

Logic, Proofs and Computation Assignment

Out: December 8

Due: Not necessary

1. Recall the satisfiability problem that we encountered in the lecture: Given a 3-CNF formula $\phi(x_1, \dots, x_n) = \bigwedge_{i=1}^m (x_{i_1} \vee x_{i_2} \vee x_{i_3})$, check if there exists $a \in \{0, 1\}^n$ such that $\phi(a_1, \dots, a_n) = 1$. Model the following Sudoku problem as a 3-CNFSAT problem:

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Use a SAT solver (for example see <https://homes.cs.washington.edu/~kevinz/sat-solver/>) to find a solution to the sudoku.

2. A (simple, undirected) graph is a pair $G = (V, E)$ where V is a finite set of vertices and $E \subseteq \binom{V}{2}$ is a set of unordered edges. A *proper 3-coloring* of G is a function $c : V \rightarrow \{1, 2, 3\}$ such that for every edge $\{u, v\} \in E$ we have $c(u) \neq c(v)$.

The decision problem **3-COLOR** is:

$$\mathbf{3-COLOR} = \{ \langle G \rangle \mid G \text{ admits a proper 3-coloring} \},$$

where $\langle G \rangle$ denotes an encoding of the graph G .

- (a) Design an algorithm that on input a 3-CNF formula

$$\Phi = C_1 \wedge C_2 \wedge \dots \wedge C_m \quad \text{with variables } x_1, \dots, x_n,$$

constructs a graph $G_\Phi = (V, E)$ such that Φ is satisfiable if and only if G_Φ is 3-colorable.

- (b) Give an explicit example: take a small 3-CNF (e.g. $(x_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee x_4)$) and draw the graph G_Φ produced by your algorithm; show a corresponding 3-coloring for a satisfying assignment.
3. Express the infinitude of primes as a logical statement. Pick your favorite proof of infinitude of primes from here: https://en.wikipedia.org/wiki/Euclid%27s_theorem and try to formalise it in Lean (https://leanprover-community.github.io/mathematics_in_lean/). Here is a great place to get started on formalizing mathematics with Lean: <https://adam.math.hhu.de/#/g/leanprover-community/nng4>