Name: $\qquad$

Entry number: $\qquad$

There are 2 questions for a total of 10 points.

1. Solve the following problems. Show the working in the space provided.
(a) (1 point) What is the last digit of $7^{100}$ ?
(a)
(b) (1 point) What is the value of $\left(2^{100} \cdot 3^{60}\right)(\bmod 5)$ ?
$\qquad$
(c) (1 point) What is the remainder when $\sum_{i=1}^{100}(i)$ ! is divided by 9 ?
(c)
(d) (2 points) Prove or disprove: $\left(2^{n}+6 \cdot 9^{n}\right)$ is divisible by 7 for every $n \geq 0$.
2. (5 points) Consider the following problem:

HALTING-INPUT: Given the description $\langle A\rangle$ of algorithm $A$, determine if there is a halting input for $A$ (that is, there exists an input on which $A$ halts).

An algorithm $P$ is said to solve the above problem if $P(\langle A\rangle)$ halts and outputs 1 when $A$ has a halting input, and it halts and outputs 0 otherwise.
Prove: There does not exist an algorithm $P$ that solves the problem HALTING-INPUT.

