COL202: Discrete Mathematical Structures

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Discrete Probability

- **The Birthday Problem**: What is the minimum number of people who need to be in a room so that the probability that at least two of them have the same birthday is greater than 1/2?

- Probabilistic algorithms: Algorithms that make random choices at one or more steps.
- Monte Carlo Algorithms: Probabilistic algorithms for decision problems that always produces an answer. The answer may be incorrect with some small probability.
  - Example: A sends 1 million apples to B. A has cleverly packed 1000 bad apples among these 1 million apples. How does B detect that A has sent 1 million good apples or not.

Theorem (The Probabilistic Method)

If the probability that an element chosen at random from a S does not have a particular property is less than 1, there exists an element in S with this property.

- An existence proof based on the probabilistic method is nonconstructive because it does not find a particular element with the desired property.

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- Example: *Ramsey number*
  - Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group.

Theorem (The Probabilistic Method)

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- Example: *Ramsey number*
  - The Ramsey number $R(m, n)$, where m and n are positive integers greater than or equal to 2, denotes the minimum number of people at a party such that there are either m mutual friends or n mutual enemies, assuming that every pair of people at the party are friends or enemies.

Theorem (The Probabilistic Method)

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Definition (Ramsey number)

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Theorem

If k is an integer with $k \geq 2$, then $R(k, k) \geq 2^{k/2}$.

End