COL863: Quantum Computation and Information

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Introduction: Quantum Algorithms

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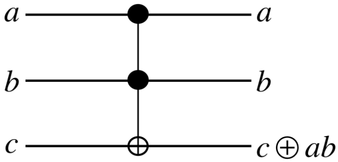
Quantum algorithms

- Can we simulate classical logic circuit using a quantum circuit?
- Claim: Any classical logic circuit can be implemented using just `NAND` and `COPY` gates.
- If we can build a quantum analogue of `NAND` and `COPY` gates, then we will be done.
- The following three-qubit gate, called the Toffoli gate, can be used to implement both `NAND` and `COPY`.



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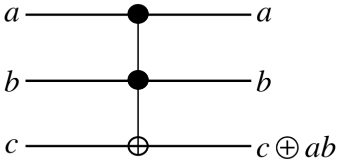

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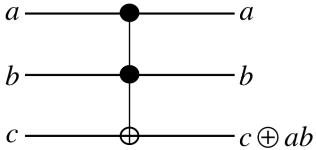

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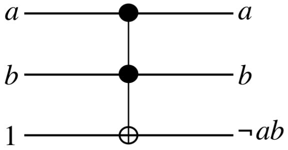

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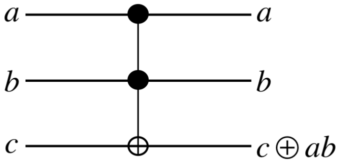

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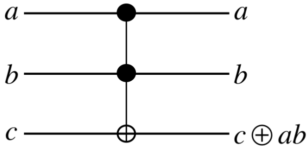

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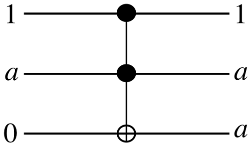
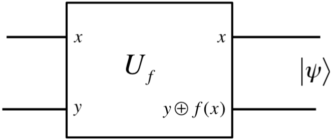


- Can we simulate classical logic circuit using a quantum circuit?
  Yes
- Can quantum circuits do more than just simulating classical ones?
  - We will introduce the idea of quantum parallelism. The main idea is simultaneous evaluation of a function over various inputs.
  - We will look at Deutsch's Algorithm which is a prototypical example used to demonstrate the idea of quantum parallelism.

- Consider any boolean function over one-bit inputs $f : \{0,1\} \to \{0,1\}$.
- Claim: It is possible to construct the following quantum gate U_f (using basic gates):



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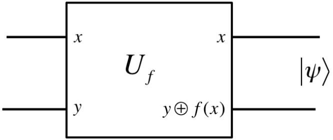
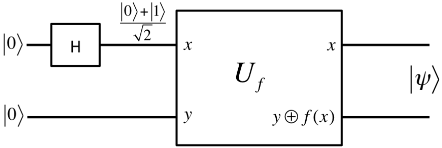


- By feeding inputs $|00\rangle$ and $|10\rangle$, we can compute $f(0)$ and $f(1)$.
- What happens when we feed the input $|+\rangle|0\rangle$ in this circuit? What is the output state $|\psi\rangle$?



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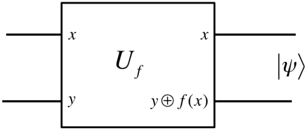

- By feeding inputs $|00\rangle$ and $|10\rangle$, we can compute $f(0)$ and $f(1)$.
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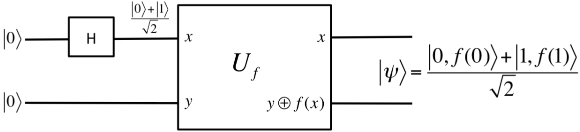

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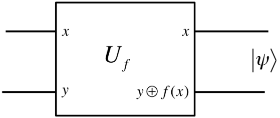

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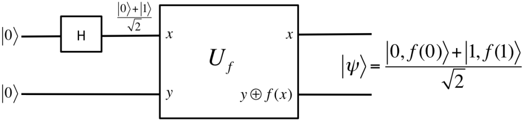

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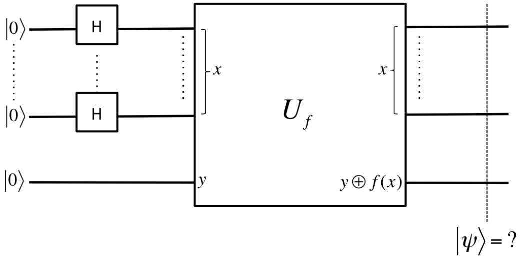

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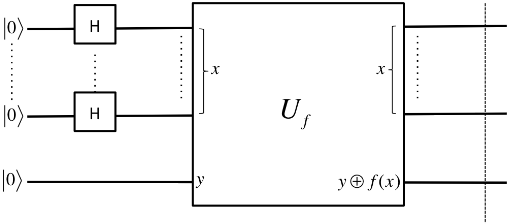

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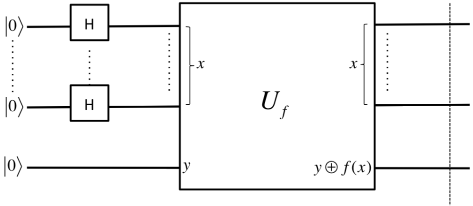

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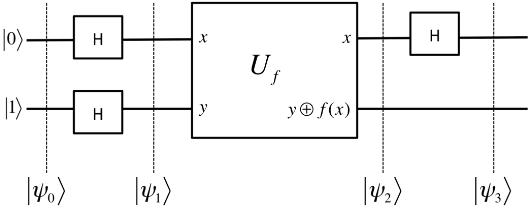
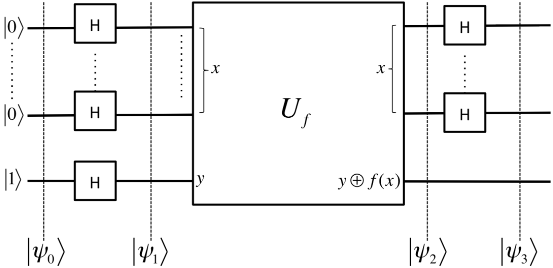
- Exploiting the parallelism in more realistic way is the key challenge while designing quantum algorithms.
- Consider the case of a boolean function on single-bit inputs $f : \{0, 1\} \rightarrow \{0, 1\}$. Suppose we would want to know if $f(0) = f(1)$. Here is a quantum circuit that solves this.



- The previous problem was a specific case of the more general Deutsch's problem that further demonstrates the power of quantum algorithms.
- Deutsch's problem: Bob has a function $f : \{0,1\}^n \rightarrow \{0,1\}$ that is either a constant function or a balanced function (i.e., f is 0 on $2^n/2$ inputs). Alice wants to determine what kind of function Bob has but can make a query to the function only once.
- The following circuit does this:



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