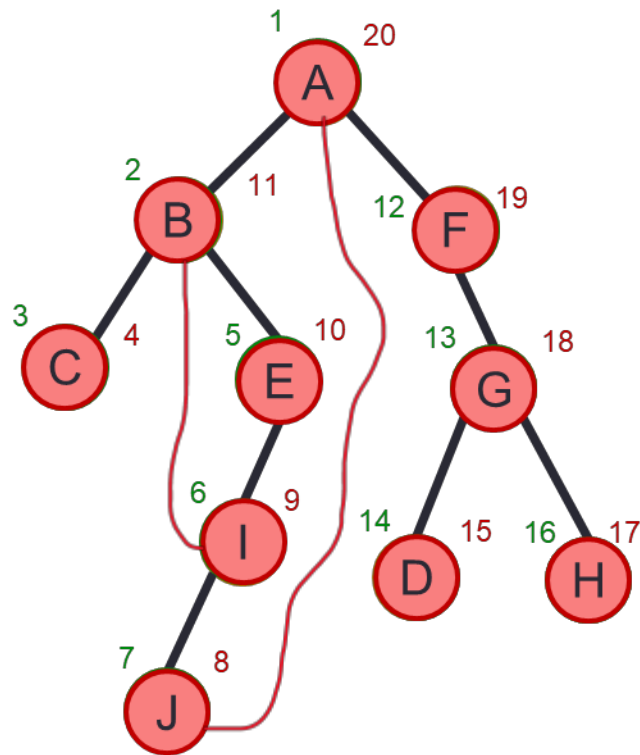
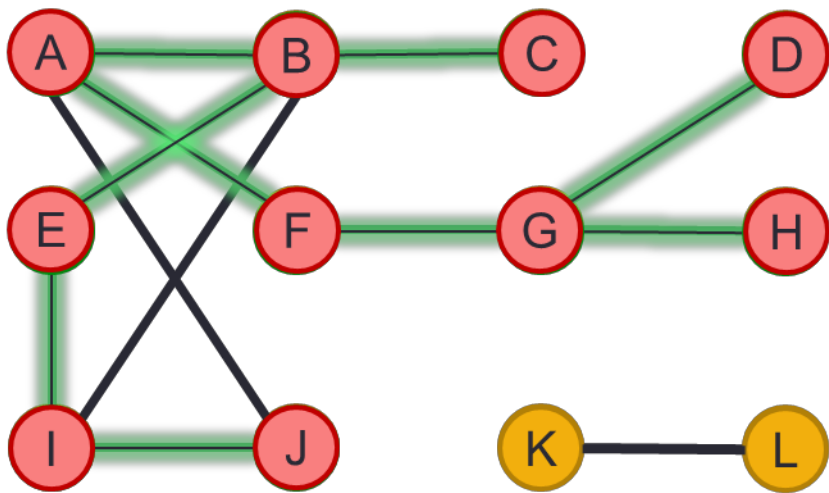


DFS augmtd. with pre, post numbers

- procedure $\text{explore}(G, v)$
- Input: graph $G = (V, E)$; node $v \in V$ output: count starts at 1
- Output: array $\text{visited}[u]$; $\text{parent}[u]$; $\text{pre}[u]$; $\text{post}[u]$
- 1. $\text{visited}[v] = \text{true}$;
- 2. for each edge $(v, u) \in E$ do:
 - if not $\text{visited}[u]$: $\text{parent}[u]=v$; $\text{pre}[u]=\text{count}$; $\text{count}++$; $\text{explore}(G, u)$;
- 3. $\text{post}[v] = \text{count}$, $\text{count}++$

keeping track of paths

Example:



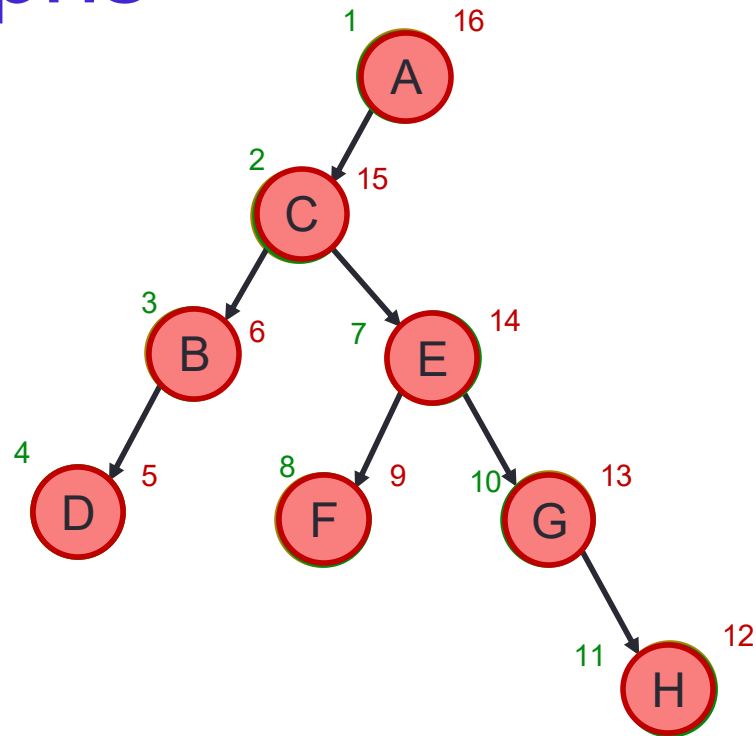
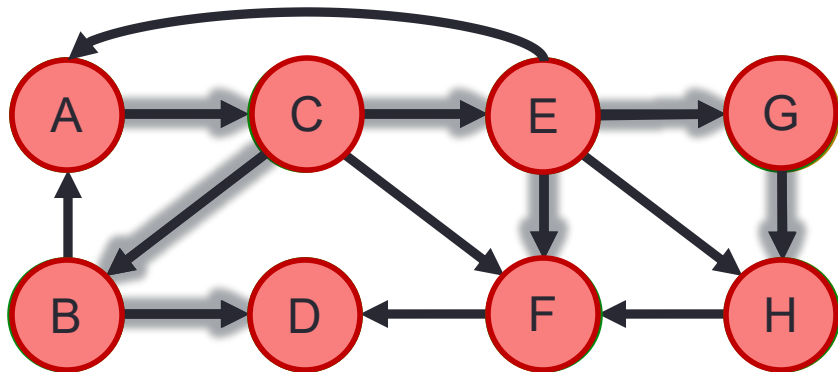
Inferring relative position in tree

- u is below v in the DFS tree iff $\text{pre}(v) < \text{pre}(u)$ and $\text{post}(u) < \text{post}(v)$.
 - In this case, an edge from u to v creates a cycle
- u is to the right of v iff $\text{pre}(v) < \text{pre}(u)$ and $\text{post}(v) < \text{post}(u)$

Edge types (directed graph)

- Tree edge: solid edge included in the DFS output tree
- Back edge: leads to an ancestor
- Forward edge: leads to a descendent
- Cross edge: leads to neither anc. or des.: always from right to left

DFS on Directed Graphs

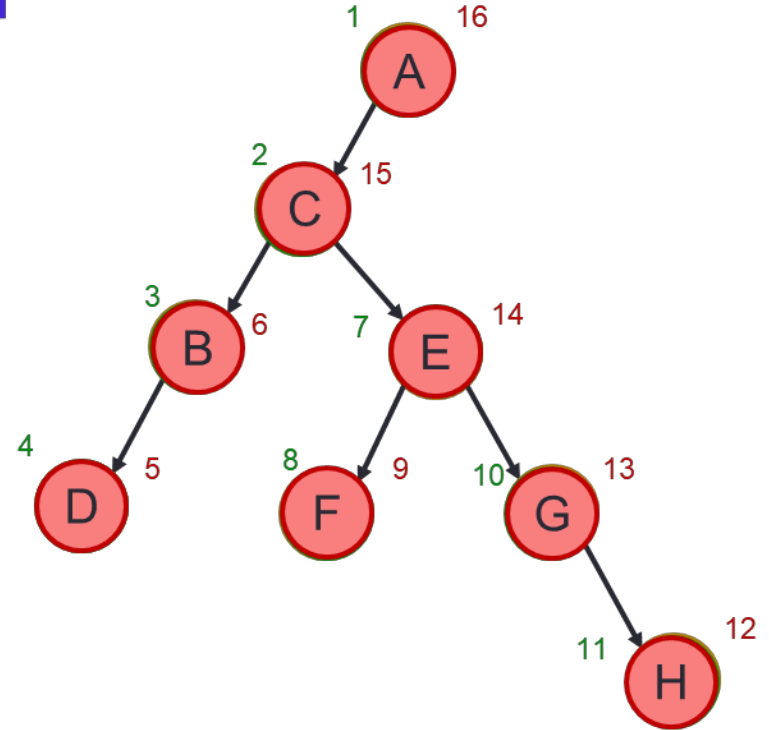
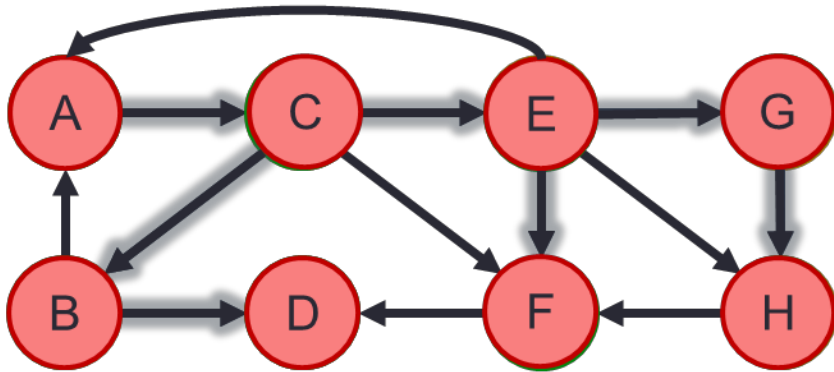


Edge types and pre/post numbers

The different types of edges can be determined from the pre/post numbers for the edge (u, v)

- (u, v) is a tree/forward edge then $pre(u) < pre(v) < post(v) < post(u)$
- (u, v) is a back edge then $pre(v) < pre(u) < post(u) < post(v)$
- (u, v) is a cross edge then $pre(v) < post(v) < pre(u) < post(u)$

DFS on Directed Graphs



Cycles in Directed Graphs

- A cycle in a directed graph is a path that starts and ends with the same vertex

$$v_0 \rightarrow v_1 \rightarrow v_2 \rightarrow \cdots \rightarrow v_k \rightarrow v_0$$

$$A \rightarrow C \rightarrow E \rightarrow A$$