

Name: _____

Entry number: _____

There are 4 questions for a total of 10 points.

1. (3 points) Construct the truth table for the compound proposition:

$$((p \vee q) \wedge (\neg p \vee r)) \rightarrow \neg(\neg q \wedge \neg r)$$

by completing the table below.

p	q	r	$(p \vee q)$	$(\neg p \vee r)$	$\neg(\neg q \wedge \neg r)$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow \neg(\neg q \wedge \neg r)$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	T	T	T	T
F	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	T	F	T

2. Let p , q , and r be the propositions:

- p : Your algorithm is efficient.
- q : Your algorithm is correct.
- r : Your algorithm is accepted for publication.

Write the propositions below using p , q , and r and logical connectives (including negations).

- (a) ($\frac{1}{2}$ point) *Your algorithm is accepted for publication but it is not efficient.*

(a) _____ $r \wedge \neg p$ _____

- (b) ($\frac{1}{2}$ point) *For your algorithm to be accepted for publication, it is necessary that it is correct.*

(b) _____ $r \rightarrow q$ _____

- (c) ($\frac{1}{2}$ point) *Your algorithm being efficient and correct is sufficient for it to be accepted for publication.*

(c) _____ $(p \wedge q) \rightarrow r$ _____

- (d) ($\frac{1}{2}$ point) *Your algorithm will not be accepted for publication unless it is efficient.*

(d) _____ $\neg p \rightarrow \neg r$ _____

3. (a) ($\frac{1}{2}$ point) State True or False: The following two compound propositions are logically equivalent:

- $(p \rightarrow q) \vee (r \rightarrow q)$
- $(p \wedge r) \rightarrow q$

(a) True

- (b) (2 points) Give reason for your answer to part (a).

Solution: We show that the compound propositions are logically equivalent using the following series of equivalences:

$$\begin{aligned}
 (p \rightarrow q) \vee (r \rightarrow q) &\equiv (\neg p \vee q) \vee (\neg r \vee q) && \text{(using conditional-disjunction)} \\
 &\equiv \neg p \vee q \vee \neg r \vee q && \text{(using associative law)} \\
 &\equiv \neg p \vee \neg r \vee q \vee q && \text{(using commutative law)} \\
 &\equiv \neg p \vee \neg r \vee q && \text{(using Idempotence law)} \\
 &\equiv \neg(p \wedge r) \vee q && \text{(using DeMorgan's law)} \\
 &\equiv (p \wedge r) \rightarrow q && \text{(using conditional disjunction)}
 \end{aligned}$$

4. (a) ($\frac{1}{2}$ point) State True or False: $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ is a tautology.

(a) True

- (b) (2 points) Give reason for your answer to part (a).

Solution: We show this using a truth table.

p	q	r	$(p \vee q) \wedge (\neg p \vee r)$	$q \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
F	T	T	T	T
T	F	F	F	F
F	T	F	T	T
F	F	T	T	T
F	F	F	F	F