

-
1. For each of these arguments, determine whether the argument is correct or incorrect and explain why.
 - a) Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.
 - b) A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.
 - c) Quincy likes all action movies. Quincy likes the movie Eight Men Out. Therefore, Eight Men Out is an action movie.
 - d) All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore, Hamilton sets at least a dozen traps.
 2. Identify the error or errors in this argument that supposedly shows that if $\exists xP(x) \wedge \exists xQ(x)$ is true, then $\exists x(P(x) \wedge Q(x))$ is true.

1. $\exists xP(x) \vee \exists xQ(x)$	Premise
2. $\exists xP(x)$	Simplification from (1)
3. $P(c)$	Existential instantiation from (2)
4. $\exists xQ(x)$	Simplification from (1)
5. $Q(c)$	Existential instantiation from (4)
6. $P(c) \wedge Q(c)$	Conjunction from (3) and (5)
7. $\exists x(P(x) \wedge Q(x))$	Existential generalization
 3. Justify the rule of universal transitivity, which states that if $\forall x(P(x) \rightarrow Q(x))$ and $\forall x(Q(x) \rightarrow R(x))$ are true, then $\forall x(P(x) \rightarrow R(x))$ is true, where the domains of all quantifiers are the same.
 4. Use rules of inference to show that if $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.
 5. Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.
 6. Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \rightarrow \neg S(x))$, and $\exists x\neg P(x)$ are true, then $\exists x\neg R(x)$ is true.
 7. For each of these arguments, explain which rules of inference are used for each step.
 - a) "Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job."
 - b) "Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution."
 - c) "Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program."
 - d) "Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean."
 8. Use a proof by contradiction to show that there is no rational number r for which $r^3 + r + 1 = 0$.
 9. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using:
 - a) a proof by contraposition

b) a proof by contradiction

10. Prove that at least one of the real numbers $a_1, a_2, \dots, a_n$ is greater than or equal to the average of these numbers. What kind of proof did you use?
11. Use a direct proof to show that every odd integer is a difference of two squares.