Name: ____________________

Entry number: ____________________

There are 3 questions for a total of 10 points.

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1. Let X be a random variable denoting the number of people attending a conference. We know that $\mathbf{E}[X] = 100$. Everyone shakes hand with everyone else at the conference and let Y denote the total number of handshakes. We know that $\mathbf{E}[Y] = 5000$. Answer the following questions.

   (a) (1 ½ points) What is the variance of X? Show calculations in the space below.

   (a) ____________

   (b) (1 ½ points) Use Chebychev's inequality to give an upper bound on the probability that less than 80 people attend the conference?

   (b) ____________

2. (4 points) Consider executing the following algorithm on an array $A[1...n]$ containing n distinct numbers $\{N_1, N_2, ..., N_n\}$ permuted randomly.

   FindMax(A, n)
   - $Max \leftarrow A[1]$
   - For $i = 2$ to n
     - If $(A[i] > Max) Max \leftarrow A[i]$
   - return(Max)

   Let X be the random variable denoting the number of times the variable Max is updated within the for loop of the FindMax algorithm. What is $\mathbf{E}[X]$ as a function of n? Express your answer concisely using big-Theta notation. Show your calculations in the space below.

   2. ____________

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3. Consider the following randomized quick-sort algorithm for sorting an array A containing distinct numbers:

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Randomized-Quick-Sort(A)
  - If (|A| = 1)return(A)
  - Randomly pick an index i in the array A
  - Use A[i] as a pivot to partition A into A_L and A_R
    // That is, A_L denotes the array of elements that are smaller than A[i], and A_R denotes the
    //array of elements that are larger than A[i]. The relative ordering of elements in A_L (and A_R)
    //is the same as that in A
  - B_L ← Randomized-Quick-Sort(A_L)
  - B_R ← Randomized-Quick-Sort(A_R)
  - return(B_L|A[i]|B_R)
```

We will try to compute the expected number of pairwise comparisons performed by the algorithm during its execution. Note that comparisons are done during the pivoting operation.

(a) (1 ½ points) For $i < j$, let X_{ij} denote the indicator random variable that is 1 if a comparison between $A[i]$ and $A[j]$ is done during the execution of the algorithm and 0 otherwise. What is the value of $\mathbf{E}[X_{ij}]$ in terms of i and j? You do not need to give reasons.

(a) ______________

(b) (1 ½ points) Let $X = \sum_{i<j} X_{ij}$. Note that X denotes the total number of pairwise comparisons. Use part (a) to give $\mathbf{E}[X]$ as a function of n. Express your answer concisely using big-Theta notation. You do not need to show calculations.

(b) ______________