

ACOL1002: Discrete Mathematical Structures

Ragesh Jaiswal, CSE, IIT Delhi

Discrete Probability

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Probability Theory

- Each performance of an experiment with two possible outcomes is called a *Bernoulli trial*.
- In general, a possible outcome of a Bernoulli trial is called a success or a failure. If p is the probability of a *success* and q is the probability of a *failure*, it follows that $p + q = 1$.
- We are interested in the probability of success in k trials in an experiment that consists of n mutually independent Bernoulli trials.

Theorem

The probability of exactly k successes in n independent Bernoulli trials, with probability of success p and probability of failure $q = 1 - p$, is $C(n, k)p^k q^{n-k}$.

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Definition (Random variable)

A random variable is a function from the sample space of an experiment to the set of real numbers. That is, a random variable assigns a real number to each possible outcome.

- Example: Suppose that a coin is flipped three times. Let $X(t)$ be the random variable that equals the number of heads that appear when t is the outcome. Then $X(t)$ takes on the following values:
 - $X(HHH) = 3$
 - $X(HHT) = X(HTH) = X(THH) = 2$
 - $X(TTH) = X(THT) = X(HTT) = 1$
 - $X(TTT) = 0$

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Definition (Random variable)

A random variable is a function from the sample space of an experiment to the set of real numbers. That is, a random variable assigns a real number to each possible outcome.

Definition (Distribution of random variable)

The *distribution* of a random variable X on a sample space S is the set of pairs $(r, \mathbf{Pr}[X = r])$ for all $r \in X(S)$, where $\mathbf{Pr}[X = r]$ is the probability that X takes the value r . (The set of pairs in this distribution is determined by the probabilities $\mathbf{Pr}[X = r]$ for $r \in X(S)$.)

- Example: Suppose that a coin is flipped three times. Let $X(t)$ be the random variable that equals the number of heads that appear when t is the outcome. What is the distribution of X ?

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- **The Birthday Problem:** What is the minimum number of people who need to be in a room so that the probability that at least two of them have the same birthday is greater than $1/2$?

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Probabilistic Algorithms

- Probabilistic algorithms: Algorithms that make random choices at one or more steps.
- Monte Carlo Algorithms: Probabilistic algorithms for decision problems that always produces an answer. The answer may be incorrect with some small probability.
 - Example: A sends 1 million apples to B . A has cleverly packed 1000 bad apples among these 1 million apples. How does B detect that A has sent 1 million good apples or not.

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