

ARGL112: Game Theory Assignment

Question 1. Matching Pennies Game with Non-symmetric Payoffs

Consider the following matching pennies game between Alice and Bob. Both can choose either Head (H) or Tail (T), and their payoffs are as follows:

	Bob: H	Bob: T
Alice: H	2, -2	-1, 1
Alice: T	-1, 1	1, -1

- (i) Explain why always playing H or always playing T is not a good strategy for Alice.
- (ii) Argue that if Alice plays H with probability $2/5$ and T with probability $3/5$, then the best strategy for Bob is to play H/T with any arbitrary probability as his payoff will not change.
- (iii) Argue that if Bob plays H with probability $2/5$ and T with probability $3/5$, then the best strategy for Alice is to play H/T with any arbitrary probability as her payoff will not change.
- (iv) Argue that the Nash equilibrium is when Alice and Bob both play H with probability $2/5$ and T with probability $3/5$. Moreover, argue that under this scenario, the expected payoff of Alice is $1/5$ and Bob is $-1/5$.

Question 2. Hotelling Problem on a Circle with 3 Vendors

Three ice-cream vendors are to choose locations on a circle of length 1 (the points are denoted by the interval $[0, 1)$ where 0 and 1 refer to the same point). A continuum of customers is uniformly distributed around the circle, and any customer chooses to go to the closest vendor (breaking ties equally if distances are equal). If each vendor wants to maximize their share of customers, then find a Nash equilibrium configuration for vendor locations.