② For every vertex $v \in V - R$, choose all edges (v, x) to include in E_s s.t. $w(v, x) \leq w(v, n_v)$



nearest sampled vertex

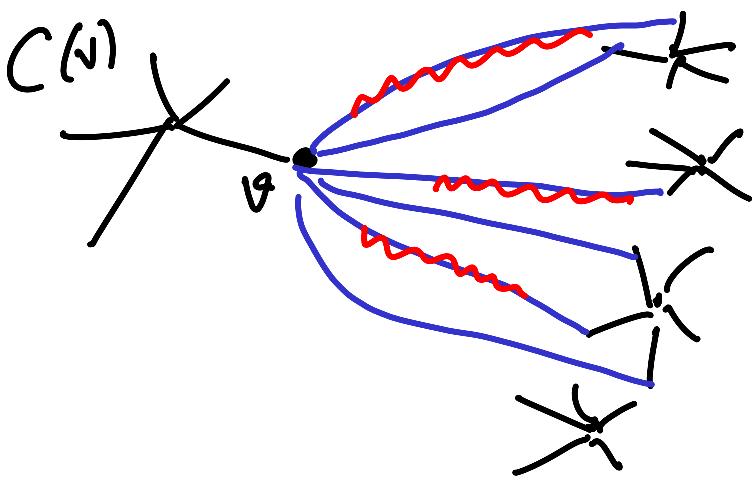
If no neighbour of v is sampled, add all edges out of v

Phase 2



$\sqrt{n}$ clusters

For any vertex v, consider the "inter-cluster" edges



$C(v)$ is the cluster of v

Among the intercluster edges, choose the least weighted for every cluster and add to E_s

Total # of edges added to E_s in phase 2 : $n \times \sqrt{n}$ (# clusters)

Expected # edges added to E_s in phase I $= \sqrt{n}$ per vertex using linearity of Exp.

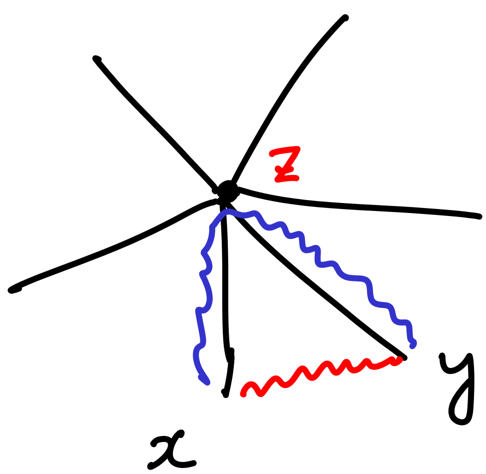
Total : $n \cdot \sqrt{n}$

Sketch of the graph $G_s = (V, E_s)$

Consider an edge $(x, y) \in E - E_s$

__Case I__ x, y belong to the same cluster

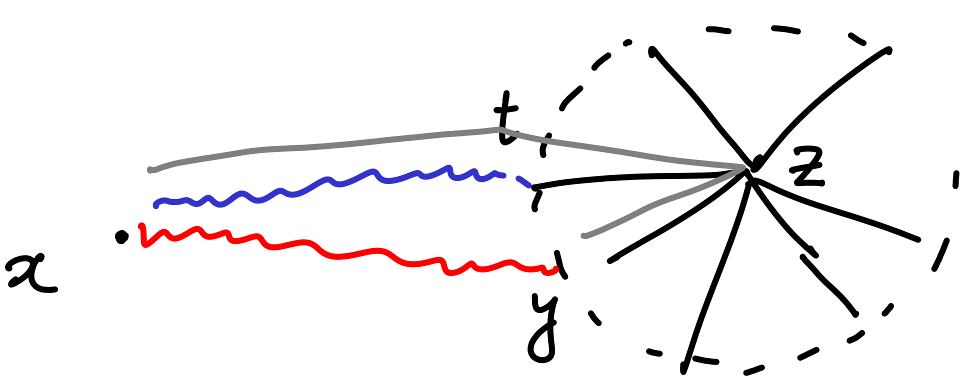
$C(y) = C(x)$



$n_x = n_y = z$

$\omega(x, z)$

__Case 2__ $C(x) \neq C(y)$



$\omega(x,t) \leq \omega(x,y)$ $\omega(z,t) \leq \omega(x,t) \leq \omega(x,y)$ $\omega(y,z) \leq \omega(x,y)$

What is the running-time of
Phases I and II ?