Name: ______________________________

ID number: ______________________________

There are 2 questions for a total of 10 points.

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1. Answer the following questions related to set equality. For this question we will use the set identity $A - B = A \cap \overline{B}$ that holds for all sets A, B. We will call this "set-difference law".

   (a) (2 ½ points) For all sets A, B show that $(A - B) \cup (B - A) = (A \cup B) \cap (\overline{A \cap B})$ using set identities given in the lecture slides along with the "set-difference law".

   Solution: The equality follows from the following chain of equalities:

$$
\begin{aligned}
(A - B) \cup (B - A) &= (A \cap \overline{B}) \cup (B \cap \overline{A}) && \text{(set-difference law)}\\
&= ((A \cap \overline{B}) \cup B) \cap ((A \cap \overline{B}) \cup \bar{A}) && \text{(Distributive law)}\\
&= ((A \cup B) \cap (\overline{B} \cup B)) \cap ((A \cup \overline{A}) \cap (\overline{B} \cup \overline{A})) && \text{(Distributive law)}\\
&= ((A \cup B) \cap U) \cap (U \cap (\overline{B} \cup \overline{A})) && \text{(Domination law)}\\
&= (A \cup B) \cap (\overline{B} \cup \overline{A}) && \text{(Identity law)}\\
&= (A \cup B) \cap (\overline{B \cup A}) && \text{(De Morgan's law)}\\
&= (A \cup B) \cap (\overline{A \cup B}) && \text{(Commutative law)}
\end{aligned}
$$

   (b) (2 ½ points) For all sets A, B, C show that $\overline{(A - B) - (B - C)} = \overline{A} \cup B$ using set identities given in the lecture slides along with the "set difference law".

   Solution: The equality follows from the following chain of equalities:

$$
\begin{aligned}
\overline{(A - B) - (B - C)} &= \overline{(A \cap \overline{B}) \cap \overline{(B \cap \overline{C})}} && \text{(set-difference law)}\\
&= \overline{(A \cap \overline{B})} \cup \overline{\overline{(B \cap \overline{C})}} && \text{(De Morgan law)}\\
&= \overline{(A \cap \overline{B})} \cup (B \cap \overline{C}) && \text{(Complementation law)}\\
&= (\overline{A} \cup \overline{\overline{B}}) \cup (B \cap \overline{C}) && \text{(De Morgan law)}\\
&= (\overline{A} \cup B) \cup (B \cap \overline{C}) && \text{(Complementation law)}\\
&= (\overline{A} \cup B \cup B) \cap (\overline{A} \cup B \cup \overline{C}) && \text{(Distributive law)}\\
&= (\overline{A} \cup B) \cap (\overline{A} \cup B \cup \overline{C}) && \text{(Idempotent law)}\\
&= (\overline{A} \cup B) && \text{(Absorption law)}
\end{aligned}
$$

2. (5 points) Argue that the set $\mathbb{Z}^+ \times \mathbb{Z}^+$ is countable.

Solution: An infinite set is countable if and only if its elements can be listed in a sequence. We list the elements of the set $\mathbb{Z}^+ \times \mathbb{Z}^+$ in a sequence by first listing those tuples $(p, q) \in \mathbb{Z}^+ \times \mathbb{Z}^+$ such that $p + q = 2$ (there is only one such tuple), then list those tuples $(p, q) \in \mathbb{Z}^+ \times \mathbb{Z}^+$ such that $p + q = 3$ (there are two such tuples), and so on. So, the first few elements of the sequence is $(1, 1), (1, 2), (2, 1), (3, 1), (2, 2), (1, 3), ...$ Since all the elements of the the set $\mathbb{Z}^+ \times \mathbb{Z}^+$ are in the list, we can conclude that the set is countable.