CSL202: Discrete Mathematical Structures

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Induction and Recursion

- Mathematical induction is used to prove statements that assert that $P(n)$ is true for all positive integers n, where $P(n)$ is a propositional function.
- A proof by mathematical induction has two parts:
  - **Basis step**: Here we show that $P(1)$ is true.
  - **Inductive step**: Here we show that if $P(k)$ is true, then $P(k+1)$ is true.

Definition (Principle of mathematical induction)

To prove that $P(n)$ is true for all positive integers n, where $P(n)$ is a propositional function, we complete two steps:

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- In the inductive step, we assume that for arbitrary positive integer $P(k)$ is true and then show that $P(k+1)$ must also be true. The assumption that $P(k)$ is true is called the *inductive hypothesis*.
- Induction may be expressed as the following rule of inference:

$$(P(1) \wedge \forall k(P(k) \rightarrow P(k+1))) \rightarrow \forall n\ P(n)$$

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- Why is mathematical induction valid?
  - Well-ordering principle: Every nonempty subset of the set of positive integers has a least element.
  - Argue the validity of mathematical induction using the axiom above.

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- Prove the following generalization of De Morgan's laws:

$$\overline{\cap_{j=1}^{n} A_j} = \cup_{j=1}^{n} \overline{A_j}$$

whenever $A_1, A_2, ..., A_n$ are subsets of a universal set U and $n \geq 2$.

End