

Name: _____

Entry number: _____

There are 2 questions for a total of 10 points.

1. Answer the following MCQ-type questions:

(a) (1 point) Suppose the jug capacities are 6 liters and 10 liters. Which target amount B is **not** possible?

(A) 2 (B) 4 (C) 5 (D) 8

(b) (1 point) What is $\mathbb{Z}_{10}^*$?

(A) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ (B) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ (C) $\{1, 3, 7, 9\}$ (D) $\{2, 4, 6, 8\}$

(c) (1 point) If $p = 5$, $q = 11$, and $e = 3$, then $\phi(N) = 40$. Which value of d satisfies $3d \equiv 1 \pmod{40}$?

(A) 7 (B) 13 (C) 27 (D) 40

(d) (1 point) If $p = 5$, $q = 11$, and $e = 3$, then $\phi(N) = 40$. What is the encryption of $M = 10$?

(d) _____

(e) (1 point) Which one is a generator for $\mathbb{Z}_7^*$?

(A) 1 (B) 2 (C) 3 (D) 6

2. Let S be the subset of the set of ordered pairs of integers defined recursively by:

Basis step: $(0, 0) \in S$

Recursive step: If $(a, b) \in S$, then $(a + 3, b + 2) \in S$ and $(a + 2, b + 3) \in S$.

Answer the questions that follow:

(a) (1 point) State True or False: $(12, 13)$ is in the set S .

(a) _____

(b) (1 point) State True or False: $(12, 12)$ is in the set S .

(b) _____

(c) (1 point) State True or False: For every $(a, b) \in S$, $a + b$ is a multiple of 5.

(c) _____

(d) (1 point) State True or False: For every (a, b) where $a + b$ is a positive multiple of 5, $(a, b) \in S$.

(d) _____

(e) (1 point) Give a reason for your answer to part (d).