

ACOL1002: Discrete Mathematical Structures

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Logic

Rules of inference: Propositional logic

Rule of inference	Tautology	Name
$\frac{p \quad p \rightarrow q}{\therefore q}$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus ponens
$\frac{\neg q \quad p \rightarrow q}{\therefore \neg p}$	$[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$	Modus tollens
$\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$\frac{p \vee q \quad \neg p}{\therefore q}$	$[(p \vee q) \wedge \neg p] \rightarrow (q)$	Disjunctive syllogism

Table: Rules of inference.

Logic

Rules of inference: Propositional logic

Rule of inference	Tautology	Name
$\frac{p}{\therefore ?}$	$p \rightarrow ?$	Addition
$\frac{p \wedge q}{\therefore ?}$	$(p \wedge q) \rightarrow ?$	Simplification
$\frac{p}{q}$ $\frac{q}{\therefore ?}$	$[(p) \wedge (q)] \rightarrow ?$	Conjunction
$\frac{p \vee q}{\neg p \vee r}$ $\therefore ?$	$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow ?$	Resolution

Table: Rules of inference.

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Rule of inference	Tautology	Name
$\frac{p}{\therefore p \vee q}$	$p \rightarrow (p \vee q)$	Addition
$\frac{p \wedge q}{\therefore p}$	$(p \wedge q) \rightarrow p$	Simplification
$\frac{p}{q}$ $\frac{q}{\therefore p \wedge q}$	$[(p) \wedge (q)] \rightarrow (p \wedge q)$	Conjunction
$\frac{p \vee q}{\neg p \vee r}$ $\frac{\neg p \vee r}{\therefore q \vee r}$	$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$	Resolution

Table: Rules of inference.

Logic

Rules of inference: Propositional logic

- Which rule of inference is used in the argument:
If it rains today, then we will not have a barbecue today.
If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, *if it rains today, then we will have barbecue tomorrow.*

- Which rule of inference is used in the argument:
*If it rains today, then we will not have a barbecue today.
If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, if it rains today, then we will have barbecue tomorrow.*
- Show that the hypothesis:
 - *"It is not sunny this afternoon and it is colder than yesterday."*
 - *"We will go swimming only if it is sunny."*
 - *"If we do not go swimming, then we will take a canoe trip."*
 - *"If we take a canoe trip, then we will be home by sunset."*
- lead to the conclusion:
 - *"We will be home by sunset."*

Logic

Rules of inference: Propositional logic

- Show that the following argument is valid. If today is tuesday, I have a test in Mathematics or economics. If my Economics Professor is sick, I will not have a test in Economics. Today is tuesday and my Economics Professor is sick. Therefore I have a test in Mathematics.
- Show that the following argument is valid. If Mohan is a lawyer, then he is ambitious. If Mohan is an early riser, then he does not like idlies. If Mohan is ambitious, then he is an early riser. Then if Mohan is a lawyer, then he does not like idlies.

End