

Linearization aka Topological Sort

- Is it possible to order the vertices such that all edges go in only one direction?
- For what types of DAGs is this possible?
- How do we find such an ordering?

Property of DAGS

Theorem: every edge in a DAG goes from a higher post number to lower post number.

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proof:

suppose (u, v) is an edge in a DAG then it can't be a back edge, therefore it can only be a forward edge/tree edge or a cross edge.

All of which have the property that $\text{post}(v) < \text{post}(u)$.

Corollary: Sorting by post numbers is a topological sort

Property of DAGS

Linearization of a DAG:

Since we know that edges go in the direction of decreasing post numbers, if we order the vertices by decreasing post numbers then we will have a linearization

procedure **linearize**(DAG $G = (V, E)$)

run DFS(G)

return list of vertices in decreasing order of post numbers
(by putting at start of list when post number assigned)

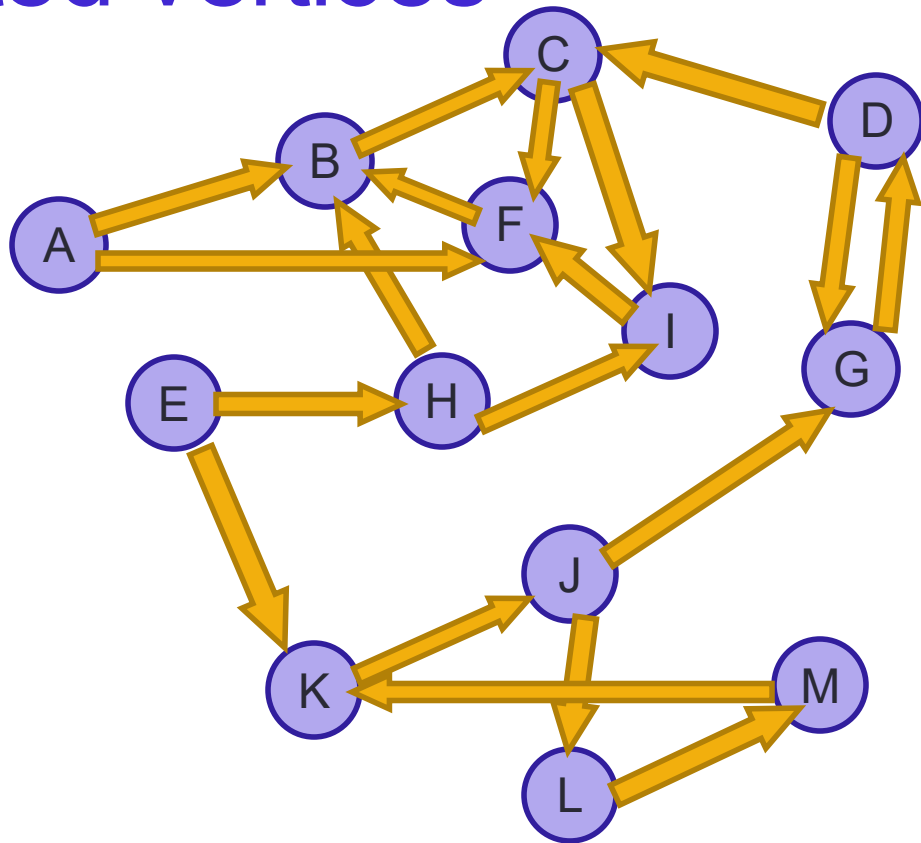
Sources and sinks

- Since all DAGs can be linearized, that means the first vertex in the ordering does not have any edges coming in and the last vertex does not have any edges going out.
- **Definitions:**
 - A vertex with no incoming edges is called a **source**
 - A vertex with no outgoing edges is called a **sink**
- **Theorem:** All DAGs have at least one source and one sink.

Strongly connected vertices

Two vertices u and v in a directed graph are strongly connected if there exists a path from u to v and a path from v to u .

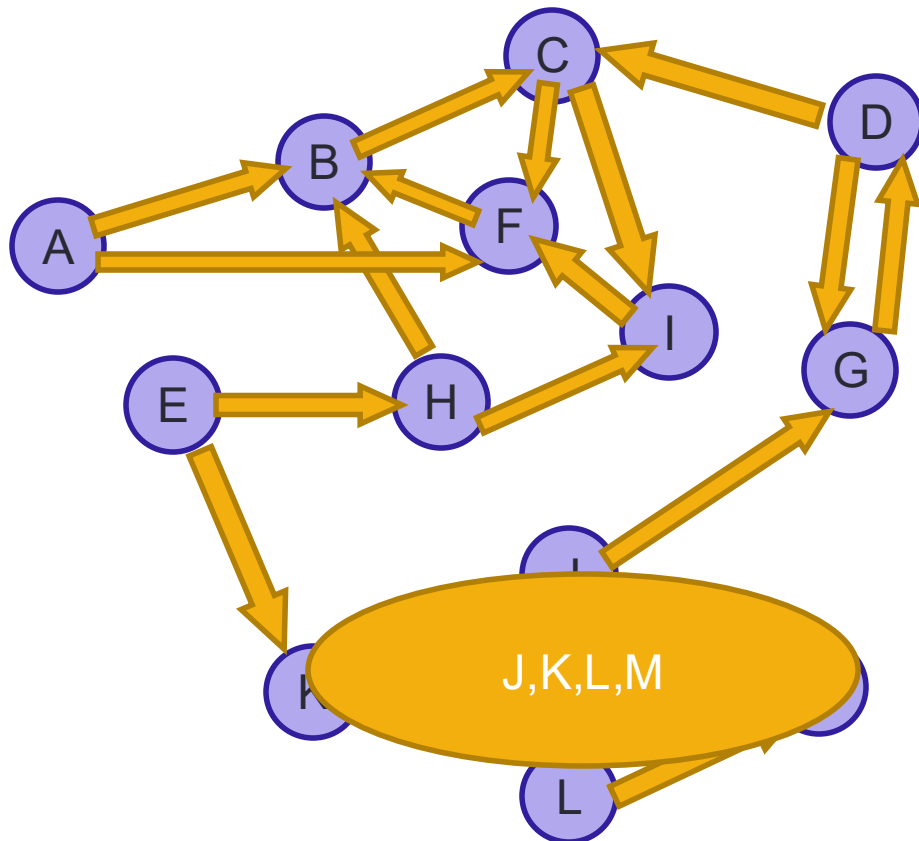
Which vertices are strongly connected to J?



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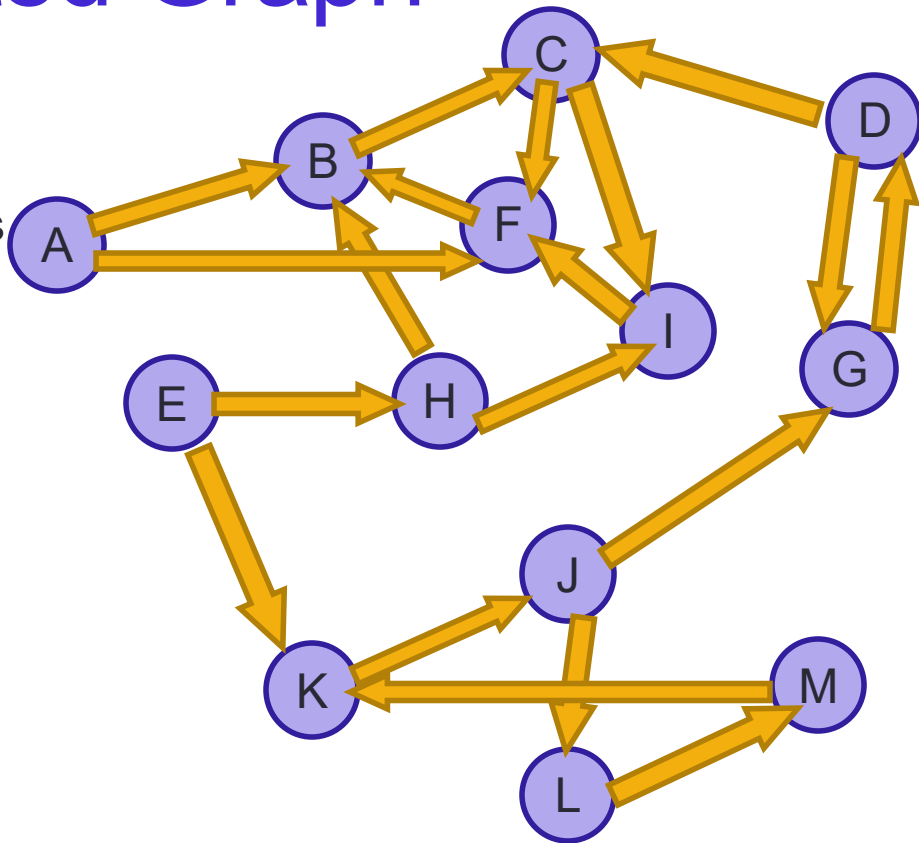
Which vertices are strongly connected to J? {J, K, L, M}



Strongly connected Graph

A graph is called strongly connected if for each pair of vertices v, u there is a path from v to u and a path from u to v .

Is this a strongly connected graph?



Strongly connected components

Consider the relation uRv if u is strongly connected to v .

Then R is an equivalence relation. It is reflexive, symmetric and transitive.

So R partitions V , the set of vertices into equivalence classes.

These equivalence classes are called strongly connected components.

Strongly connected component

What are the strongly connected components of this graph?

