

ACOL1002: Discrete Mathematical Structures

Ragesh Jaiswal, CSE, IIT Delhi

Counting

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Generating Permutations and Combinations

- How do you generate a permutation of n distinct objects?

Counting

Generating Permutations and Combinations

- How do you generate a permutation of n distinct objects?
 - This is the same as generating permutations of $\{1, 2, \dots, n\}$.
- Total ordering on permutations of $\{1, 2, 3, \dots, n\}$:
 - $(a_1, a_2, \dots, a_n) < (b_1, b_2, \dots, b_n)$ iff there is a j such that $a_1 = b_1, a_2 = b_2, \dots, a_{j-1} = b_{j-1}$, and $a_j < b_j$.
- Question: What is the next permutation after $(a_1, a_2, \dots, a_n)$?

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Generating Permutations and Combinations

- How do you generate a combination of n distinct objects?

Discrete Probability

Discrete Probability

Introduction

- An *experiment* is a procedure that yields one of a given set of possible outcomes.
- The *sample space* of the experiment is the set of possible outcomes.
- An *event* is a subset of the sample space.

Definition (Probability)

If S is a finite nonempty sample space of equally likely outcomes, and E is an event, that is, a subset of S , then the probability of E is $\Pr[E] = \frac{|E|}{|S|}$.

- What is the probability that when two dice are rolled, the sum of the numbers on the two dice is 7?

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- What is the probability that when two dice are rolled, the sum of the numbers on the two dice is 7?
- What is the probability that the numbers 11, 4, 17, 39, and 23 are drawn in that order from a bin containing 50 balls labeled with the numbers 1, 2, ..., 50 if (a) the ball selected is not returned to the bin before the next ball is selected and (b) the ball selected is returned to the bin before the next ball is selected?

Discrete Probability

Introduction

Theorem

Let E be an event in a sample space S . The probability of the event $\bar{E} = S - E$, the complementary event of E , is given by $\Pr[\bar{E}] = 1 - \Pr[E]$.

- A sequence of 10 bits is randomly generated. What is the probability that at least one of these bits is 0?

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Theorem

Let E_1 and E_2 be events in the sample space S . Then

$$\Pr[E_1 \cup E_2] = \Pr[E_1] + \Pr[E_2] - \Pr[E_1 \cap E_2].$$

- What is the probability that a positive integer selected at random from the set of positive integers not exceeding 100 is divisible by either 2 or 5?

End