

ACOL106: Data Structures and Algorithms

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Data Structures: **Balanced** Binary Search Trees

Data Structures

Binary Search Trees → Multiway Search Trees

- Binary search trees allows storage and access of data in time proportional to the height of the tree.
- The number of children of a node binary search trees is upper bounded by 2.
- Removing this restriction might provide us more flexibility without costing us in terms of performance.
- **Multiway Search Trees** are generalisation of Binary Search Trees where internal nodes are allowed to have more than two children.

Data Structures

Binary Search Trees $\rightarrow$ Multiway Search Trees

- **Multiway Search Trees** are generalisation of Binary Search Trees where internal nodes are allowed to have more than two children.
- A node in an ordered tree is said to be a d -node iff it has d children.

Definition (Multiway Search Tree)

A Multiway Search Tree is an ordered tree that has the following properties:

- Each internal node is a d -node with $d \geq 2$.
- Each internal d -node with children $c_1, \dots, c_d$ stores an ordered set of $(d - 1)$ key-value pairs $(k_1, v_1), (k_2, v_2), \dots, (k_{d-1}, v_{d-1})$, where $k_1 \leq k_2 \leq \dots \leq k_{d-1}$.
- Consider any d -node w . Let $k_0 = -\infty$ and $k_d = +\infty$ by convention. For every entry (k, v) stored in the subtree rooted at c_i , we have $k_{i-1} \leq k < k_i$.

Data Structures

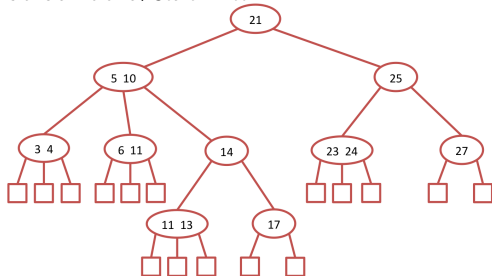
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- Is this a Multiway Search Tree?



Data Structures

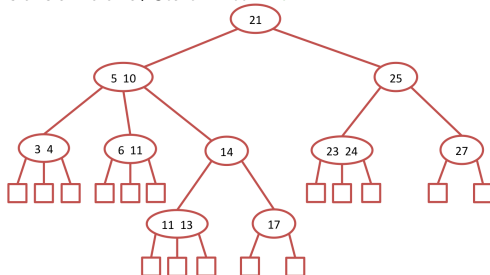
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- Is this a Multiway Search Tree? No



Data Structures

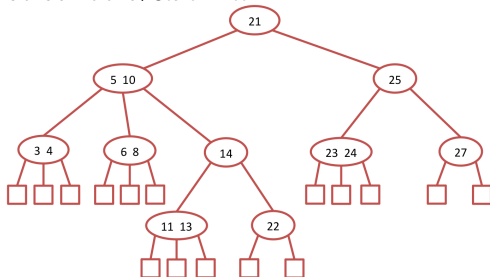
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Data Structures

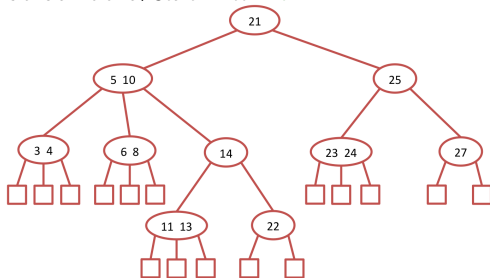
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Data Structures

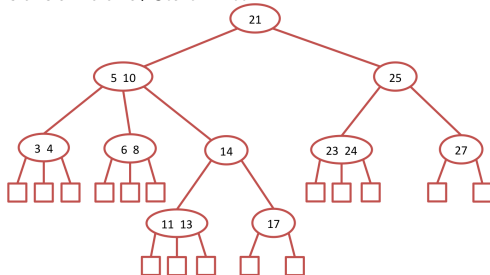
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- Is this a Multiway Search Tree?



Data Structures

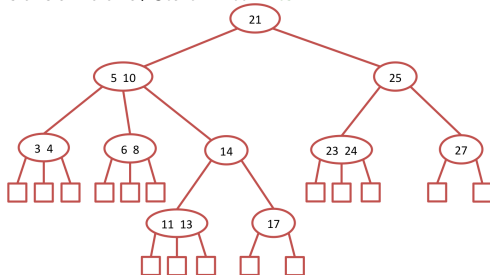
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- Is this a Multiway Search Tree? **Yes**



Data Structures

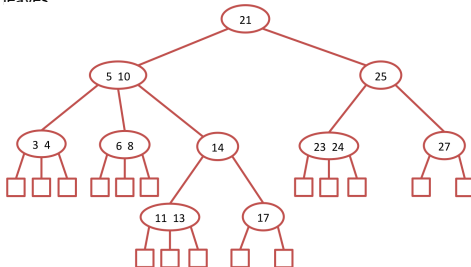
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- Claim 1: Any multiway search tree containing n entries has $n + 1$ leaves



Data Structures

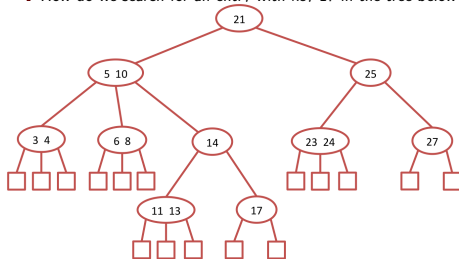
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- Question: How do we perform search operation on multiway search trees?
 - How do we search for an entry with key 17 in the tree below?



Data Structures

Multiway Search Trees $\rightarrow$ (2,4)-Trees

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Definition ((2,4)-Tree)

A (2, 4)-Tree is a multiway search tree with the following two additional properties:

- ① Size property: Every internal node has **at most 4 children**.
- ② Depth property: All leaves have the **same depth**.

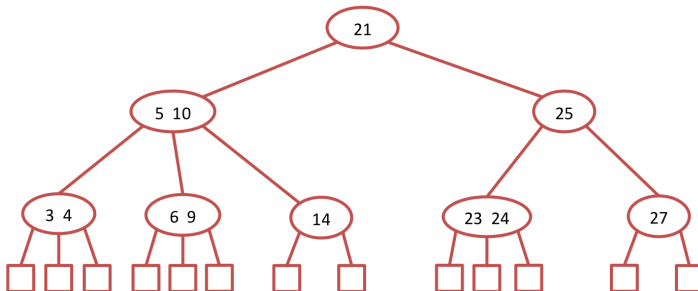
Data Structures

Multiway Search Trees $\rightarrow$ (2,4)-Trees

Definition ((2,4)-Tree)

A (2, 4)-Tree is a multiway search tree with the following two additional properties:

- 1 Size property: Every internal node has **at most 4 children**.
- 2 Depth property: All leaves have the **same depth**.



Data Structures

Multiway Search Trees $\rightarrow$ (2,4)-Trees

Definition ((2-4)-Tree)

A (2, 4)-Tree is a multiway search tree with the following two additional properties:

- ① Size property: Every internal node has **at most 4 children**.
- ② Depth property: All leaves have the **same depth**.

- Claim 1: The height of any (2, 4)-tree storing n entries is $O(\log n)$.

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Multiway Search Trees $\rightarrow$ (2,4)-Trees

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A (2, 4)-Tree is a multiway search tree with the following two additional properties:

- ① Size property: Every internal node has **at most 4 children**.
 - ② Depth property: All leaves have the **same depth**.
- Claim 1: The height of any (2, 4)-tree storing n entries is $O(\log n)$.
 - Question: What is the running time of the search operation on a (2, 4)-Tree?

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Multiway Search Trees $\rightarrow$ (2,4)-Trees

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A (2, 4)-Tree is a multiway search tree with the following two additional properties:

- ① Size property: Every internal node has **at most 4 children**.
- ② Depth property: All leaves have the **same depth**.

- Claim 1: The height of any (2, 4)-tree storing n entries is $O(\log n)$.
- Question: What is the running time of the search operation on a (2, 4)-Tree? $O(\log n)$

End