

ACOL1002: Discrete Mathematical Structures

Ragesh Jaiswal, CSE, IIT Delhi

Logic

Propositional Logic: logical operators

Definition (Conditional statement)

Let p and q be propositions. The conditional statement $p \rightarrow q$ is the proposition that is false when p is true and q is false, and true otherwise.

- q is true on the condition that p is true.
- This is also called an *implication*.
- There are various ways to express $p \rightarrow q$:
 - “ p is sufficient for q ” or “a sufficient condition for q is p ”
 - “ q if p ”
 - “ q when p ”
 - “ q is necessary for p ” or “a necessary condition for p is q ”
 - “ q unless $\neg p$ ”
 - “ p implies q ”
 - “ p only if q ”
 - “ q whenever p ”
 - “ q follows from p ”

Logic

Propositional Logic: logical operators

Definition (Conditional statement)

Let p and q be propositions. The conditional statement $p \rightarrow q$ is the proposition that is false when p is true and q is false, and true otherwise.

Definition (Converse)

The converse of a proposition $p \rightarrow q$ is the proposition $q \rightarrow p$.

Definition (Contrapositive)

The contrapositive of a proposition $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$.

Definition (Inverse)

The inverse of a proposition $p \rightarrow q$ is the proposition $\neg p \rightarrow \neg q$.

Logic

Propositional Logic: logical operators

Definition (Conditional statement)

Let p and q be propositions. The conditional statement $p \rightarrow q$ is the proposition that is false when p is true and q is false, and true otherwise.

Definition (Converse)

The converse of a proposition $p \rightarrow q$ is the proposition $q \rightarrow p$.

Definition (Contrapositive)

The contrapositive of a proposition $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$.

Definition (Inverse)

The inverse of a proposition $p \rightarrow q$ is the proposition $\neg p \rightarrow \neg q$.

- Show that the contrapositive of $p \rightarrow q$ always has the same truth value as $p \rightarrow q$.

Logic

Propositional Logic: logical operators

Definition (Conditional statement)

Let p and q be propositions. The conditional statement $p \rightarrow q$ is the proposition that is false when p is true and q is false, and true otherwise.

Definition (Converse)

The converse of a proposition $p \rightarrow q$ is the proposition $q \rightarrow p$.

Definition (Contrapositive)

The contrapositive of a proposition $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$.

Definition (Inverse)

The inverse of a proposition $p \rightarrow q$ is the proposition $\neg p \rightarrow \neg q$.

- Show that the contrapositive of $p \rightarrow q$ always has the same truth value as $p \rightarrow q$.
- Show that, neither converse nor inverse of $p \rightarrow q$ has the same truth value as $p \rightarrow q$ for all truth values of p and q .

Logic

Propositional Logic: logical operators

- Negation ($\neg$)
- Conjunction ($\wedge$)
- Disjunction ($\vee$).
- Exclusive or ($\oplus$)
- Conditional statement ($\rightarrow$)
- Bi-conditional statement ($\leftrightarrow$): Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition “ p if and only if q ”. The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called bi-implications.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Table: Truth table for $p \leftrightarrow q$.

Logic

Propositional Logic: logical operators

- Negation ($\neg$)
- Conjunction ($\wedge$)
- Disjunction ($\vee$).
- Exclusive or ($\oplus$)
- Conditional statement ($\rightarrow$)
- Bi-conditional statement ($\leftrightarrow$): Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition “ p if and only if q ”. The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called bi-implications.
 - $p \leftrightarrow q$ is also expressed as:
 - “ p is necessary and sufficient for q ”
 - “ p iff q ”
 - “if p , then q and conversely”
 - Show that $p \leftrightarrow q$ always has the same truth value as $(p \rightarrow q) \wedge (q \rightarrow p)$.

- Logical operators
 - Negation ($\neg$)
 - Conjunction ($\wedge$)
 - Disjunction ($\vee$).
 - Exclusive or ($\oplus$)
 - Conditional statement ($\rightarrow$)
 - Bi-conditional statement ($\leftrightarrow$)
- A compound proposition is formed by applying these operators on simpler propositions. E.g. $(p \vee q \wedge r)$.
- Operator Precedence (in decreasing order): $\neg, \wedge, \oplus, \vee, \rightarrow, \leftrightarrow$.
- Construct the truth table for $p \vee \neg q \rightarrow p \wedge q$.

- Simplify complex sentences and enable to logically analyze them.
 - “You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.”
 - p : “You can ride the roller coaster.”
 - q : “You are under 4 feet tall.”
 - r : “You are older than 16 years old.”
 - Express the sentence in terms of propositions p , q , and r .

- Simplify complex sentences and enable to logically analyze them.
 - “You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.”
 - p : “You can ride the roller coaster.”
 - q : “You are under 4 feet tall.”
 - r : “You are older than 16 years old.”
 - Express the sentence in terms of propositions p , q , and r .
 - $(q \wedge \neg r) \rightarrow \neg p$.

- Simplify complex sentences and enable to logically analyze them.
- Translate system specification expressed in natural language into unambiguous logical expressions.
 - Example:
 - “The diagnostic message is stored in the buffer or is retransmitted.”
 - “The diagnostic message is not stored in the buffer.”
 - “If the diagnostic message is stored in the buffer, then it is retransmitted.”
 - “The diagnostic message is not retransmitted.”
 - Consistency: Whether all the specifications can be satisfied simultaneously.
 - Are these specifications consistent?

End