Name: _____

Entry number: _____

There are 3 questions for a total of 10 points.

1. Answer the following questions:

  (a) (2 points) Let $M = HTHT$. Show that the action of operator M on any qubit is the same as rotating the Bloch sphere about an axis along $\vec{n} = (\cos\frac{\pi}{8}, \sin\frac{\pi}{8}, \cos\frac{\pi}{8})$ with corresponding unit vector $\hat{n}$ through an angle θ defined by $\cos\frac{\theta}{2} \equiv \cos^2\frac{\pi}{8}$.

  (b) (2 points) Show that $HTH = R_x(\pi/4)$.

(c) (1 point) Draw the circuit for swapping two qubits using only CNOT gates.

2. (2 points) Let U be a single qubit operator with eigenvalues ± 1 so that U can also be treated as an observable. Show that measuring any qubit using the observable U has the same measurement statistics as measuring the first qubit in the computational basis in the circuit below.
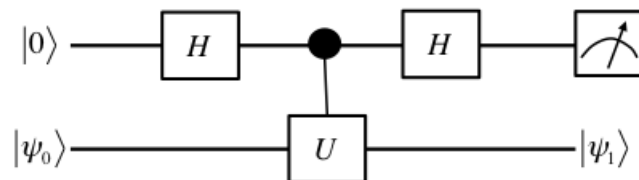
3. (3 points) Provide a decomposition of the transform

$$\frac{1}{\sqrt{2}} \begin{bmatrix} i & 0 & 0 & i \\ 0 & -i & -i & 0 \\ -i & 0 & 0 & i \\ 0 & i & -i & 0 \end{bmatrix}$$

into a product of two-level unitaries.

Space for rough work