

COL106: Data Structures and Algorithms

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Data Structures

Tree → Tree Traversal

- One of the most basic operations on Trees is Tree Traversal.
- Here are two ways in which the nodes of a rooted tree may be traversed.
 - Pre-order Traversal: Visit the root *before* visiting the children.
 - Post-order Traversal: Visit the root *after* visiting the sub-trees.

Data Structures

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Algorithm

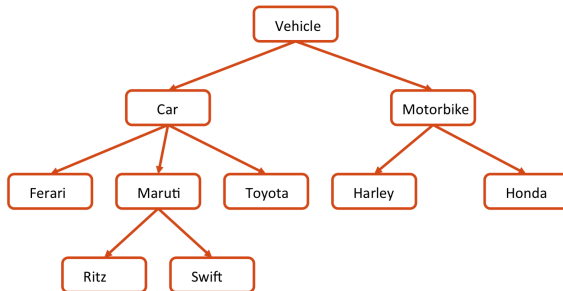
PreorderVisit(v)

- Visit the root node v
- For every child u of v :
 - PreorderVisit(u)

PostorderVisit(v)

- For every child u of v :
 - PostorderVisit(u)
- Visit the root node v

- Question: Output the nodes visited while doing a pre-order traversal in the tree below.



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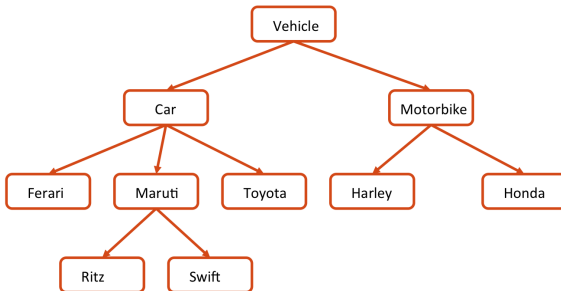
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PostorderVisit(v)

- For every child u of v :
 - PostorderVisit(u)
- Visit the root node v

- Question: Output the nodes visited while doing a post-order traversal in the tree below.



Data Structures

Tree $\rightarrow$ Binary Tree

- A binary tree is a an ordered tree where all the nodes have at most two children.
- Each node is either is labeled as either being a left child or a right child.
- A binary tree is proper if each internal node has exactly two children or improper otherwise.

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- A binary tree is proper if each internal node has exactly two children or improper otherwise.
- For any given binary tree T , let:
 - N denote the number of nodes in the T .
 - L denote the number of external nodes (or leaves) in T .
 - I denote the number of internal nodes in T .
 - H denote the height of T . Height of a tree is equal to the height of the root.
- Show that:
 - 1 $H + 1 \leq N \leq 2^{H+1} - 1$
 - 2 $1 \leq L \leq 2^H$
 - 3 $H \leq I \leq 2^H - 1$
 - 4 $\log(N + 1) - 1 \leq H \leq N - 1$

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 - ⑤ The number of edges is equal to $(N - 1)$.

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 - 5 The number of edges is equal to $(N - 1)$.
 - 6 In any proper binary tree with n nodes, the number of leaves equals the number of internal nodes plus one.

End