COL 702 Advanced Data Structures and Algorithms

Quiz 2 Sem I 2018-19, Max 20, Time 20 mins

Name ____________________ **Entry No.** ____________________

Your description should be in a pseudo-language - not a code. Mention clearly any data structures that you use including how the input and output is stored.
Write only in the space provided below each question.

1. Suppose the cost of *Empty-Stack* operation is $\lceil\sqrt{k}\rceil$ when there are k elements in the stack, whereas cost of *Push* and *Pop* are are $O(1)$. Analyze the cost of n stack operations where each operation is one of $\{Push, Pop, Empty - Stack\}$ beginning from an empty stack. **(8 marks)**

   Note that the cost of Empty-Stack is less than k, so the analysis done in class for the cost $= k$ will upperbound this. So the overall cost is bounded by $O(n)$ and it cannot be any better. There is no need to define any special potential function, since the previous potential function ensures that the amortized cost of Empty-Stack is ≤ 0 and the worst case amortized cost for Push is still 2.

2. Given an undirected graph $G = (V, E)$, consider the set system (E, M), where M consists of those subsets of edges which induce a subgraph of G with *at most* one cycle. Given any non-negative weight function $w : E \leftarrow \mathbb{R}^+$, justify whether the generic_greedy algorithm can be used for finding the maximum weighted subset from M. **(12 marks)**

   First note that the present definition of the feasible (independent) subsets M satisfies the properties of a subset system, i.e., if $S \in M$, then $S - \{e\} \in M$.
   (It is important to establish this to invoke the matroid theorem).

   Let us try to establish the rank property. Consider any $A \subset E$ and the set of vertices $V(A)$ which consists of the end-points of A. Note that A need not be connected even if the graph G is connected. Suppose the subgraph $G(A) = (V(A), A)$ has k components. Then any maximal subgraph of A that does not contain a cycle has exactly $n - k$ edges (as discussed in class). Since we are allowing at most 1 cycle, it can have at most $n - k + 1$ edges. Therefore it satisfies the rank property and hence it is a matroid.

   More explanation in case it is not clear - Suppose the connected components in $G(A)$ are
   $G_1(A), G_2(A) \ldots G_k(A)$. Clearly these have disjoint sets of edges and vertices (since they are not connected). Suppose component $G_i(A)$ has n_i vertices, and m_i edges. Then the maximal sized independent subset of $G_i(A)$ has n_i edges if $m_i \geq n_i$, else it is $n_i - 1$. Note that $m_i \geq n_i - 1$ as it is a connected component. By definition of the independent subsets of M, only one component is allowed to have a cycle, so all maximal subsets have size $1 + \sum_{i=1}^{k}(n_i - 1) = |V(A)| - k + 1$.

   In a special case where all the $G_i(A)$ are trees, then the maximal size is $|V(A)| - k$, i.e., there is a unique maximal subset.