

ACOL1002: Discrete Mathematical Structures

Ragesh Jaiswal, CSE, IIT Delhi

Induction and Recursion

Induction and Recursion

Strong Induction

Definition (Strong induction)

To prove that $P(n)$ is true for all positive integers n , where $P(n)$ is a propositional function, we complete two steps:

- Basis step: We verify that $P(1)$ is true.
 - Inductive step: We show that the conditional statement $[P(1) \wedge P(2) \wedge \dots \wedge P(k)] \rightarrow P(k+1)$ is true for all positive integers k .
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- Strong induction is sometimes called *second principle of mathematical induction* or *complete induction*.

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- If the inductive step is valid only for integers greater than a particular integer.
 - Basis step: We verify that the propositions $P(b), P(b+1), \dots, P(b+j)$ are true.
 - Inductive step: We show that $[P(b) \wedge P(b+1) \wedge \dots \wedge P(k)] \rightarrow P(k+1)$ is true for every integer $k \geq b+j$.

Induction and Recursion

Strong Induction: Examples

- Show that if n is an integer greater than 1, then n can be written as the product of primes.

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Strong Induction: Examples

- Show that if n is an integer greater than 1, then n can be written as the product of primes.
- Prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps.

Induction and Recursion

Recursion

- The process of defining objects in terms of itself is called *recursion*.
- Examples
 - Recursively defined functions.
 - Recursively defined sets and structures

Induction and Recursion

Recursion

- Recursively defined functions.
 - Functions with non-negative integers as its domain.
 - Basis step: Specify the value of the function at 0.
 - Recursive step: Give a rule for finding its value at an integer from its values at smaller integers.
 - Such functions may alternatively be interpreted as a sequence.
- Example: Fibonacci sequence
 - Basis step: $f_0 = 0, f_1 = 1$.
 - Recursive step: $f_n = f_{n-1} + f_{n-2}$ for $n = 2, 3, 4, \dots$
 - Show that whenever $n \geq 3, f_n > \alpha^{n-2}$, where $\alpha = (1 + \sqrt{5})/2$.

Induction and Recursion

Recursion

- Recursively defined sets and structures
 - Basis step: An initial collection of elements is specified.
 - Recursive step: Rules for forming new elements in the set from those already known to be in the set are provided.
 - *Exclusion Rule*: Recursively defined set contains nothing other than those elements specified in the basis step or generated by applications of the recursive step.
- Examples:
 - Multiples of 3:
 - Basis step: $3 \in S$
 - Recursive step: If $x \in S$ and $y \in S$, then $x + y \in S$.
 - The set Σ^* of strings over the alphabet Σ :
 - Basis step: ?
 - Recursive step: ?

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Recursion

- Examples:
 - Multiples of 3:
 - Basis step: $3 \in S$
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 - The set Σ^* of strings over the alphabet Σ :
 - Basis step: $\lambda \in \Sigma^*$ (where λ is the empty string containing no symbols).
 - Recursive step: If $w \in \Sigma^*$ and $x \in \Sigma$, then $wx \in \Sigma^*$.
 - Well-formed formula in propositional logic:
 - Basis step: T , F , and s , where s is a propositional variable, are well-formed formulae.
 - Recursive step: If E and F are well-formed formulae, then $(\neg E)$, $(E \wedge F)$, $(E \vee F)$, $(E \rightarrow F)$, and $(E \leftrightarrow F)$ are well-formed formulae.

End