

ACOL1002: Discrete Mathematical Structures

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Counting

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Pigeonhole Principle

Theorem (The Generalized Pigeonhole Principle)

If N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects.

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- What is the least number of area codes needed to guarantee that the 25 million phones in a state can be assigned distinct 10-digit telephone numbers? (Assume that telephone numbers are of the form NXX-NXX-XXXX, where the first three digits form the area code, N represents a digit from 2 to 9 inclusive, and X represents any digit.)

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- Suppose that a computer science laboratory has 15 workstations and 10 servers. A cable can be used to directly connect a workstation to a server. For each server, only one direct connection to that server can be active at any time. We want to guarantee that at any time any set of 10 or fewer workstations can simultaneously access different servers via direct connections. What is the minimum number of direct connections needed to achieve this goal?

Counting

Pigeonhole Principle: Example Applications

- During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games.
- Show that among any $n + 1$ positive integers not exceeding $2n$ there must be an integer that divides one of the other integers.
- Every sequence of $n^2 + 1$ distinct real numbers contains a subsequence of length $n + 1$ that is either strictly increasing or strictly decreasing.

End