Name: ______________________

Entry number: ______________________

There are 6 questions for a total of 15 points.

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1. (1 point) Fill the truth-table below:

P	Q	R	$P \leftrightarrow Q$	$\neg Q \vee R$	$(P \leftrightarrow Q) \rightarrow (\neg Q \vee R)$
T	T	T			
T	T	F			
T	F	T			
F	T	T			
T	F	F			
F	T	F			
F	F	T			
F	F	F			

2. Let the domain of discourse consist of all real numbers and let $P(x, y)$ mean $yx^2 = y^3$.
   (a) (½ point) State whether the following quantified statement is true or false:

$$(\exists x \forall y P(x, y)) \vee (\exists y \forall x P(x, y))$$

(a) ______________

   (b) (1 point) Give reasons for your answer to part (a).

3. (2 ½ points) Let $Q(p, s, z)$ be the statement "the price of product p in store s is z rupees", where the domain of variable p consists of all products, s consists of all stores, and z consists of all valid product prices. You may assume for this question that all stores carry all products. Use quantifiers to express the following statement: "Store A is the cheapest store for all products".

3. ______________________

4. Let A, B, C be non-empty sets, and let $g : A \rightarrow B$ and $h : A \rightarrow C$ and let $f : A \rightarrow B \times C$ defined as:

$$f(x) = (g(x), h(x)).$$

Answer the following:

(a) (½ point) <u>State true or false</u>: If f is onto, then both g and h are onto.

(a) ______________

(b) (½ point) <u>State true or false</u>: If g and h are onto, then f is onto.

(b) ______________

(c) (½ point) <u>State true or false</u>: If at least one of g, h is one-to-one, then f is one-to-one.

(c) ______________

(d) (½ point) <u>State true or false</u>: If g and h are not one-to-one, then f is not one-to-one.

(d) ______________

(e) (2 points) Give reasons for your answer to part (b).

(f) (2 points) Give reasons for your answer to part (d).

5. Answer the following:

   (a) (½ point) State true or false: Let $f(n) = 5n2^n + 3^n$ and $g(n) = n3^n$. Then $f(n) = O(g(n))$.

   (a) ________________

   (b) (½ point) State true or false: Let $f(n) = 5n2^n + 3^n$ and $g(n) = n3^n$. Then $g(n) = O(f(n))$.

   (b) ________________

6. (3 points) Prove or disprove: The function $f : \mathbb{N} \rightarrow \mathbb{N}$ defined as:

$$f(n) = \begin{cases} n-1 & \text{if } n \text{ is odd} \\ n+1 & \text{if } n \text{ is even} \end{cases}$$

   is one-to-one and onto. (*Note that* 0 *is an even number*)

Space for rough work