Axiomatic Semantic Models

TSO Model

We define store order so $\subseteq O \times O$ where $O = \mathsf{W} \cup \mathsf{U} \cup \mathsf{F}$ such that so is total.

Axioms

- hb is irreflexive. (ConsHB)
- so; hb is irreflexive. (ConsHBSO)
- fr; hb is irreflexive. (ConsFRHB)
- fr; so is irreflexive. (ConsFRSO)
- fr; so; rfe; po is irreflexive where rfe $=$ rf $\setminus$ po (ConsFSRP)
- fr; so; $[\mathsf{U} \cup \mathsf{F}]$; po is irreflexive. (ConsFSUFP)

Observation.

Allows reordering of $\mathsf{W}_x; \mathsf{R}_y$ only where $x, y \in$ Loc and $x \neq y$.

Transition on Execution Graph

We can construct an execution along with the execution.

At each step of execution we append an event to the execution graph.

$$\mathsf{Append}(G, a) = \langle G.\mathsf{E} \uplus \{a\}, \mathsf{AddPO}(G, a), \mathsf{AddRF}(G, a), \mathsf{AddMO}(G, a)\rangle$$

where

$$\mathsf{AddPO}(G, a) = G.\mathsf{po} \cup (G.\mathsf{E}^0 \cup G.\mathsf{E}^{a.\mathsf{Tid}} \times \{a\})$$

$$\mathsf{AddRF}(G, a) = \textbf{if } a \in \mathsf{R} \textbf{ then } \exists w \in G.\mathsf{WU}_x.$$
$$G'.\mathsf{rf} = G.\mathsf{rf} \cup \{(w, a)\} \wedge w.\mathsf{wVal} = a.\mathsf{rVal} \textbf{ else} G'.\mathsf{rf} = G.\mathsf{rf}$$

$$\mathsf{AddMO}(G, a) = \textbf{if } a \in \mathsf{WU} \textbf{ then } \exists O \subseteq G.\mathsf{WU}_x.$$
$$G'.\mathsf{mo} = G.\mathsf{mo} \cup (O \times \{a\}) \cup (\{a\} \times G.\mathsf{WU}_x \setminus O) \textbf{ else } G'.\mathsf{mo} = G.\mathsf{mo}$$

Consistent Execution Graph construction

Steps

- Graph G is consistent.
- Append an event a to G and create a graph G'
- G' is consistent.

$$\frac{\begin{array}{c} i \in \mathsf{Tid} \qquad \ell = \langle \_, X, \_\_ \rangle \\ a = \langle \_, i, \ell \rangle \qquad P(i), s \xrightarrow{\ell} P'(i), s' \\ \mathsf{Cons}_M(G) \qquad \mathsf{Append}(G, a) \qquad \mathsf{Cons}_M(G') \end{array}}{P, G \Rightarrow P', G'}$$

$$P, G_{\mathsf{init}} \Rightarrow \cdots \Rightarrow \mathsf{skip} \mid\mid \mathsf{skip} \mid\mid \cdots \mid\mid \mathsf{skip}, G_{\mathsf{final}}$$

Consistent Execution Graph construction

$$COH \wedge Atomicity \iff acy(G.\mathrm{po}_x \cup G.\mathrm{rf} \cup G.\mathrm{mo} \cup G.\mathrm{fr})$$

Proof Sketch

($\Leftarrow$) Follows immediately. (How?)

($\Rightarrow$) Not so obvious.

COH and *Atomicity* has fixed number of relation compositions.
But, a $G.\mathrm{po}_x \cup G.\mathrm{rf} \cup G.\mathrm{mo} \cup G.\mathrm{fr}$ path can be arbitrarily long.

- Follow the graph construction step.
- The condition holds at initial state.
- Assume the condition holds on consistent graph G.
- Append an event a to get consistent $G' = \mathrm{Append}(G, a)$.
- Show that the condition holds on G'
  - all cases of a (i.e. W/R/U/F)
  - possible relations among the events