Name: ____________________

Entry number: ____________________

There are 3 questions for a total of 10 points.

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1. A *P-matrix* is a matrix $\Pi \in \mathbb{C}^{n \times n}$ such that $\Pi^2 = \Pi$. Answer the following question:
   (a) (½ point) State true or false: For any Hermitian matrix $\Pi \in \mathbb{C}^{n \times n}$, Π is a *P-matrix* if and only if $\Pi = \sum_{i=1}^{k} |v_i\rangle \langle v_i|$ for some orthonormal vectors $|v_1\rangle, |v_2\rangle, ..., |v_k\rangle \in \mathbb{C}^n$.

   (a) ____________

   (b) (2 points) Give reasons for your answer to part (a).

2. An operator M on a finite dimensional vector space V with inner products is said to be *norm preserving* if for every $|w\rangle \in V$, $|| \, |w\rangle \, || = ||M \, |w\rangle \, ||$. Answer the following question.

   (a) ($2\frac{1}{2}$ points) Let $|v_1\rangle, ..., |v_n\rangle$ be an ~~orthogonal~~ orthonormal basis for V. What conditions on numbers $a_1, ..., a_n \in \mathbb{C}$ are necessary and sufficient for $M \equiv \sum_{i=1}^{n} a_i \, |v_i\rangle \langle v_i|$ to be norm preserving? Give reasons.

3. (5 points) Show that any positive operator on a finite dimensional inner product space is necessarily a Hermitian.