

-
1. Show that if $|A| = |B|$ and $|B| = |C|$, then $|A| = |C|$.
 2. Show that if A, B, C are sets such that $|A| \leq |B|$ and $|B| \leq |C|$, then $|A| \leq |C|$.
 3. Suppose that A is a countable set. Show that the set B is also countable if there is an onto function f from A to B .
 4. Show that if A is an infinite set, then it contains a countably infinite subset.
 5. Show that there is no infinite set A such that $|A| < |\mathbb{Z}^+| = \aleph_0$.
(we will use $\aleph_0$ to denote the cardinality of positive integers.)
 6. Show that the set of real numbers that are solutions of quadratic equations $ax^2 + bx + c = 0$, where a , b , and c are integers, is countable.
 7. Show that $\mathbb{Z}^+ \times \mathbb{Z}^+$ is countable by showing that the polynomial function $f : \mathbb{Z}^+ \times \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ with $f(m, n) = (m + n - 2)(m + n - 1)/2 + m$ is one-to-one and onto.
 8. Use the Schröder-Bernstein theorem to show that $(0, 1)$ and $[0, 1]$ have the same cardinality.
 9. Show that $(0, 1)$ and $\mathbb{R}$ have the same cardinality by:
 - (a) showing that $f(x) = \frac{2x-1}{2x(1-x)}$ is a bijection from $(0, 1)$ to $\mathbb{R}$.
 - (b) using the Schröder-Bernstein theorem.
 10. Show that there is a one-to-one correspondence from the set of subsets of the positive integers to the set of real numbers between 0 and 1. Use this to show that $\aleph_0 < |\mathcal{P}(\mathbb{Z}^+)| = \mathbb{R}$.
 11. Show that the set of all computer programs in a particular programming language is countable.
 12. Show that the set of functions from the positive integers to the set $\{0, 1\}$ is uncountable.
 13. We say that a function is computable if there is a computer program that finds the values of this function. Use this to show that there are functions that are not computable.