COL863: Quantum Computation and Information

Ragesh Jaiswal, CSE, IIT Delhi

- Instructor
 - Ragesh Jaiswal
 - Email: rjaiswal@cse.iitd.ac.in
 - Office: SIT Building, Room no. 403

- Grading Scheme
 - Quizzes (announced): 25%
 - @ Minor 1 and 2: 20% each.
 - **6** *Major*: 35%
- Policy on cheating:
 - Anyone found using unfair means in the course will receive an F grade.

• <u>Textbook</u>: Quantum Computation and Quantum Information by *Michael A. Nielsen and Isaac L. Chuang*.

- Gradescope: A paperless grading system. Use the course code
 M8E8YG to register in the course on Gradescope. Use only your IIT Delhi email address to register on Gradescope.
- Course webpage: http://www.cse.iitd.ac.in/ ~rjaiswal/Teaching/2019/COL863.
 - The site will contain course information, references, problems. Please check this page regularly.



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- What is quantum mechanics?
 - Mathematical framework for constructing physical theories.

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 - Mathematical framework of for designing quantum algorithms and information processing.
 - Examples where quantum information processing systems have gone beyond classical ones.
 - Factoring, discrete logarithm, superdense coding, quantum search...

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 - Mathematical framework of for designing quantum algorithms and information processing.
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 - Factoring, discrete logarithm, superdense coding, quantum search...
- This is not a Quantum Mechanics course!
 - We will start and build from a purely mathematical abstraction without going into the details of how the mathematical framework was arrived at or why such a framework might be reasonable.

Computation: A historical perspective

- Church-Turing Thesis
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- What about quantum mechanical processes? Can they be simulated efficiently by Turing Machines?
 - There are examples where this is not known.
 - So, quantum computation may be the (only) candidate counterexample to the extended Church-Turing Thesis.



Shannon's noiseless channel coding theorem

- Quantifies the physical resources required to store the output of an information source.
- Shannon's noisy channel coding theorem
 - Quantifies the amount of information that is possible to reliably transmit through a noisy channel.
- What is the quantum analogue of the physical resource for encoding information? Qubit
- Some surprising results:
 - Superdense coding: Two classical bits can be communicated using a single quantum bit.
 - <u>Distributed quantum computation</u>: Quantum computers can require exponentially less communication to solve certain problems compared to classical computers.



- Private key cryptography
 - It is assumed that Alice and Bob share a secret key and protocols are designed using this assumption.

Private key cryptography

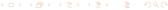
- It is assumed that Alice and Bob share a secret key and protocols are designed using this assumption.
- Main issue: How do Alice and Bob share a secret key?
- Quantum key distribution (Weisner,1960; Bennett and Brassard, 1984): Alice and Bob can communicate over a quantum channel to share a secret key even in presence of an adversary.

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Public key cryptography:

- Alice and Bob both have a pair of public-private keys.
- Messages are encoded using public key (that everyone knows) and can be decoded using the corresponding private key (that only the owner knows).
- Such protocols exist. However, some popular ones become insecure if efficient algorithms for factoring and discrete logarithm problems are built.
- Quantum algorithms: There are efficient quantum algorithms for both discrete logarithm and factoring.



- What is a qubit?
 - Qubit is to quantum computation as bit is to classical computation.
- Classical bit can be realised in real physical systems. Does it hold for qubits?
 - Yes but with a lot of ifs and buts. People would not have started talking about this concept if it were completely imaginary.
 - Since we do not have the expertise to go deeper into how qubits can be realised, we will treat it as a mathematical object.

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- Okay ... the classical bit has two states 0 and 1 (and that is pretty much the full description of the bit). Is qubit similar?



- What is a qubit? Quantum analogue of classical bit.
- Classical bit can be realised in real physical systems. Does it hold for qubits? We will work with yes.
- The classical bit has two states 0 and 1. Is qubit similar?
 - Yes and no. A qubit can be in states |0> and |1>. However, these are not the only two states of the qubit.
 - A qubit can be in a superposition or linear combination of states:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

where α and β are complex numbers.



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 - Yes and no. A qubit can be in states $|0\rangle$ and $|1\rangle$. However, these are not the only two states of the qubit.
 - A qubit can also be in a superposition or linear combination of states such as: $|\psi\rangle=\alpha\,|0\rangle+\beta\,|1\rangle$, where α and β are complex numbers.
- Then is it true that there are infinitely many possible states for a qubit?
 - Yes this is true.
- Can all these infinitely many states be recognised or measured? In other words, can one determine the state of a qubit (i.e., α , β)?
 - No. A measurement results in either 0 or 1 as output.
 - For a qubit in state α $|0\rangle + \beta$ $|1\rangle$, the probability of 0 is $|\alpha|^2$ and 1 is $|\beta|^2$ (Note that this means $|\alpha|^2 + |\beta|^2 = 1$)
 - Measurements changes the state of the qubit. If the measurement results in $x \in \{0,1\}$, then the post-measurement state is $|x\rangle$.



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 - Summary: The state of a qubit is a *unit* vector in a two-dimensional complex vector space with $|0\rangle$ and $|1\rangle$ as the orthonormal basis (interpreted as computational basis states).
- Doesn't this mean that a qubit can encode infinite amount of information?
 - This is tricky. Even though α and β may encode a lot of information, the information available to us is only through a measurement and we can only extract a single bit of information from a measurement.
 - However, note that nature keeps track of α, β .

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- Doesn't this mean that a qubit can encode infinite amount of information? No
- What about multiple qubit systems?
 - A two qubit system can be written as a superposition of computational basis states $|00\rangle\,, |01\rangle\,, |10\rangle\,, |11\rangle$:

$$|\psi\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{10} |10\rangle + \alpha_{11} |11\rangle$$



End