Axiomatic Semantic Models

Notations

Let $R, S \subseteq \mathsf{E} \times \mathsf{E}$ be binary relations on events. $R; S$ denotes the relational composition of R and S.

R^{-1} denotes the inverse of relation R.

$\mathrm{dom}(R)$ and $\mathrm{range}(R)$ are domain and range of R respectively.

$R^?, R^+, R^*$ are the reflexive, the transitive, the reflexive-transitive closures of R respectively

$[A]$ denotes an identity relation on set A i.e.
$[A](x, y) \triangleq x = y \wedge x \in A$

R_x denotes relation R on events on location x i.e.
$R_x = \{(a, b) \mid a.\mathsf{loc} = b.\mathsf{loc} = x\}$

Modification Order

We introduce modification order in execution $\cdots$

Now execution graph is $G = \langle \mathsf{E}, \mathsf{po}, \mathsf{rf}, \mathsf{mo} \rangle$ where mo denotes modification order and $G.\mathsf{mo}$ is the set of mo edges in G.

Axiom: mo is a total order on the same-location writes or updates in G.

For a pair of same-location writes or updates a and b in G, either $G.\mathsf{mo}(a, b)$ or $G.\mathsf{mo}(b, a)$ holds.

$$\forall a, b \in G.\mathsf{WU}.\ a.\mathsf{loc} = b.\mathsf{loc} \implies G.\mathsf{mo}(a, b) \oplus G.\mathsf{mo}(b, a)$$

Derived Relation

From-read $\mathsf{fr} \triangleq \mathsf{rf}^{-1}; \mathsf{mo} \setminus [\mathsf{E}]$

Coherence

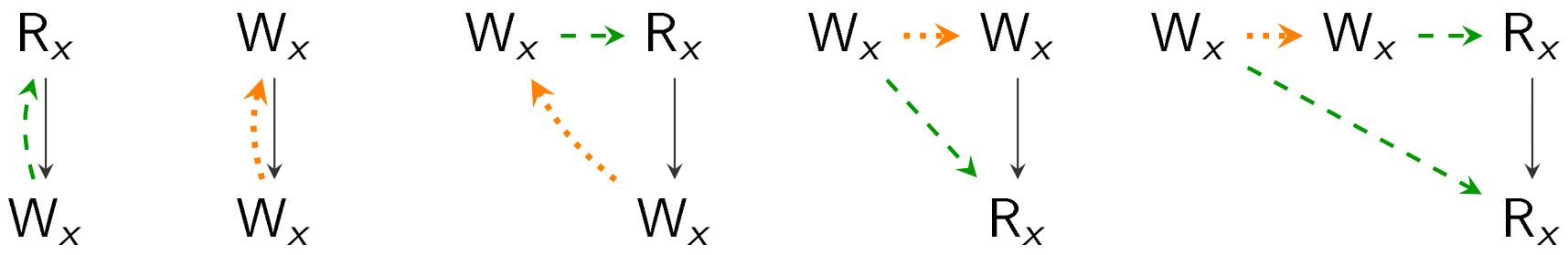
Restrictions on same-location events in an execution.

Axioms

- rf; po is irreflexive. (ConsRFPO)
- mo; po is irreflexive. (ConsWW)
- mo; rf; po is irreflexive. (ConsRW)
- rf^{-1}; mo; po is irreflexive. (ConsWR)
- rf^{-1}; mo; rf; po is irreflexive. (ConsRR)

COH = ConsRFPO ∧ ConsWW ∧ ConsRW ∧ ConsWR ∧ ConsRR

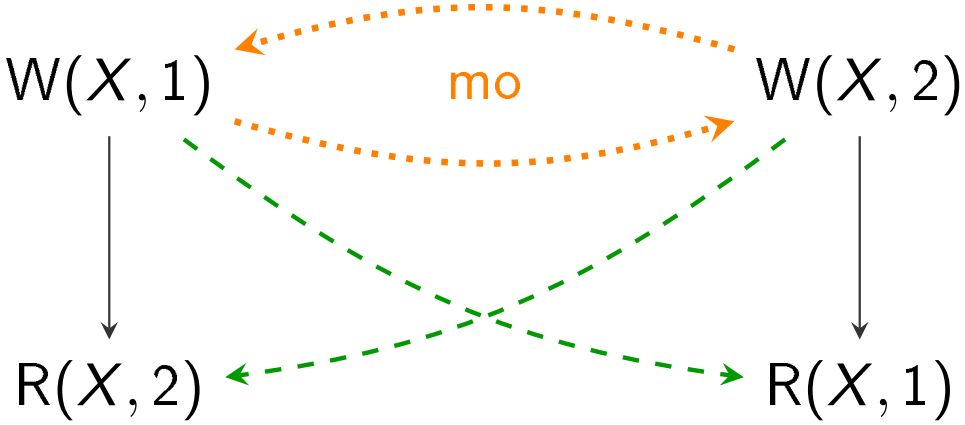
Restricted Patterns



Coherence Example

$X := 1;$	$X := 2;$
$a := X;$ // 2	$b := X;$ // 1

Disallow $a = 2, b = 1$ outcome.



Atomicity

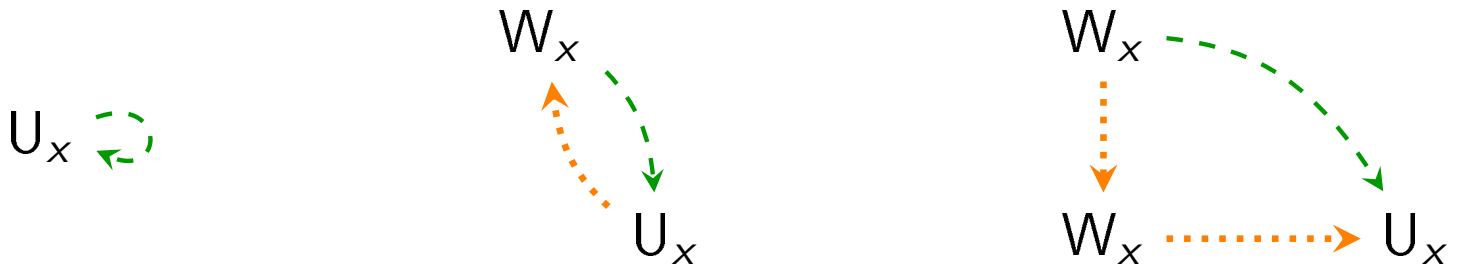
In an update event the read-write operations are atomic.

Axioms

- rf is irreflexive. (ConsIrrRF)
- mo; rf is irreflexive. (ConsMORF)
- rf^{-1}; mo; mo is irreflexive. (ConsRFMOMO)

RMW = ConsIrrRF ∧ ConsMORF ∧ ConsRFMOMO

Restricted Patterns

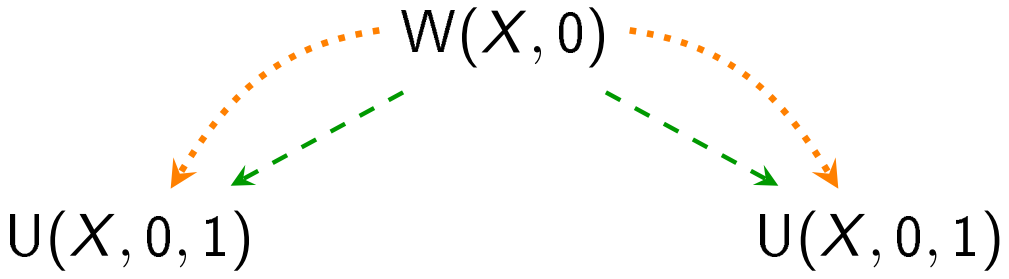


Example

Initially $X = 0$;

$a := \mathrm{CAS}(X, 0, 1); \;\|\; b := \mathrm{CAS}(X, 0, 1);$

Both CASes cannot be successful in an execution.



Release-Acquire (RA) Model

Use *happen-before* relation: $\mathsf{hb} \triangleq (\mathsf{po} \cup \mathsf{rf})^+$

Axioms

- hb is irreflexive. (ConsHB)
- mo; hb is irreflexive. (ConsHBMO)
- rf^{-1}; mo; hb is irreflexive. (ConsHBMORF)
- rf^{-1}; mo; mo is irreflexive. (ConsRFMOMO)

Example:

$$X = Y = 0;$$

$$\begin{array}{l} X := 1; \\ Y := 1; \end{array} \quad \Bigg\| \quad \begin{array}{l} a := Y; \ // \ 1 \\ \textbf{if}(a == 1) \\ \quad b := X; \ // \ 0 \end{array}$$

RA disallows $a = 1 \wedge b = 0$.

SC Model

Axioms

- $(\mathsf{po} \cup \mathsf{rf} \cup \mathsf{mo} \cup \mathsf{fr})$ is irreflexive. (SC)

Example: SC disallows $a = 0 \wedge b = 0$.

$$X = Y = 0;$$

$$\begin{array}{l|l} X := 1; & Y := 1; \\ a := Y; & b := X; \end{array}$$