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1. Write each of these statements in the form “if p , then q ” in English.
(*Hint: Refer to the list of common ways to express conditional statements provided in this section.*)
 - a) I will remember to send you the address only if you send me an e-mail message.
 - b) To be a citizen of this country, it is sufficient that you were born in the United States.
 - c) If you keep your textbook, it will be a useful reference in your future courses.
 - d) The Red Wings will win the Stanley Cup if their goalie plays well.
 - e) That you get the job implies that you had the best credentials.
 - f) The beach erodes whenever there is a storm.
 - g) It is necessary to have a valid password to log on to the server.
 - h) You will reach the summit unless you begin your climb too late.
 - i) You will get a free ice cream cone, provided that you are among the first 100 customers tomorrow.
 2. Construct a truth table for each of these compound propositions.
 - a) $p \rightarrow \neg q$
 - b) $\neg p \leftrightarrow q$
 - c) $(p \rightarrow q) \vee (\neg p \rightarrow q)$
 - d) $(p \rightarrow q) \wedge (\neg p \rightarrow q)$
 - e) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
 - f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$
 3. An explorer is captured by a group of cannibals. There are two types of cannibals – *those who always tell the truth and those who always lie*. The cannibals will barbecue the explorer unless he can determine whether a particular cannibal always lies or always tells the truth. He is allowed to ask the cannibal exactly one question.
 - a) Explain why the question “*Are you a liar?*” does not work.
 - b) Find a question that the explorer can use to determine whether the cannibal always lies or always tells the truth.
 4. Express these system specifications using the propositions p : “*The user enters a valid password,*” q : “*Access is granted,*” and r : “*The user has paid the subscription fee*” and logical connectives (including negations).
 - a) “*The user has paid the subscription fee, but does not enter a valid password.*”
 - b) “*Access is granted whenever the user has paid the subscription fee and enters a valid password.*”
 - c) “*Access is denied if the user has not paid the subscription fee.*”
 - d) “*If the user has not entered a valid password but has paid the subscription fee, then access is granted.*”
 5. Are these system specifications consistent? “*If the file system is not locked, then new messages will be queued. If the file system is not locked, then the system is functioning normally, and conversely. If new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, then new messages will be sent to the message buffer.*”
 6. Steve would like to determine the relative salaries of three coworkers using two facts. First, he knows that if Fred is not the highest paid of the three, then Janice is. Second, he knows that if Janice is not the lowest paid, then Maggie is paid the most. Is it possible to determine the relative salaries of Fred, Maggie, and Janice from what Steve knows? If so, who is paid the most and who the least? Explain your reasoning.

7. The police have three suspects for the murder of Mr. Cooper: Mr. Smith, Mr. Jones, and Mr. Williams. Smith, Jones, and Williams each declare that they did not kill Cooper. Smith also states that Cooper was a friend of Jones and that Williams disliked him. Jones also states that he did not know Cooper and that he was out of town the day Cooper was killed. Williams also states that he saw both Smith and Jones with Cooper the day of the killing and that either Smith or Jones must have killed him. Can you determine who the murderer was if
 - a) one of the three men is guilty, the two innocent men are telling the truth, but the statements of the guilty man may or may not be true?
 - b) innocent men do not lie?
8. Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known? Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin. Explain your reasoning.
9. Show that $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology.
10. Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logically equivalent.
11. Show that $\neg$ and $\wedge$ form a functionally complete collection of logical operators. (*Hint: First use a De Morgan law to show that $p \vee q$ is logically equivalent to $\neg(\neg p \wedge \neg q)$.*)
12. Show that $\neg$ and $\vee$ form a functionally complete collection of logical operators.
13. How many of the disjunctions $(p \vee \neg q)$, $(\neg p \vee q)$, $(q \vee r)$, $(q \vee \neg r)$, and $(\neg q \vee \neg r)$ can be made simultaneously true by an assignment of truth values to p , q , and r ?