C/C++ Concurrency and Program Transformations

Recap: Transformations

Redundant access elimination such as

- $Y_{\mathsf{rlx}} = v; Y_{\mathsf{rlx}} = v' \rightsquigarrow Y_{\mathsf{rlx}} = v'$ (OW)
- $Y_{\mathsf{rlx}} = v; a = Y_{\mathsf{rlx}} \rightsquigarrow Y_{\mathsf{rlx}} = v; a = v;$ (RAW)
- $a = Y_{\mathsf{rlx}}; a = Y_{\mathsf{rlx}} \rightsquigarrow a = Y_{\mathsf{rlx}}$ (RAR)

Fence Insertion such as

- $C_1; C_2 \rightsquigarrow C_1; \mathsf{F}_{\mathsf{rel,acq,sc}}; C_2$

Access strengthening such as

- $Y_{\mathsf{rlx}} = v \rightsquigarrow Y_{\mathsf{rel}} = v$
- $a = Y_{\mathsf{rlx}} \rightsquigarrow a = Y_{\mathsf{acq}}$

Reordering of accesses such as

- $a = Y_{\mathsf{rlx}}; Z_{\mathsf{rlx}} = v \rightsquigarrow Z_{\mathsf{rlx}} = v; a = Y_{\mathsf{rlx}}$

Recap: Correct Compilation

For each consistent execution of the target program there exists a corresponding consistent execution of the source program with same outcome.

$$X \in [\![\mathbb{P}_{\mathsf{tgt}}]\!].\ \exists X' \in [\![\mathbb{P}_{\mathsf{src}}]\!].\ O(X) = O(X')$$

Recap: Proof Strategy

1. For each consistent execution X' of target program $\mathbb{P}_{\mathsf{tgt}}$, define an execution X of $\mathbb{P}_{\mathsf{src}}$.
2. Show that X is consistent.
3. Show that $O(X) = O(X')$

Recap: Overwritten Write Elimination

We proved the correctness of overwritten write transformation in C11.

Overwritten write:

$$Y_o = v; Y_{o'} = v' \rightsquigarrow Y_{o'} = v' \qquad \text{(OW)}$$

where $o, o' \in \{\mathsf{na}, \mathsf{rlx}, \mathsf{rel}, \mathsf{sc}\}$

Theorem

(OW) is correct when $o \sqsubseteq o'$.

Correctness of Read-after-Write Elimination

$$Y_o = v; a = Y_{o'} \rightsquigarrow Y_o = v; a = v; \qquad \text{(RAW)}$$

Theorem

(RAW) is correct when $o' \sqsubseteq o$

Remember the proof steps:

1. For each consistent execution X' of target program $\mathbb{P}_{\text{tgt}}$, define an execution X of $\mathbb{P}_{\text{src}}$.
2. Show that X is consistent.
3. Show that $O(X) = O(X')$

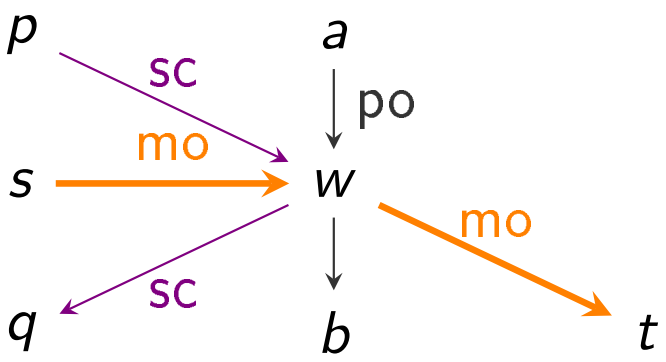
Correctness of Read-after-Write Elimination

Let X' be a target execution where $Y_o = v$ results in event $w = \mathrm{W}_o(Y, v)$.
We define source execution X where $a = Y_{o'}$ results in event $r = \mathrm{R}_{o'}(Y, v)$ such that

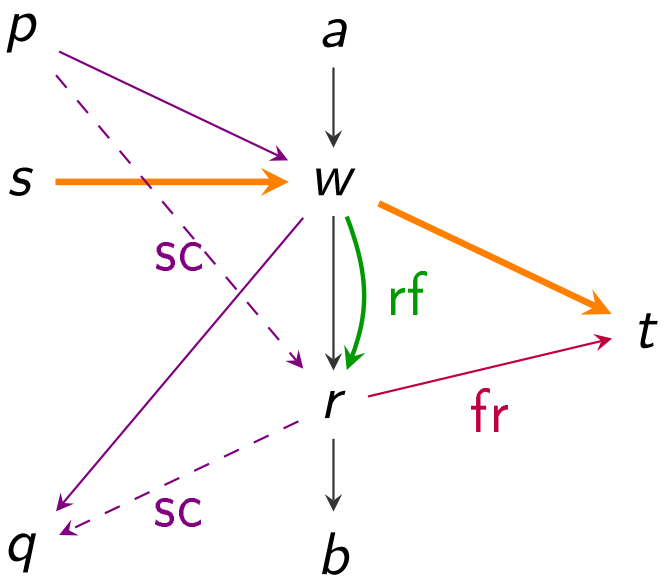
- $X.\mathrm{E} = X'.\mathrm{E} \cup \{r\}$
- $X.\mathrm{po} = X'.\mathrm{po} \cup \{(w, r)\} \cup \{(a, r) \mid X'.\mathrm{po}(a, w)\} \cup \{(r, b) \mid X'.\mathrm{po}(w, b)\}$
- $X.\mathrm{rf} = X'.\mathrm{rf} \cup \{(w, r)\}$
- $X.\mathrm{mo} = X'.\mathrm{mo}$
- $X.\mathrm{sc} = X'.\mathrm{sc} \cup \{(w, r) \mid o' = \mathrm{sc}\} \cup \{(p, r) \mid X'.(p, w) \wedge o' = \mathrm{sc}\} \cup \{(r, q) \mid X'.\mathrm{sc}(w, q) \wedge o' = \mathrm{sc}\}$

Correctness of Read-after-Write Elimination

- $X.\mathsf{E} = X'.\mathsf{E} \cup \{r\}$
- $X.\mathsf{po} = X'.\mathsf{po} \cup \{(w, r)\} \cup \{(a, r) \mid X'.\mathsf{po}(a, w)\} \cup \{(r, b) \mid X'.\mathsf{po}(w, b)\}$
- $X.\mathsf{rf} = X'.\mathsf{rf} \cup \{(w, r)\}$
- $X.\mathsf{mo} = X'.\mathsf{mo}$
- $X.\mathsf{sc} = X'.\mathsf{sc} \cup \{(w, r) \mid o' = \mathsf{sc}\} \cup \{(p, r) \mid X'.(p, w) \wedge o = \mathsf{sc}\} \cup \{(r, q) \mid X'.\mathsf{sc}(w, q) \wedge o = \mathsf{sc}\}$



Target X'



Source X

RAW Elimination: Proof of Correctness

Now we prove the theorem similar to OW theorem.
Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathrm{hb} = X'.\mathrm{hb} \cup \{(w, r)\} \cup \{(a, r) \mid X'.\mathrm{hb}(a, w)\} \cup \{(r, b) \mid X'.\mathrm{hb}(w, b)\}$

RAW Elimination: Proof of Correctness

Now we prove the theorem similar to OW theorem.
Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathsf{hb} = X'.\mathsf{hb} \cup \{(w, r)\} \cup \{(a, r) \mid X'.\mathsf{hb}(a, w)\} \cup \{(r, b) \mid X'.\mathsf{hb}(w, b)\}$
(irrHB)
Assume there exists a $X.\mathsf{hb}$ cycle.
It implies that there exists a $(X.\mathsf{po} \cup X.\mathsf{sw})^+$ cycle.
It implies that the $(X.\mathsf{po} \cup X.\mathsf{sw})^+$ cycle passes through event r.
However, from definition, $(w, r) \in X.\mathsf{sw}; [r] \subseteq X.\mathsf{po}$ and there is no outgoing $[\{r\}]; X.\mathsf{sw}$ edge.
Therefore $X.\mathsf{hb}$ through r contains $X.\mathsf{po}(-, r)$ and $X.\mathsf{po}(r, -)$.
It implies $X'.\mathsf{hb}$ is a cycle which is a contradiction.
Therefore $X.\mathsf{hb}$ satisfies (irrHB).

RAW Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathsf{hb} = X'.\mathsf{hb} \cup \{(w, r)\} \cup \{(a, r) \mid X'.\mathsf{hb}(a, w)\} \cup \{(r, b) \mid X'.\mathsf{hb}(w, b)\}$

Assume X is not coherent.

RAW Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathsf{hb} = X'.\mathsf{hb} \cup \{(w, r)\} \cup \{(a, r) \mid X'.\mathsf{hb}(a, w)\} \cup \{(r, b) \mid X'.\mathsf{hb}(w, b)\}$

Assume X is not coherent.
Assume there exists a $X.\mathsf{rf}; X.\mathsf{hb}$ cycle.
It involves event r.
It implies $X.\mathsf{rf}; [\{r\}]; X.\mathsf{po}$ cycle.
However, from definition, this is a contradiction in X.
Therefore X satisfies (CohHBRF).

RAW Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathsf{hb} = X'.\mathsf{hb} \cup \{(w, r)\} \cup \{(a, r) \mid X'.\mathsf{hb}(a, w)\} \cup \{(r, a) \mid X'.\mathsf{hb}(w, a)\}$

Assume X is not coherent.

RAW Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathsf{hb} = X'.\mathsf{hb} \cup \{(w, r)\} \cup \{(a, r) \mid X'.\mathsf{hb}(a, w)\} \cup \{(r, a) \mid X'.\mathsf{hb}(w, a)\}$

Assume X is not coherent.
Assume there exists a $X.\mathsf{mo}; X.\mathsf{rf}; X.\mathsf{hb}$ cycle.
It involves event r.
Considering the outgoing edges from r:
$X.\mathsf{po}(r, b) \implies X.\mathsf{po}(w, b)$.
Therefore a $X.\mathsf{mo}; X.\mathsf{rf}; X.\mathsf{hb}$ cycle implies that there already exists a a $X'.\mathsf{mo}; X'.\mathsf{rf}; X'.\mathsf{hb}$ cycle which is a contradiction.
Therefore X satisfies (Coh-RW).

RAW Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathsf{hb} = X'.\mathsf{hb} \cup \{(w, r)\} \cup \{(a, r) \mid X'.\mathsf{hb}(a, w)\} \cup \{(r, b) \mid X'.\mathsf{hb}(w, b)\}$

Assume X is not coherent.

Exercise: Prove rest of the coherence axioms.

RAW Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

(Atomicity)
Assume X does not preserve (Atomicity).
However, from definition, it implies X' violates (Atomicity) which is a contradiction.
Therefore X preserves atomicity.

RAW Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

(ConsSC)
Two possibilities:
Case $X.\text{sc} = X'.\text{sc}$ when $'o \neq \text{sc}$
In this case X preserves (ConsSC) as X' preserves (ConsSC).
Case $X'.\text{sc} \subseteq X.\text{sc}$ when $o' = \text{sc}$
Assume X does not preserve (ConsSC) due to r.
However, from definition, it implies X' violates (ConsSC) which is a contradiction. (Why?)
Therefore X preserves (ConsSC).

RAW Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

(SCread)
Assume X does not preserve (SCread).
Two possibilities:
Case $o' \neq \mathsf{sc}$
From the definition X preserves (SCread).

Case $o' = \mathsf{sc}$
In this case $\mathsf{sc}_{\mathsf{imm}}(w, r)$ holds and therefore (SCread) holds.

Therefore source execution X is consistent.

RAW Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that $O(X) = O(X')$

From definition, $X.\text{mo} = X'.\text{mo}$ and hence $O(X) = O(X')$.

Therefore the transformation is correct.

Correctness of Read-after-Read Elimination

$$a = Y_{o'}; b = Y_o \rightsquigarrow a = Y_{o'}; b = a; \qquad \text{(RAR)}$$

Theorem

(RAR) is correct when $o \sqsubseteq o'$

Remember the proof steps:

1. For each consistent execution X' of target program $\mathbb{P}_{\text{tgt}}$, define an execution X of $\mathbb{P}_{\text{src}}$.
2. Show that X is consistent.
3. Show that $O(X) = O(X')$

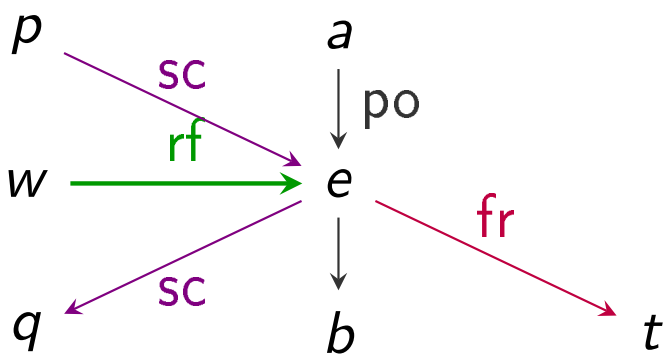
Correctness of Read-after-Read Elimination

Let X' be a target execution where $a = Y_{o'}$ results in event $e = \mathrm{R}_{o'}(Y, v)$ reading from the write event event w.
We define source execution X where $b = Y_o$ results in event $r = \mathrm{R}_o(Y, v)$ such that

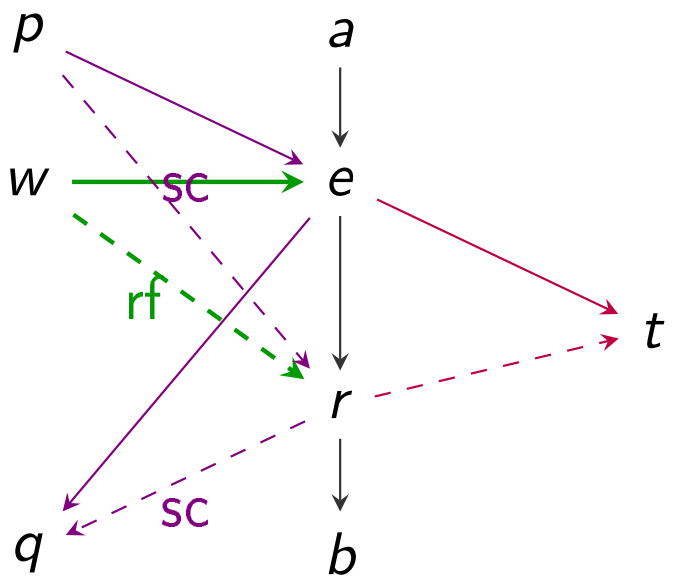
- $X.\mathrm{E} = X'.\mathrm{E} \cup \{r\}$
- $X.\mathrm{po} = X'.\mathrm{po} \cup \{(e, r)\} \cup \{(a, r) \mid X'.\mathrm{po}(a, e)\} \cup \{(r, b) \mid X'.\mathrm{po}(e, b)\}$
- $X.\mathrm{rf} = X'.\mathrm{rf} \cup \{(w, r) \mid X'.\mathrm{rf}(w, e)\}$
- $X.\mathrm{mo} = X'.\mathrm{mo}$
- $X.\mathrm{sc} = X'.\mathrm{sc} \cup \{(e, r) \mid o = \mathrm{sc}\} \cup \{(p, r) \mid X'.(p, e) \wedge o = \mathrm{sc}\} \cup \{(r, q) \mid X'.\mathrm{sc}(e, q) \wedge o = \mathrm{sc}\}$

Correctness of Read-after-Read Elimination

- $X.\mathsf{E} = X'.\mathsf{E} \cup \{r\}$
- $X.\mathsf{po} = X'.\mathsf{po} \cup \{(e, r)\} \cup \{(a, r) \mid X'.\mathsf{po}(a, e)\} \cup \{(r, b) \mid X'.\mathsf{po}(e, b)\}$
- $X.\mathsf{rf} = X'.\mathsf{rf} \cup \{(w, r) \mid X'.\mathsf{rf}(w, e)\}$
- $X.\mathsf{mo} = X'.\mathsf{mo}$
- $X.\mathsf{sc} = X'.\mathsf{sc} \cup \{(e, r) \mid o = \mathsf{sc}\} \cup \{(p, r) \mid X'.(p, e) \wedge o = \mathsf{sc}\} \cup \{(r, q) \mid X'.\mathsf{sc}(e, q) \wedge o = \mathsf{sc}\}$



Target X'



Source X

RAR Elimination: Proof of Correctness

Now we prove the theorem similar to OW and RAR theorems. Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathsf{hb} = X'.\mathsf{hb} \cup \{(e, r)\} \cup \{(a, r) \mid X'.\mathsf{hb}(a, e)\} \cup \{(r, b) \mid X'.\mathsf{hb}(e, b)\}$

RAR Elimination: Proof of Correctness

Now we prove the theorem similar to OW and RAR theorems.
Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathsf{hb} = X'.\mathsf{hb} \cup \{(e, r)\} \cup \{(a, r) \mid X'.\mathsf{hb}(a, e)\} \cup \{(r, b) \mid X'.\mathsf{hb}(e, b)\}$
(irrHB)
Assume there exists a $X.\mathsf{hb}$ cycle.
It implies that there exists a $(X.\mathsf{po} \cup X.\mathsf{sw})^+$ cycle.
It implies that the $(X.\mathsf{po} \cup X.\mathsf{sw})^+$ cycle passes through event r.
However, from definition, $X.\mathsf{sw}(a, r) \implies X.\mathsf{sw}(a, e)$ for any event a.
Therefore $X.\mathsf{hb}$ cycle implies $X'.\mathsf{hb}$ is a cycle which is a contradiction.
Therefore $X.\mathsf{hb}$ satisfies (irrHB).

RAR Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathsf{hb} = X'.\mathsf{hb} \cup \{(e, r)\} \cup \{(a, r) \mid X'.\mathsf{hb}(a, e)\} \cup \{(r, b) \mid X'.\mathsf{hb}(e, b)\}$

Assume X is not coherent.

RAR Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathsf{hb} = X'.\mathsf{hb} \cup \{(e, r)\} \cup \{(a, r) \mid X'.\mathsf{hb}(a, e)\} \cup \{(r, b) \mid X'.\mathsf{hb}(e, b)\}$

Assume X is not coherent.
Assume there exists a $X.\mathsf{rf}; X.\mathsf{hb}$ cycle.
It involves event r.
It implies $X.\mathsf{rf}; [\{r\}]; X.\mathsf{po}$ cycle.
However, from definition, in this case there is a this is a $X'.\mathsf{rf}; X'.\mathsf{hb}$ cycle through e which is a contradiction.
Therefore X satisfies (CohHBRF).

RAR Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathsf{hb} = X'.\mathsf{hb} \cup \{(e, r)\} \cup \{(a, r) \mid X'.\mathsf{hb}(a, e)\} \cup \{(r, b) \mid X'.\mathsf{hb}(e, b)\}$

Assume X is not coherent.

RAR Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathsf{hb} = X'.\mathsf{hb} \cup \{(e, r)\} \cup \{(a, r) \mid X'.\mathsf{hb}(a, e)\} \cup \{(r, b) \mid X'.\mathsf{hb}(e, b)\}$

Assume X is not coherent.
Assume there exists a $X.\mathsf{mo}; X.\mathsf{rf}; X.\mathsf{hb}$ cycle.
It involves event r.
Considering the outgoing edges from r:
$X.\mathsf{po}(r, b) \implies X.\mathsf{po}(e, b)$.
Therefore a $X.\mathsf{mo}; X.\mathsf{rf}; X.\mathsf{hb}$ cycle implies that there already exists a a $X'.\mathsf{mo}; X'.\mathsf{rf}; X'.\mathsf{hb}$ cycle which is a contradiction.
Therefore X satisfies (Coh-RW).

RAW Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

Note that
$X.\mathsf{hb} = X'.\mathsf{hb} \cup \{(e, r)\} \cup \{(a, r) \mid X'.\mathsf{hb}(a, e)\} \cup \{(r, b) \mid X'.\mathsf{hb}(e, b)\}$

Assume X is not coherent.

Exercise: Prove rest of the coherence axioms.

RAR Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

(Atomicity)
Assume X does not preserve (Atomicity).
However, from definition, it implies X' violates (Atomicity) which is a contradiction.
Therefore X preserves atomicity.

RAR Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

(ConsSC)
Two possibilities:
Case $X.\mathsf{sc} = X'.\mathsf{sc}$ when $o \neq \mathsf{sc}$
In this case X preserves (ConsSC) as X' preserves (ConsSC).
Case $X'.\mathsf{sc} \subseteq X.\mathsf{sc}$ when $o = \mathsf{sc}$
Assume X does not preserve (ConsSC) due to r.
However, from definition, it implies X' violates (ConsSC) which is a contradiction. (Why?)
Therefore X preserves (ConsSC).

RAR Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that X is consistent.

We show X is consistent by contradiction.

(SCread)
Assume X does not preserve (SCread).
Two possibilities:
Case $o \neq \mathsf{sc}$
From the definition X preserves (SCread).

Case $o = \mathsf{sc}$
In this case $\mathsf{sc}_{\mathsf{imm}}(e, r)$ holds and $e.\mathsf{ord} = \mathsf{sc}$ and therefore (SCread) holds.

Therefore source execution X is consistent.

RAW Elimination: Proof of Correctness

Remember the proof strategy steps:

1. Show that $O(X) = O(X')$

From definition, $X.\text{mo} = X'.\text{mo}$ and hence $O(X) = O(X')$.

Therefore the transformation is correct.

Exercise

Show that the following transformation is safe.

$$a = Y_{\mathsf{na}}; b = Y_o \rightsquigarrow a = Y_{\mathsf{na}}; b = a; \qquad \text{(RAR)}$$