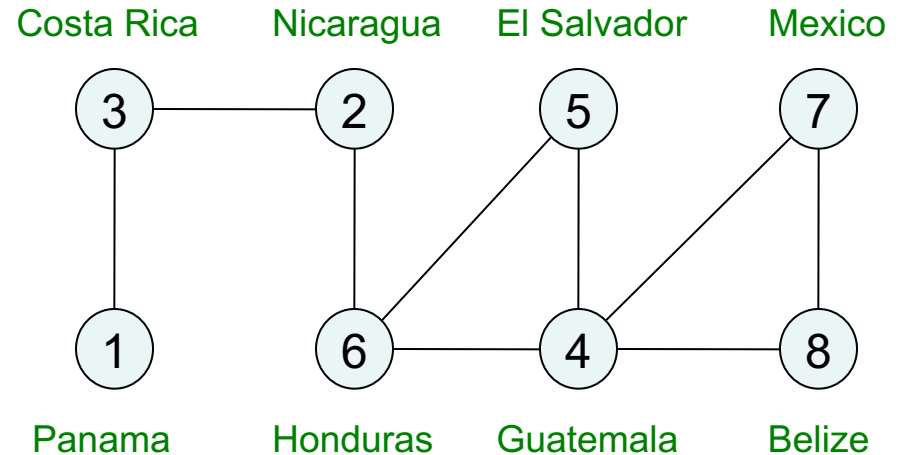


Why graphs?

A cartographer's problem



Graph specified by *nodes* and *edges*.

node

=

country

edge

=

neighbors

Graph coloring problem: color nodes of graph with as few colors as possible, so that there is no edge between nodes of the same color.

Exam scheduling

The registrar's problem

Pitchers (15)						
#	Player	MTL Team	B	T	HR	Residence
1	Bryan Augenstein	South Bend	R	R	6-6	232
2	Randy Boone	Lansing	R	R	6-3	215
3	Mark Depoules	Quad Cities	R	R	6-2	200
4	Edgar Estanga	Lansing	L	L	5-10	185
5	Alfredo Figaro	West Michigan	R	R	6-0	173
6	Jeff Jeffords	Dayton	R	R	6-1	200
7	Jon Kiser	West Michigan	L	L	6-4	215
8	Steven Johnson	Great Lakes	R	R	6-1	212
9	Joseph Krebs	Dayton	L	L	6-0	200
10	Derek McCard	Fort Wayne	R	R	6-1	200
11	Brad Mills	Lansing	L	L	6-0	185
12	Luis Montano	Dayton	R	R	6-0	180
13	Jarrod Parker	South Bend	R	R	6-1	190
14	Miguel Ramirez	Great Lakes	R	R	5-11	165
15	Evan Schreiber	South Bend	R	R	6-3	190
Catchers (2)						
#	Player	MTL Team	B	T	HR	Residence
1	Sean Coughlin*	South Bend	L	L	6-1	206
2	Kerley Jansen	Great Lakes	B	R	6-4	225
Infielders (11)						
#	Player	MTL Team	B	T	HR	Residence
1	Kevin Ahrens	Lansing	B	R	6-1	190
2	Chris Carlson*	West Michigan	R	R	6-4	225
3	Felix Carrasco	Fort Wayne	B	R	6-1	244
4	Justin Jackson	Lansing	R	R	6-2	175
5	Pete Kopra*	Quad Cities	R	R	6-0	170
6	Mike Moe	South Bend	L	L	6-0	188
7	Andy Parrino*	Fort Wayne	B	R	6-0	177
8	Manny Rodriguez*	Lansing	L	L	6-3	190
9	John Toliano	Lansing	B	R	5-11	180
10	Joe Tubbie	West Michigan	R	R	5-11	170
11	Brandon Waring*	Dayton	R	R	6-4	195
Outfielders (2)						
#	Player	MTL Team	B	T	HR	Residence
1	Evan Frey*	South Bend	L	L	5-11	171
2	Charlie Kingray*	Quad Cities	L	L	6-2	210
3	Andrew Lamb*	Great Lakes	L	L	6-3	200
4	Dennis Phipps	Dayton	R	R	6-2	176
5	Casper Wells	West Michigan	R	R	6-2	210

Schedule final exams:

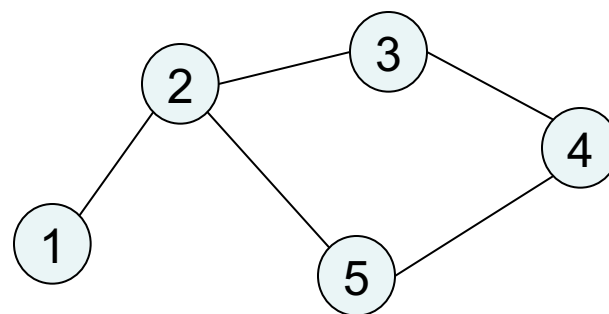
- use as few time slots as possible
- can't schedule two exams in the same slot if there's a student taking both classes.

This is also graph coloring!

Node = exam

Edge = some student is taking both endpoint-exams

Color = time slot



Animal crossing

Animals need to be ferried across a river

- Use as few boats as possible
- Cannot put two animals in the same boat if one will eat the other

This is, yet again, graph coloring!

Node = animal

Edge = one endpoint-animal will eat the other

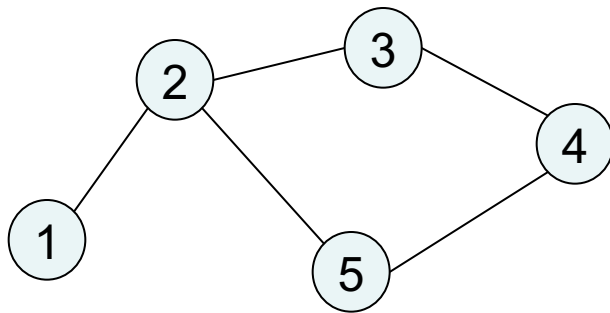
Color = boat

Graph representations

$G = (V, E)$ where

V : vertices/nodes

E : edges



$V = \{1, 2, 3, 4, 5\}$

$E = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{2, 5\}, \{4, 5\}\}$

Undirected edges: symmetric relationship

Directed graphs

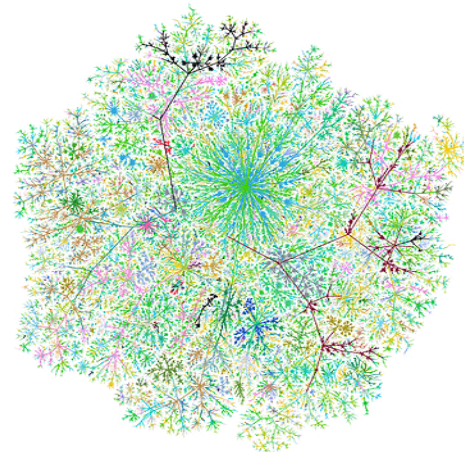
(x, y) : edge *from* x *to* y

e.g. World wide web

node URL

edge (u, v) u points to v

Billions of nodes and edges!



How are graphs stored on a computer?

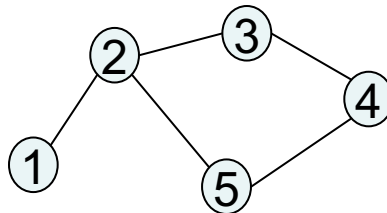
Adjacency matrix

$V \times V$ matrix A

$$A(i,j) = \begin{cases} 1 & \text{if } (i,j) \text{ is in } E \\ 0 & \text{otherwise} \end{cases}$$

Symmetric if G undirected

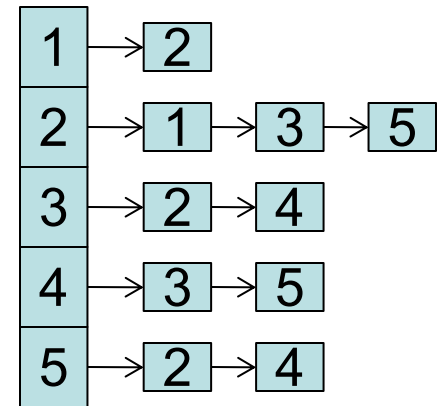
$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$



PRO check for an edge in $O(1)$ time
CON uses up $O(V^2)$ space

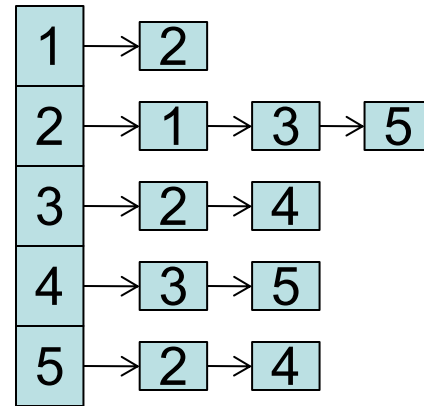
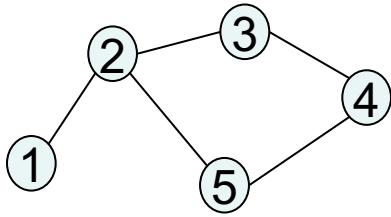
Adjacency list

For each node, list of outgoing edges

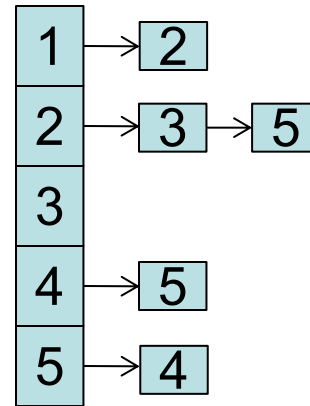
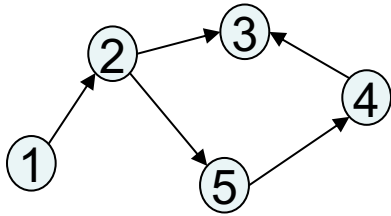


PRO just $O(V + E)$ space
CON check for an edge in $O(V)$ time
PRO easily iterate through node's neighbors

Undirected graphs: adjacency list

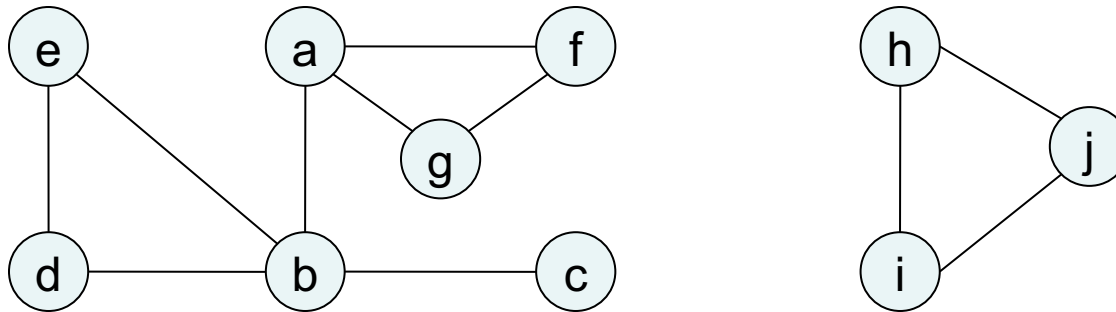


Directed graphs: adjacency list



Reachability in undirected graphs

What parts of a graph are reachable from a given vertex?



With an adjacency list representation, this is like navigating a maze...

Potential difficulty	Don't go round in circles	Don't miss anything
Classical solution	Piece of chalk to mark visited junctions	Ball of string – leads back to starting point
Cyber-analog	Boolean variable for each vertex: visited or not	STACK