Name: _____

Entry number:

There are 2 questions for a total of 20 points.

1. (10 points) Given a 4-to-1 function $f : \{0,1\}^n \to \{0,1\}^n$ such that $f(x) = f(x \oplus a) = f(x \oplus b) = f(x \oplus a \oplus b)$ for some $a, b \neq 0^n$ and $a \neq b$. Give an efficient Quantum algorithm for finding a and b. Discuss running time. You may use the following Quantum gate:



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- 2. (10 points) Suppose you are given the following quantum gates:
 - 1. QFT_n : *n*-qubit QFT
 - 2. $InvQFT_n$: *n*-qubit inverse QFT
 - 3. $R_k \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{2\pi i}{2^k}} \end{bmatrix}$ for k = 1, ..., n.

Given two *n*-qubit registers that are initialized to $|x\rangle$ and $|y\rangle$ respectively, describe how you would compute $|(x + y) \pmod{2^n}\rangle$ using just the gates given above. You may also use the controlled operations.

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