CSL202: Discrete Mathematical Structures

Ragesh Jaiswal, CSE, IIT Delhi

Administrative Information

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- Instructor
  - Ragesh Jaiswal
  - Office Hours: 11-12, Sun.
  - *Email*: `rjaiswal@cse.iitd.ac.in`
- Teaching Assistants
  - Gagan Madan (*email*: `me1130015@mech.iitd.ac.in`)
  - Dishant Goyal (*email*: `csz178060@cse.iitd.ac.in`)

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- Grading Scheme
  1. *Quizzes* (weekly) : 40%
  2. *Minor 1 and 2*: 15% each.
  3. *Major*: 30%
- Important points:
  - There will be homework given every week that you are expected to finish before the beginning of the next week class.
  - Homework will not be graded and so you are not supposed to submit the homework.
  - There will be a quiz based on the material of the homework and tutorial of past week.
  - You will be given tutorial sheet in addition to the homework that you should attempt before attending the tutorial. The tutor will only lead the discussions.
- Policy on cheating:
  - Anyone found using unfair means in the course will receive an **F** grade.

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- Textbook: Discrete Mathematics and its Applications by *Kenneth H. Rosen*.

- Gradescope: A paperless grading system. Use the course code **9BPDNE** to register. Please use your formal email address from IIT Jammu.

- Course webpage: `http://www.cse.iitd.ac.in/~rjaiswal/Teaching/2018/CSL202`.
  - The site will contain course information, references, homework problems, tutorial problems. Please check this page regularly.

Introduction

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- What are *Discrete* Mathematical *Structures*?
  - Discrete: Separate or distinct.
  - Structures: Objects built from simpler objects as per some rules/patterns.
- Discrete Mathematics: Study of discrete mathematical objects and structures.

Introduction

- Why study Discrete Mathematics?
  - Information processing and computation may be interpreted as manipulation of discrete structures.
  - Enable you to think logically and argue about correctness of computer programs and analyze them.
- What you should expect to learn from this course:
  - **Rigorous thinking!**
  - Mathematical foundations of Computer Science.

Introduction

- Topics:
  - Logic: propositional logic, predicate logic, proofs. mathematical induction etc.
  - Fundamental Structures: sets functions, relations, recursive functions etc.
  - Counting: Pigeonhole principle, permutation and combination, recurrence relations, generating functions, inclusion-exclusion etc.
  - Graphs: representing graphs, connectivity, shortest paths etc.

Logic: Propositional Logic

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    - Argue correctness of a computer program.
    - Automatic verification.
    - Check security of a cryptographic protocol.
    - ...

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- Propositional logic: Basic form of logic.

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- Are these statements propositions?
  - New Delhi is the capital of India.
  - What time is it?
  - Please read the first two sections of the book after this lecture.
  - $2 + 2 = 5$.
  - $x + 1 = 2$.

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- Propositional variable: Variables that represent propositions.
- Truth value: The truth value of a proposition is true (denoted by **T**) if it is a true proposition and false (denoted by **F**) if it is a false proposition.
- The area of logic that deals with propositions is called *propositional logic* or *propositional calculus*.
- Compound proposition: Proposition formed from existing proposition using *logical operators*.

- Negation ($\neg$): Let p be a proposition. The negation of p (denoted by $\neg p$), is the statement "it is not the case that p." The proposition $\neg p$ is read as "not p". The truth value of the $\neg p$ is the opposite of the truth value of p.

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    - p: A Tiger has been seen in this area.
      $\neg p$: ?

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  - Examples:
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      $\neg p$: It is not the case that a tiger has been seen in this area.

p	**¬p**
T	F
F	T

Table: Truth table for $\neg p$.

- Negation ($\neg$)
- Conjunction ($\wedge$): Let p and q be propositions. The conjunction of p and q (denoted by $p \wedge q$) is the proposition "p and q". The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

p	**q**	**p ∧ q**
T	T	T
T	F	F
F	T	F
F	F	F

Table: Truth table for $p \wedge q$.

- Negation ($\neg$)
- Conjunction ($\wedge$)
- Disjunction ($\vee$): Let p and q be propositions. The disjunction of p and q (denoted by $p \vee q$) is the proposition "p or q". The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

p	**q**	**p ∨ q**
T	T	T
T	F	T
F	T	T
F	F	F

Table: Truth table for $p \vee q$.

- Negation ($\neg$)
- Conjunction ($\wedge$)
- Disjunction ($\vee$).
- Exclusive or ($\oplus$): Let p and q be propositions. The exclusive or of p and q (denoted by $p \oplus q$) is the proposition that is true when exactly one of p and q is true and is false otherwise.

p	**q**	**p ⊕ q**
T	T	F
T	F	T
F	T	T
F	F	F

Table: Truth table for $p \oplus q$.

- Negation ($\neg$)
- Conjunction ($\wedge$)
- Disjunction ($\vee$).
- Exclusive or ($\oplus$)
- Conditional statement ($\rightarrow$): Let p and q be propositions. The conditional statement $p \rightarrow q$ is the proposition "if p, then q." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

p	**q**	$\mathbf{p \rightarrow q}$
T	T	T
T	F	F
F	T	T
F	F	T

Table: Truth table for $p \rightarrow q$.

Definition (Conditional statement)

Let p and q be propositions. The conditional statement $p \rightarrow q$ is the proposition that is is false when p is true and q is false, and true otherwise.

- q is true on the condition that p is true.
- This is also called an *implication*.
- There are various ways to express $p \rightarrow q$:
  - "p is sufficient for q" or "a sufficient condition for q is p"
  - "q if p"
  - "q when p"
  - "q is necessary for p" or "a necessary condition for p is q"
  - "q unless $\neg p$"
  - "p implies q"
  - "p only if q"
  - "q whenever p"
  - "q follows from p"

Definition (Conditional statement)

Let p and q be propositions. The conditional statement $p \rightarrow q$ is the proposition that is is false when p is true and q is false, and true otherwise.

Definition (Converse)

The converse of a proposition $p \rightarrow q$ is the proposition $q \rightarrow p$.

Definition (Contrapositive)

The contrapositive of a proposition $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$.

Definition (Inverse)

The inverse of a proposition $p \rightarrow q$ is the proposition $\neg p \rightarrow \neg q$.

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- Show that the contrapositive of $p \rightarrow q$ always has the same truth value as $p \rightarrow q$.
- Show that, neither converse nor inverse of $p \rightarrow q$ has the same truth value as $p \rightarrow q$ for all truth values of p and q.

- Negation ($\neg$)
- Conjunction ($\wedge$)
- Disjunction ($\vee$).
- Exclusive or ($\oplus$)
- Conditional statement ($\rightarrow$)
- Bi-conditional statement ($\leftrightarrow$): Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q". The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called bi-implications.

p	**q**	$\mathbf{p \leftrightarrow q}$
T	T	T
T	F	F
F	T	F
F	F	T

Table: Truth table for $p \leftrightarrow q$.

- Negation ($\neg$)
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- Bi-conditional statement ($\leftrightarrow$): Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q". The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called bi-implications.
  - $p \leftrightarrow q$ is also expressed as:
    - "p is necessary and sufficient for q"
    - "p iff q"
    - "if p, then q and conversely"
  - Show that $p \leftrightarrow q$ always has the same truth value as $(p \rightarrow q) \wedge (q \rightarrow p)$.

- Logical operators
  - Negation ($\neg$)
  - Conjunction ($\wedge$)
  - Disjunction ($\vee$).
  - Exclusive or ($\oplus$)
  - Conditional statement ($\rightarrow$)
  - Bi-conditional statement ($\leftrightarrow$)
- A compound proposition is formed by applying these operators on simpler propositions. E.g. $(p \vee q \wedge r)$.
- Operator Precedence (in decreasing order): $\neg, \wedge, \oplus, \vee, \rightarrow, \leftrightarrow$.
- Construct the truth table for $p \vee \neg q \rightarrow p \wedge q$.

- Simplify complex sentences and enable to logically analyze them.
  - "You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."
    - p: "You can ride the roller coaster."
    - q: "You are under 4 feet tall."
    - r: " You are older than 16 years old."
  - Express the sentence in terms of propositions p, q, and r.

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  - Express the sentence in terms of propositions p, q, and r.
    - $(q \wedge \neg r) \rightarrow \neg p$.

- Simplify complex sentences and enable to logically analyze them.
- Translate system specification expressed in natural language into unambiguous logical expressions.
  - Example:
    - "The diagnostic message is stored in the buffer or is retransmitted."
    - "The diagnostic message is not stored in the buffer."
    - "If the diagnostic message is stored in the buffer, then it is retransmitted."
    - "The diagnostic message is not retransmitted."
  - Consistency: Whether all the specifications can be satisfied simultaneously.
  - Are these specifications consistent?

- Simplify complex sentences and enable to logically analyze them.
- Translate system specification expressed in natural language into unambiguous logical expressions.
- Resolve complex puzzling scenarios.
  - Example:
    - An island has two kinds of inhabitants, knights and knaves. Knights always tell the truth and Knaves always lie. You meet two people on this island A and B. What are A and B if A says "B is a knight" and B says "The two of us are opposite types"?

End