

1. Use set identities to show that $(\overline{A \cup B}) \cap (\overline{B \cup C}) \cap (\overline{A \cup C}) = \bar{A} \cap \bar{B} \cap \bar{C}$.
2. The symmetric difference of A and B , denoted by $A \oplus B$, is the set containing those elements in either A or B , but not in both A and B .
 - (a) Find the symmetric difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$.
 - (b) Draw a Venn diagram for the symmetric difference of the sets A and B .
 - (c) Show that $A \oplus B = (A \cup B) - (A \cap B)$.
 - (d) Show that $A \oplus B = (A - B) \cup (B - A)$.
 - (e) Show that $(A \oplus B) \oplus B = A$.
 - (f) Determine whether $(A \oplus B) \oplus C$ equals $A \oplus (B \oplus C)$.
 - (g) Suppose that A , B , and C are sets such that $A \oplus C = B \oplus C$. Must it be the case that $A = B$?
3. Let $A_i = \{1, 2, 3, \dots, i\}$ for $i = 1, 2, 3, \dots$. Find
 - a) $\bigcup_{i=1}^n A_i$
 - b) $\bigcap_{i=1}^n A_i$
4. Let $A_i = \{\dots, -2, -1, 0, 1, 2, \dots, i\}$ for $i = 1, 2, 3, \dots$. Find
 - a) $\bigcup_{i=1}^n A_i$
 - b) $\bigcap_{i=1}^n A_i$
5. Let f be a function from A to B . Let S and T be subsets of B . Show that
 - a) $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$.
 - b) $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$.
6. Let f be a function from A to B . Let S be a subset of B . Show that $f^{-1}(\bar{S}) = \overline{f^{-1}(S)}$.
7. Show that if x is a real number and m is an integer, then $[x + m] = [x] + m$.
8. Derive the formula for $\sum_{k=1}^n k^3$.
9. Give a tight estimate on $\sum_{i=1}^n \log_2 n$.
10. Show that if A and B are sets, A is uncountable, and $A \subseteq B$, then B is uncountable.
11. Show that the union of a countable number of countable sets is countable.
12. Show that the set of all finite bit strings is countable.