

-
1. Write the numbers $1, 2, \dots, 2n$ on a blackboard, where n is an odd integer. Pick any two of the numbers, j and k , write $|j - k|$ on the board, and erase j and k . Continue this process until only one integer is written on the board. Prove that this integer must be odd. Does the statement hold if n were even?
 2. Suppose that five ones and four zeros are arranged around a circle. Between any two equal bits, you insert a 0, and between any two unequal bits, you insert a 1 to produce nine new bits. Then you erase the nine original bits. Show that when you iterate this procedure, you can never get nine zeros. [*Hint: Work backward, assuming that you did end up with nine zeros.*]
 3. Prove or disprove: You can use dominoes to tile a standard checkerboard (i.e., 8×8 chessboard) with all 4 corners removed.
 4. Prove that given a nonnegative integer n , there is a unique nonnegative integer m such that $m^2 \leq n < (m + 1)^2$.
 5. Assuming the truth of the statement that $\sqrt{n}$ is irrational whenever n is a positive integer that is not a perfect square, prove that $\sqrt{2} + \sqrt{3}$ is irrational.
 6. Determine whether each of these statements is True or False:
 - a) $x \in \{x\}$
 - b) $\{x\} \subseteq \{x\}$
 - c) $\{x\} \in \{x\}$
 - d) $\{x\} \in \{\{x\}\}$
 - e) $\emptyset \subseteq \{x\}$
 - f) $\emptyset \in \{x\}$
 7. Prove or disprove: If A and B are sets, then $\mathcal{P}(A \times B) = \mathcal{P}(A) \times \mathcal{P}(B)$.
 8. Find the truth set of each of these predicates, where the domain is the set of integers:
 - a) $P(x) : x^3 \geq 1$
 - b) $Q(x) : x^2 = 1$
 - c) $R(x) : x < x^2$
 9. A tree is an undirected graph that is connected and does not have any cycles. A cycle is a sequence of nodes $v_1, \dots, v_k$ such that for every i , there is an edge between v_i and v_{i+1} , and there is an edge between v_1 and v_k .
Prove or disprove: Any tree with n nodes has $n - 1$ edges.